Cryptography V: Digital Signatures

Computer Security Lecture 6

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Outline

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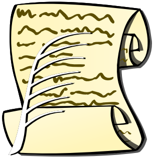
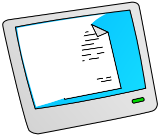
- Digital signatures are the asymmetric analogue of MACs, with a crucial difference. MACs can't disinguish which of A or B provided integrity to a message (so no non-repudiation or independent verifiability).
- NB: **electronic signature** is a more general notion.

Handwritten versus Digital Signatures





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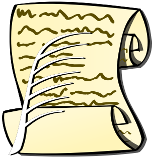

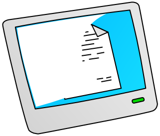

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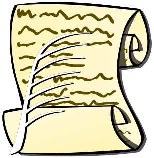

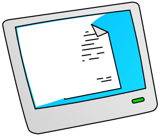

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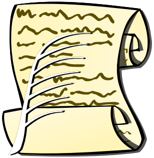

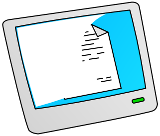

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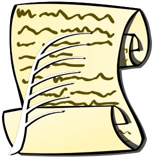

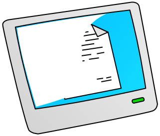

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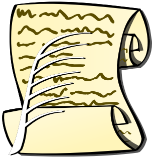

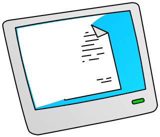

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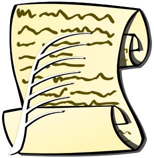

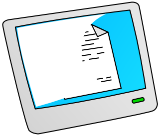

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Signature mechanism

A signature mechanism for principal A is given by:

- A message space $\mathcal{M}$ of messages for signing
- A set $\mathcal{S}$ of *signatures* (e.g. strings $\{0,1\}^n$)
- A secret **signing function** $S_A : \mathcal{M} \to \mathcal{S}$
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2. For any principal other than A, it is computationally infeasible to find for any $m \in \mathcal{M}$, an $s \in \mathcal{S}$ such that $V_A(m, s) = \text{true}$.

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Remark: nobody has proved a signature mechanism satisfying 2 exists, although there are good candidates.

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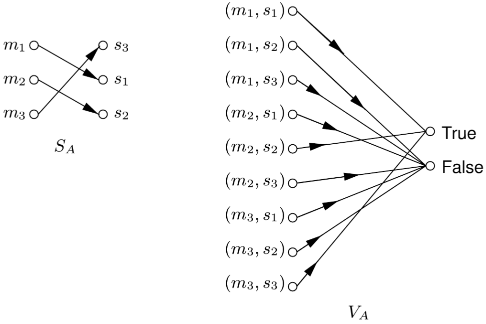

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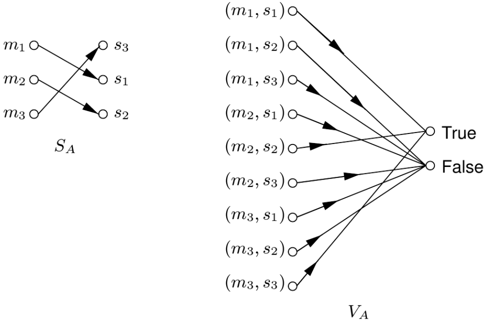

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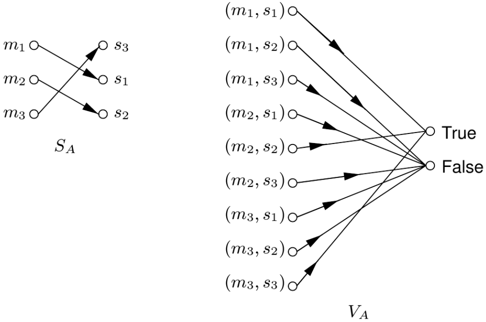

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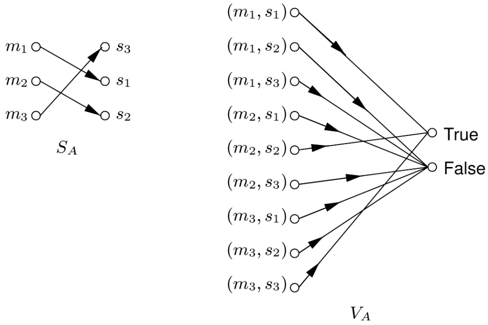

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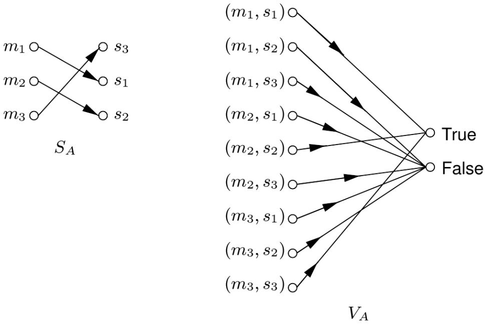

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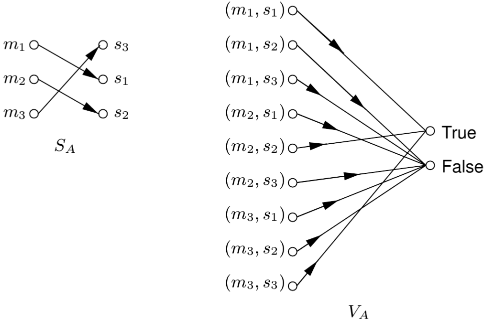

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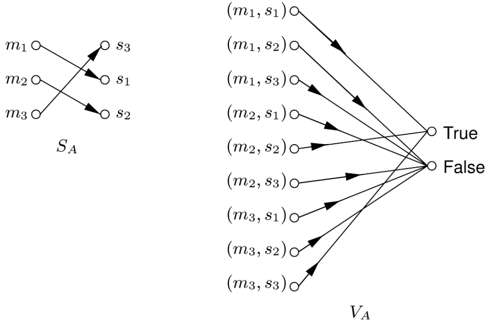

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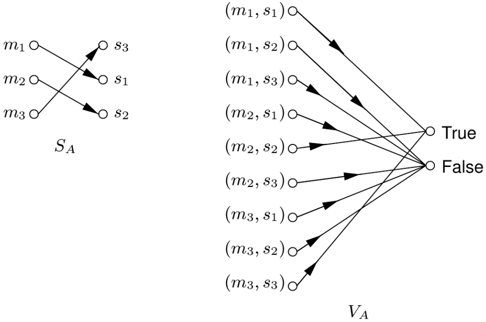

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  3. **Accepts** the signature if $u =$ true,
     Rejects it if $u =$ false.

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- If A and B disagree about a signature, a judge Judy can verify the contracts also using S:

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Digital signatures from PK encryption

- Suppose we have a public-key encryption scheme with $\mathcal{M} = \mathcal{C}$, and (d, e) a key-pair. Then because E_e and D_d are both permutations on $\mathcal{M}$, we have that:

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  - the signing function $S_A = D_d$
  - the verification function V_A is defined by

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  3. **Chosen-message attack**: adversary can obtain signatures for messages of his choosing. Messages may be determined in advance or in **adaptive** way, using signer as oracle.

Existential forgery

- The previous scheme is too simple because signatures are forgeable: a principal B can generate a random $s \in \mathcal{S}$ as a signature, apply the public encryption function to get a message $m = E_e(s)$, and transmit (m, s).

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- But this ability violates property 2 given earlier.

Signatures with redundancy

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- Now A only transmits the signature s, since the message $m = E_e(s)$ can be recovered by the verification function.

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- Now A only transmits the signature s, since the message $m = E_e(s)$ can be recovered by the verification function.
- This property is **message recovery**, the scheme is called a **signature scheme with recovery**.

Signatures with redundancy

- A fix to reduce likelihood of existential forgery is to take $\mathcal{M}' \subset \mathcal{M}$ to be messages with a special *redundant* structure, which is publicly known e.g., messages padded to an even length, surrounded with a fixed bit pattern.
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- Existential forgery is now less likely.

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- This is called a **signature scheme with appendix**.

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- To verify the signature, upon receipt of (m, s), compute $s^e \bmod n$ and verify whether it equals $h(m)$

Distributed RSA Signatures

- Signatures can optionally be *distributed* so that each of t users contributes to the signature. A trusted party T computes t *shares* such that

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- *Secret sharing* can also be used so that $l < t$ users could be used to construct a signature.

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- Verification works because for a correct signature,

$$(g^d)^e e^s \equiv g^{de+rs} \equiv g^m \pmod{p}.$$

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- **Security** of both ElGamal and DSA schemes relies on the intractability of the DLP.
- Comparison with RSA signature scheme: key generation is faster; signature generation is about the same; DSA verification is slower. Verification is the most common operation in general.

Outline

Summary: Digital Signature Schemes

- **RSA**, **ElGamal**, **DSA** already described. There are several variants of ElGamal, including schemes with message recovery.
- Notice difference between *randomized* and *deterministic* schemes.
- Schemes for **one-time signatures** (e.g., Rabin, Merkle), require a fresh public key for each use.
  - Typically more efficient than RSA/ElGamal methods.
  - But tedious for multiple documents
- E-cash protocols use **blind signature** schemes that prevent the signer (e.g., a bank) linking a signed message (e.g., the cash) with the user.
- For real world security guarantees:
  - **obtaining correct public key** is vital;
  - non-repudiation supposes that **private key has not been stolen**;
  - we may require **secure time stamps**.

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Recommended Reading

Chapter 14 (14.2–14.4, 14.7) of Smart (3rd Ed).