Cryptography II: Hash Functions

Computer Security Lecture 3

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Hash function basics

- A **hash function** is a *computationally efficient* function $h : \{0,1\}^* \rightarrow \{0,1\}^k$ which *compresses* any arbitrary length binary string to a fixed size k-length binary *hash value* (or *hash* for short).
- A good hash function distributes values uniformly: the probability that a randomly chosen string s gets mapped to a particular hash y is $\frac{1}{2^k}$
- A **cryptographic hash function** must satisfy some further properties, e.g.:
  1. it should be difficult to invert;
  2. it should be difficult to find a second input that hashes to the same value as another input;
  3. it should be difficult to find any two inputs that hash to the same value.

Hash function uses and non-uses

- **Integrity**: Alice sends $m, h(m)$ (or alternatively, $E_k(m||h(m))$) to Bob.
- Protects against *malicious* modification.
- **Confidentiality**: An *Authentication Server* stores a user's password p as $h(p)$.
- Other uses: confirming knowledge (e.g. password) without revealing, deriving keys, pseudo-random numbers. A piece of "cryptographic glue".
- On their own, hash functions don't protect against
  - Malicious repetition of data, e.g., repeating a £100 bank deposit. (**Ex.** how could you do that?)
  - Dishonest repudiation, e.g., denying sending a hashed email message with a correct hash.
- Nor do they support message recovery, i.e., recovering the original message after tampering

Properties of cryptographic hash functions

Preimage Resistance (One-way)

h is **preimage resistant** if given a hash value y, it is computationally infeasible to find an x such that $h(x) = y$.

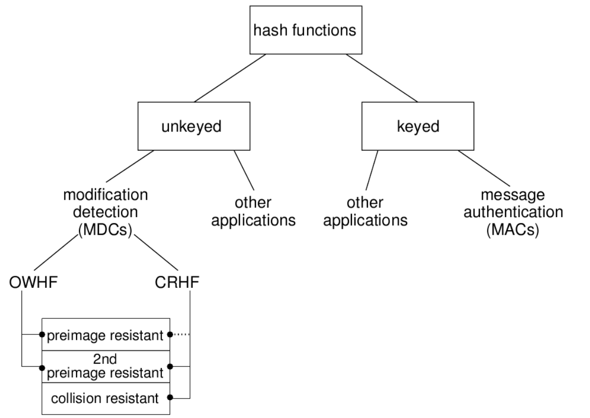
2nd Preimage Resistance (Weak Collision Resistance)

h is **2nd preimage resistant** if given a value x_1 and its hash $h(x_1)$, it is computationally infeasible to find another x_2 such that $h(x_2) = h(x_1)$.

(Strong) Collision Resistance

h is **collision resistant** if it is computationally infeasible to find *any* two inputs x_1 and x_2 such that $h(x_1) = h(x_2)$.

Hash function Classification [HAC]



Modification Detection Codes

- The main application of hash functions is as **Modification Detection Codes** to provide **data integrity**.
- A hash $h(x)$ provides a short *message digest*, a "fingerprint" of some possibly large data x. If the data is altered, the digest should become invalid.
  - This allows the data (but not the hash!) to be stored in an unsecured place.
  - If x is altered to x', we hope $h(x) \neq h(x')$, so it can be detected.
- This is useful especially where *malicious* alteration is a concern, e.g., software distribution.
- Ordinary hash functions such as CRC-checkers produce *checksums* which are not 2nd preimage resistant: an attacker could produce a hacked version of a software product and ensure the checksum remained the same.

Varieties of MDCs

- A **one-way hash function (OWHF)** is a hash function that satisfies preimage resistance and 2nd-preimage resistance.
- A **collision resistant hash function (CRHF)** is a hash function that satisfies 2nd-preimage resistance and collision resistance.
- In practice, CRHF usually satisfies preimage resistance.
- CRHFs are harder to construct than OWHFs and have longer length hash values.
- Choice between OWHF and CRHF depends on application:
  - If attacker can control input, CRHF required.
  - Otherwise OWHF suffices
- **Ex**: which is needed for password file security?

Message Authentication Codes

- **Message Authentication Codes** are *keyed* hash functions, indexed with a secret key.
  - As well as data integrity, they provide **data-origin authentication**, because it is assumed that apart from the recipient, only the sender knows the secret key necessary to compute the MAC.
- A MAC is a key-indexed family of hash functions, $\{h_k \mid k \in \mathcal{K}\}$. MACs must satisfy a *computation resistance* property.

Computation Resistance

Given a set of pairs $(x_i, h_k(x_i))$ it is computationally infeasible to find any other text-MAC pair $(x, h_k(x))$ for a new input $x \neq x_i$.

Relationships between properties

- **Collision resistance implies 2nd-preimage resistance**.
- Sketch proof [HAC]:
  - Let h be CR, but suppose it is not 2nd PI.
  - Fix some input x; compute $h(x)$.
  - Since not 2nd PI, we can find an $x' \neq x$ with $h(x') = h(x)$.
  - But now (x, x') is a collision, so h cannot be CR.
- This and similar arguments (e.g., see Smart) can be made precise using the *Random Oracle Model*.
- **Collision resistance does not imply preimage resistance**
- Contrived counterexample:

$$h(x) = \begin{cases} 1 \mid\mid x & \text{if } x \text{ has length } n \\ 0 \mid\mid g(x) & \text{otherwise} \end{cases}$$

Collision Resistance and Birthday Attacks

- To satisfy (strong) collision resistance, a hash function must be large enough to withstand a **birthday attack**. (or *square root attack*).
- Drawing random elements with replacement from a set of k elements, a repeat is likely after about $\sqrt{k}$ selections.
- Mallory has two contracts, one for £1000, the other £100,000, to be signed with a 64-bit hash. He makes 2^{32} minor variations in each (e.g spaces/control chars), and finds a pair with the same hash. Later claims second document was signed, not first.
- An n-bit unkeyed hash function has **ideal security** if producing a preimage or 2nd-preimage each requires 2^n operations, and producing a collision requires $2^{n/2}$ operations.

From one-way functions to MDCs

- **Multiplication of large primes** is a OWF
  - for appropriate choices of p and q, $f(p, q) = pq$ is a one-way function since *integer factorization [FACTORING]* is difficult.
  - Not feasible to turn into an MD function, though. (**Ex**: why?)
- **Exponentiation in finite fields** is a OWF
  - for appropriate primes p and numbers α, $f(x) = \alpha^x \bmod p$ is a one-way function, since the *discrete logarithm problem [DLP]* is difficult.
  - Main problem with turning this into a realistic MD function is that it's too slow to calculate.

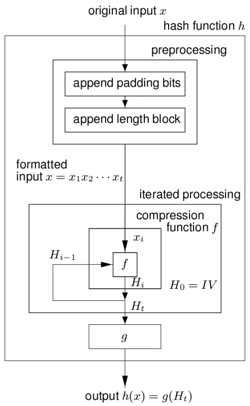
OWFs from block ciphers

- A block cipher is an encryption scheme which works on fixed length blocks of input text.
- We can construct a OWF from a block cipher such as DES, which is treated essentially as a random function:

$$h(x) = E_k(x) \oplus x$$

  for fixed key k. This *can* be turned into a MD function, by iteration...

Iterated hash function construction [HAC]



Building up hash functions

- An **iterated hash function** is constructed using a *compression function* f which converts a $t + n$-bit input into an n-bit output.
  - The input x is split into blocks $x_1 x_2, \ldots x_k$ of size t, appending padding bits and a *length block* indicating the original length.

$$H_0 = IV \qquad H_i = f(H_{i-1}, x_i), \ \ 1 \le i \le k \qquad h(x) = g(H_k).$$

  - IV: an initialization vector; g: an output transformation (often identity).
- This is **Merkle's meta-method**
  - Fact: any CR compression function f can be extended to a CRHF by the above construction, and
  - *padding*: the last block with 0s, adding a final extra block x_k which holds right-justified binary representation of $length(x)$ (this padding is called **MD strengthening**).
  - Set $IV = 0^n$, $g = id$, and compute $H_i = f(H_{i-1}, x_i)$.

MD5

- Improvement of MD4; MD4 and MD5 designed by Ron Rivest.
  - Text processed in 512-bit blocks, as 16 32-bit sub-blocks. Output is four 32-bit blocks, giving a **128-bit** hash. Message padded with 1 and then 0s until last block is 448 bits long, then a 64-bit length.
  - Main loop has four rounds, chaining 4 variables a, b, c, d. Each round uses a different operation (with a similar structure) 16 times, which computes a new value of one of the four variables using a non-linear function of the other three, chosen to preserve randomness properties of the input.
  - For example, the first round uses the operation:

$$\begin{aligned} a &= (F(b,c,d) + x_i + t_j) <<< s \\ F(b,c,d) &= (b \wedge c) \vee (\neg b \wedge d) \end{aligned}$$

  where $<<< s$ is left-circular shift of s bits, x_i is the ith sub-block of the message. Constants t_j are the integer part of $2^{32} * \text{abs}(\sin(i+1))$ where $0 \le i \le 63$ is in radians (for the 4 * 16 steps).

Secure Hash Algorithm SHA-1 (160)

SHA-1 is a NIST standard [FIPS 180] also based on MD4. An attack strategy with cost 2^{51} was found in 2011.

- Five 32-bit blocks are chained; output is 160 bits. Message blocks 512 bits. Padding like MD5.
  - Main loop has four rounds of 20 operations, chaining 5 variables a, b, c, d, e, f. Five IVs and four constants are used:

$A =$ `0x67452301`	$K_0 =$ `0x5A827999`
$B =$ `0xEFCDAB89`	$K_1 =$ `0x6ED9EBA1`
$C =$ `0x98BADCFE`	$K_2 =$ `0x8F1BBCDC`
$D =$ `0x10325476`	$K_3 =$ `0xCA62C1D6`
$E =$ `0xC3D2E1F0`	

  - The message block undergoes an *expansion transformation* from 16*32-bit words x_i to 80*32-bit words, w_i by:

$$\begin{aligned} w_i &= x_i, & \text{for } 0 \le i \le 15. \\ w_i &= (w_{i-3} \oplus w_{i-8} \oplus \\ &\qquad w_{i-14} \oplus w_{i-16}) <<< 1, & \text{for } 16 \le i \le 79. \end{aligned}$$

SHA-1 (160) continued

- 80 steps in main loop, changing Ks and Fs 4 times
  - Where $j = i/20$:

```
for( i = 0; i < 80; i++ )     {
    tmp = (a <<< 5) + F_j(b, c, d) + e + w_i + K_j;
    e = d;
    c = b <<< 30;
    b = a;
    a = tmp;
}
```

  - Each F_j combines three of the five variables:

$$\begin{aligned} F_0(X,Y,Z) &= (X \wedge Y) \vee (\neg X \wedge Z) \\ F_1(X,Y,Z) &= X \oplus Y \oplus Z \\ F_2(X,Y,Z) &= (X \wedge Y) \vee (X \wedge Z) \vee (Y \wedge Z) \\ F_3(X,Y,Z) &= X \oplus Y \oplus Z \end{aligned}$$

  - Finally a, b, c, d, e are added to tmp (all addition is modulo 2^{32}).
- **Exercise:** implement SHA-1 in your favourite language following this. Test against `sha1sum`.

Current Status

- Hash functions are versatile and powerful primitive.
- However, difficult to construct and less researched than encryption schemes.
  - ideal hash function is a "random mapping" where knowledge of previous results doesn't give knowledge of another.
  - practical fast iterative hash constructions fail this!
  - MD4 (1998), MD5 (1993/2005), SHA-1 (2005) are now **all considered broken**.
- The US National Institute of Standards and Technology (NIST) has standardised a set of newer hash functions.
  - Formerly called SHA-2, they are denoted by their output size: SHA-256, SHA-384, SHA-512.
  - However, since they are based upon the same *SHA* construction, they are not long-term solutions
  - In 2012, NIST awarded a new standard SHA-3 to the *Keccak* algorithm (a *sponge function* which has arbitrary output length).

References

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- Neils Ferguson and Bruce Schneier. *Practical Cryptography*.
  John Wiley & Sons, 2003.
- Douglas R Stinson. *Cryptography Theory and Practice*.
  CRC Press, second edition edition, 2002.
- Nigel Smart. *Cryptography: An Introduction*.
  McGraw-Hill, 2003. Third edition online: `http://www.cs.bris.ac.uk/~nigel/Crypto_Book/`

Recommended Reading

One of: Ch 9 of HAC (9.1–9.2); Ch. 10 of Smart 3rd Ed; 11.1–11.3 of Gollmann.