Cryptography II: Hash Functions

Computer Security Lecture 3

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Outline

Varieties of hash function

Properties of hash functions

Building hash functions

Standard hash functions

Conclusion

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Hash function basics

- A **hash function** is a *computationally efficient* function $h : \{0,1\}^* \rightarrow \{0,1\}^k$ which *compresses* any arbitrary length binary string to a fixed size k-length binary *hash value* (or *hash* for short).

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- A good hash function distributes values uniformly: the probability that a randomly chosen string s gets mapped to a particular hash y is $\frac{1}{2^k}$
- A **cryptographic hash function** must satisfy some further properties, e.g.:
  1. it should be difficult to invert;
  2. it should be difficult to find a second input that hashes to the same value as another input;
  3. it should be difficult to find any two inputs that hash to the same value.

Hash function uses and non-uses

- **Integrity**: Alice sends $m, h(m)$ (or alternatively, $E_K(m||h(m))$) to Bob.
- Protects against *malicious* modification.
- **Confidentiality**: An *Authentication Server* stores a user's password p as $h(p)$.
- Other uses: confirming knowledge (e.g. password) without revealing, deriving keys, pseudo-random numbers. A piece of "cryptographic glue".
- On their own, hash functions don't protect against
  - Malicious repetition of data, e.g., repeating a £100 bank deposit. (**Ex.** how could you do that?)
  - Dishonest repudiation, e.g., denying sending a hashed email message with a correct hash.
- Nor do they support message recovery, i.e., recovering the original message after tampering

Properties of cryptographic hash functions

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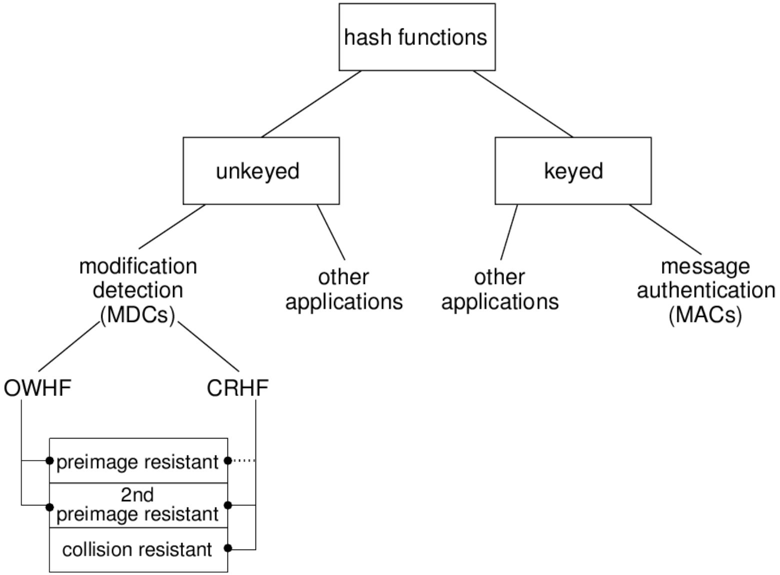
(Strong) Collision Resistance

h is **collision resistant** if it is computationally infeasible to find *any* two inputs x_1 and x_2 such that $h(x_1) = h(x_2)$.

Hash function Classification [HAC]



Modification Detection Codes

- The main application of hash functions is as **Modification Detection Codes** to provide **data integrity**.
- A hash $h(x)$ provides a short *message digest*, a "fingerprint" of some possibly large data x. If the data is altered, the digest should become invalid.
  - This allows the data (but not the hash!) to be stored in an unsecured place.
  - If x is altered to x', we hope $h(x) \neq h(x')$, so it can be detected.
- This is useful especially where *malicious* alteration is a concern, e.g., software distribution.
- Ordinary hash functions such as CRC-checkers produce *checksums* which are not 2nd preimage resistant: an attacker could produce a hacked version of a software product and ensure the checksum remained the same.

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- **Ex**: which is needed for password file security?

Message Authentication Codes

- **Message Authentication Codes** are *keyed* hash functions, indexed with a secret key.
  - As well as data integrity, they provide **data-origin authentication**, because it is assumed that apart from the recipient, only the sender knows the secret key necessary to compute the MAC.
- A MAC is a key-indexed family of hash functions, $\{h_k \mid k \in \mathcal{K}\}$. MACs must satisfy a *computation resistance* property.

Computation Resistance

Given a set of pairs $(x_i, h_k(x_i))$ it is computationally infeasible to find any other text-MAC pair $(x, h_k(x))$ for a new input $x \neq x_i$.

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- This and similar arguments (e.g., see Smart) can be made precise using the *Random Oracle Model*.
- **Collision resistance does not imply preimage resistance**
- Contrived counterexample:

$$h(x) = \begin{cases} 1 \mid\mid x & \text{if } x \text{ has length } n \\ 0 \mid\mid g(x) & \text{otherwise} \end{cases}$$

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- Mallory has two contracts, one for £1000, the other £100,000, to be signed with a 64-bit hash. He makes 2^{32} minor variations in each (e.g spaces/control chars), and finds a pair with the same hash. Later claims second document was signed, not first.

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- An n-bit unkeyed hash function has **ideal security** if producing a preimage or 2nd-preimage each requires 2^n operations, and producing a collision requires $2^{n/2}$ operations.

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  - Main problem with turning this into a realistic MD function is that it's too slow to calculate.

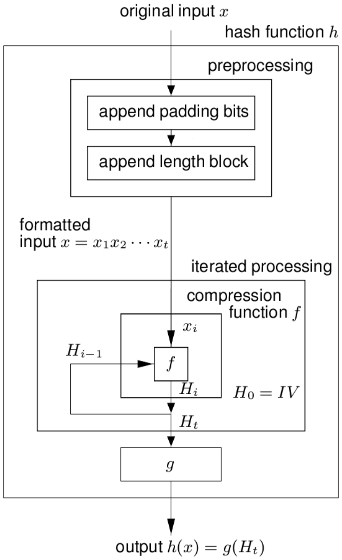
OWFs from block ciphers

- A block cipher is an encryption scheme which works on fixed length blocks of input text.
- We can construct a OWF from a block cipher such as DES, which is treated essentially as a random function:

  $$h(x) = E_k(x) \oplus x$$

  for fixed key k. This *can* be turned into a MD function, by iteration...

Iterated hash function construction [HAC]



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  - Set $IV = 0^n$, $g = id$, and compute $H_i = f(H_{i-1}, x_i)$.

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  - For example, the first round uses the operation:

$$\begin{aligned} a &= (F(b,c,d) + x_i + t_j) <<< s \\ F(b,c,d) &= (b \wedge c) \vee (\neg b \wedge d) \end{aligned}$$

    where $<<< s$ is left-circular shift of s bits, x_i is the ith sub-block of the message. Constants t_j are the integer part of $2^{32} * \mathrm{abs}(\sin(i+1))$ where $0 \leq i \leq 63$ is in radians (for the 4 * 16 steps).

Secure Hash Algorithm SHA-1 (160)

SHA-1 is a NIST standard [FIPS 180] also based on MD4.
An attack strategy with cost 2^{51} was found in 2011.

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  - Main loop has four rounds of 20 operations, chaining 5 variables a, b, c, d, e, f. Five IVs and four constants are used:

$$A = \texttt{0x67452301}$$
$$B = \texttt{0xEFCDAB89}$$
$$C = \texttt{0x98BADCFE}$$
$$D = \texttt{0x10325476}$$
$$E = \texttt{0xC3D2E1F0}$$

$$K_0 = \texttt{0x5A827999}$$
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  - The message block undergoes an *expansion transformation* from 16*32-bit words x_i to 80*32-bit words, w_i by:

$$\begin{aligned} w_i &= x_i, & \text{for } 0 \leq i \leq 15. \\ w_i &= (w_{i-3} \oplus w_{i-8} \oplus \\ & \qquad w_{i-14} \oplus w_{i-16}) <<< 1, & \text{for } 16 \leq i \leq 79. \end{aligned}$$

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&\textbf{for}(\ i = 0;\ i < 80;\ i{+}{+}\ ) \quad \{ \\
&\quad tmp = (a <<< 5) + F_j(b, c, d) + e + w_i + K_j; \\
&\quad e = d; \\
&\quad c = b <<< 30; \\
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  - Finally a, b, c, d, e are added to *tmp* (all addition is modulo 2^{32}).

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F_0(X, Y, Z) & = & (X \wedge Y) \vee (\neg X \wedge Z) \\
F_1(X, Y, Z) & = & X \oplus Y \oplus Z \\
F_2(X, Y, Z) & = & (X \wedge Y) \vee (X \wedge Z) \vee (Y \wedge Z) \\
F_3(X, Y, Z) & = & X \oplus Y \oplus Z
\end{array}
$$

  - Finally a, b, c, d, e are added to *tmp* (all addition is modulo 2^{32}).
- **Exercise:** implement SHA-1 in your favourite language following this. Test against `sha1sum`.

Outline

Varieties of hash function

Properties of hash functions

Building hash functions

Standard hash functions

Conclusion

Current Status

- Hash functions are versatile and powerful primitive.
- However, difficult to construct and less researched than encryption schemes.
  - ideal hash function is a “random mapping” where knowledge of previous results doesn’t give knowledge of another.
  - practical fast iterative hash constructions fail this!
  - MD4 (1998), MD5 (1993/2005), SHA-1 (2005) are now **all considered broken**.
- The US National Institute of Standards and Technology (NIST) has standardised a set of newer hash functions.
  - Formerly called SHA-2, they are denoted by their output size: SHA-256, SHA-384, SHA-512.
  - However, since they are based upon the same *SHA* construction, they are not long-term solutions
  - In 2012, NIST awarded a new standard SHA-3 to the *Keccak* algorithm (a *sponge function* which has arbitrary output length).

References

- A. J. Menezes, P. C. Van Oorschot, S. A. Vanstone, eds. *Handbook of Applied Cryptography*.
  CRC Press, 1997. Online: `http://www.cacr.math.uwaterloo.ca/hac`.

- Neils Ferguson and Bruce Schneier. *Practical Cryptography*.
  John Wiley & Sons, 2003.

- Douglas R Stinson. *Cryptography Theory and Practice*.
  CRC Press, second edition edition, 2002.

- Nigel Smart. *Cryptography: An Introduction*.
  McGraw-Hill, 2003. Third edition online: `http://www.cs.bris.ac.uk/~nigel/Crypto_Book/`

Recommended Reading

One of: Ch 9 of HAC (9.1–9.2); Ch. 10 of Smart 3rd Ed; 11.1–11.3 of Gollmann.