Cryptography I: Introduction

Computer Security Lecture 2

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17th January 2013

Outline

- Terminology
- Basic Definitions
- Symmetric Cryptography
- Asymmetric Cryptography

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Terminology

Basic Definitions

Symmetric Cryptography

Asymmetric Cryptography

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- **encryption scheme**, **cipher**, **cryptosystem**:
  a mechanism for encryption and decryption

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- **non-repudiation** — preventing denial of actions

We want to ensure these properties, even when another party may eavesdrop or intercept messages.

Carefully designed **cryptographic protocols** help this.

Cryptographic primitives

Protocols are built using **cryptographic primitives**, parametrised on 0, 1, or 2 **keys**.

Unkeyed	**Secret key**	**Public key**
Random sequences	Symmetric-key ciphers — block and stream	Public-key ciphers
One-way permutations	Keyed hash functions (aka MACs)	Digital signatures
Hash functions	Identification primitives	Identification primitives
	Digital signatures	
	Pseudorandom sequences	

Familiar examples:

- Hash functions: MD5, SHA-1, SHA-256
- Symmetric block ciphers: DES, 3DES, AES
- Public key ciphers: RSA, El Gammal
- Digital signature schemes: RSA, DSA

Notation and example applications

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- Digital signature schemes $S_A(m)$, $V_A(m, s)$
  - key signing: public key infrastructures (PKIs)

Choosing primitives

- Choice of primitives influenced by:
    - functionality needed
    - performance
    - implementation ease
    - *amount* of security
- Measuring of security is tricky: may consider
    - primitives are “perfect”, maybe “unbreakable”
        - what is the worst that can happen?
    - primitives are “imperfect”
        - what does attacker know?
        - how much effort can attacker spend?

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  - Formal analysis now being extended to these kinds of *computational* models.

Cryptanalysis attacks

- Setup: have $c_1 = E_k(m_1), \ldots, c_n = E_k(m_n)$, small n.
- Best outcome: find k or algorithm for D_k^{-1}.
- Try to better **brute-force** (exhaustive search).

Attack type	Attacker knowledge
Ciphertext only	the c_i (deduce at least m_i)
Known plaintext	the c_i and m_i
Chosen plaintext	c_i for *chosen* m_i
Adaptive chosen plaintext	as above, but iterative
Chosen ciphertext	$c_i, m_i = D_d(c_i)$. Find decryption key d.
“Rubber-hose”	bribery, torture, or blackmail
“Purchase-key”	(not cryptanalysis, but v. successful)

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    - Rule-of-thumb: for good symmetric encryption algorithms, a key space of 2^{128} is currently considered prudent.
    - But this is a simplistic view!

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Basic Definitions

Symmetric Cryptography

Asymmetric Cryptography

Bijections

- Recall that a **bijection** is a mathematical function which is one-to-one (injective) and onto (surjective).
- In particular, if $f : X \to Y$ is a bijection, then for all $y \in Y$, there is a unique $x \in X$ such that $f(x) = y$. This unique x is given by the *inverse* function $f^{-1} : Y \to X$.

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(Saying this, non-bijections, in fact non-functions, *are* used as encryption transformations. Can you imagine how?)

Message spaces

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- Each space is given over some *alphabet*, a set $\mathcal{A}$.
  For example, we may consider $\mathcal{A}$ to be the letters of the English alphabet A-Z, or the set of binary digits $\{0, 1\}$. (Of course, any alphabet can be encoded using words over $\{0, 1\}$).

Cryptography systems

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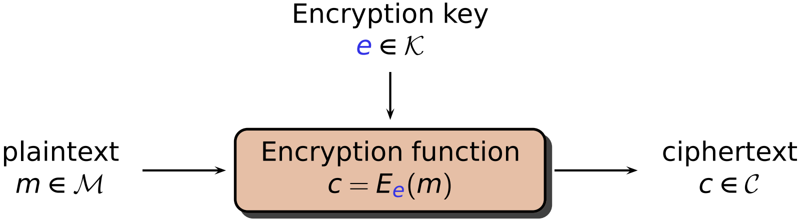
Encryption and decryption transformations are indexed using *keys*.

- The *key space* $\mathcal{K}$ is a finite set of *keys* $k \in \mathcal{K}$.
- An **encryption scheme** consists of two sets indexed by keys
  - a family of encryption functions $\{E_e \mid e \in \mathcal{K}\}$
  - a family of decryption functions $\{D_d \mid d \in \mathcal{K}\}$

  such that for each $e \in \mathcal{K}$, there is a unique $d \in \mathcal{K}$ with $D_d = E_e^{-1}$. We call such a pair (e, d) a *key pair*.
- An encryption scheme is also known as a **cryptography system** or a **cipher**.

Encryption



Encryption key

$e \in \mathcal{K}$

plaintext

$m \in \mathcal{M}$

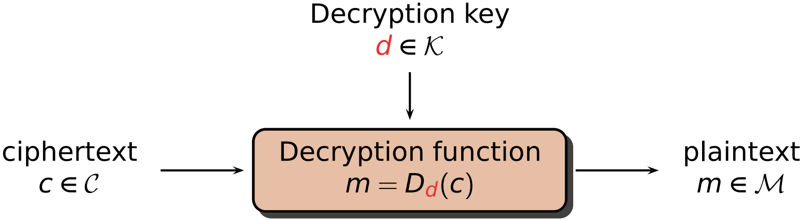
Encryption function

$c = E_e(m)$

ciphertext

$c \in \mathcal{C}$

Decryption



Decryption key

$d \in \mathcal{K}$

ciphertext

$c \in \mathcal{C}$

Decryption function

$m = D_d(c)$

plaintext

$m \in \mathcal{M}$

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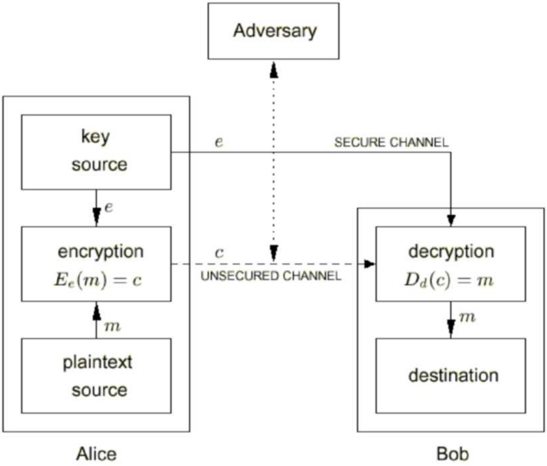
Symmetric and asymmetric cryptography

- **symmetric** cryptography
  - e and d are (essentially) the same
  - aka secret-key, shared-key, single-key, conventional
- **asymmetric** cryptography
  - Given e, it is (computationally) infeasible to find d.
  - aka public-key (PK), since e can be made public.

- Of course, the key-pair relation is not the only difference between symmetric and asymmetric cryptography. Other differences arise from characteristics of known algorithms and usage modes.

- Note: these definitions are imprecise: to be exact, one should define the meanings of “essentially” and “computationally infeasible”.

Symmetric cryptography



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- Secure channels are difficult and costly to implement. How to deliver secret keys through unsecured channels had confounded thinkers for many centuries.

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> *If you can read everything I write, I cannot rely on any secret that has gone before, how can I possibly send a confidential message to my friend which you cannot also understand?*

- The answer uses a creative leap of innovation (two keys, one public), as well relying on some clever maths in its implementation (*trapdoor one-way functions*).

One-way functions

A function $f : X \to Y$ is called a **one-way function** if

- it is feasible to compute $f(x)$ for all $x \in X$, but
- it is infeasible to find any x in the pre-image of f, such that $f(x) = y$, for a randomly chosen $y \in \operatorname{Im} f$. (If f is bijective, this means it is infeasible to compute $f^{-1}(y)$).

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The definition above is vague: to be exact, we should give precise notions of *feasible* and *infeasible*. This is possible, but so far **no-one has proved the existence of a true one-way function**. Some functions used in modern ciphers are properly called *candidate one-way functions*, which means that there is a body of belief that they are one-way.

Trapdoor one-way functions

- A **trapdoor one-way function** is a one-way function f that has a “trapdoor”: given some additional information, it is feasible to compute an x such that $f(x) = y$, for any $y \in \operatorname{Im} f$.

These are just what we need for public key crypto: the private key is the trapdoor information.

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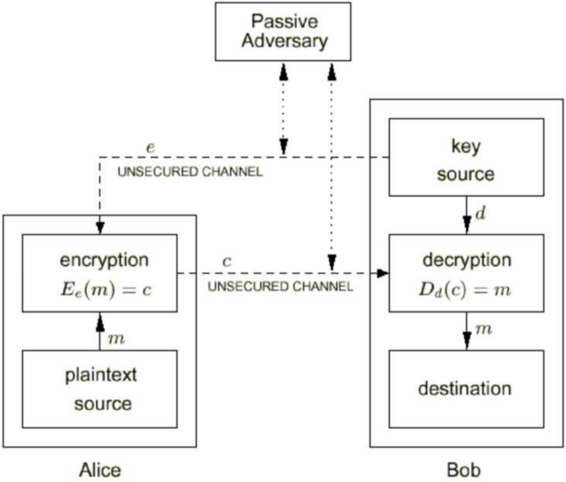
- It’s unlikely that one-way functions do *not* exist; some hash functions are as secure as NP-complete problems.
- Catastrophic failure for present functions is less common than gradual failure due to advances in computation power and (non-revolutionary but clever) algorithms or cryptanalysis, bringing some attacks closer to feasibility.

Asymmetric cryptography



References

Some content is adapted from Chapter 1 of the HAC. Schneier’s text is readable (but dated). Smart’s book is more rigorous. Kahn’s book has a detailed history.

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- Bruce Schneier. *Applied Cryptography*.
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- Nigel Smart. *Cryptography: An Introduction*.
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- David Kahn. *The Codebreakers*.
  Simon & Schuster, revised edition, 1997.

Recommended Reading

Chapter 1 of HAC. Chapter 3, Sections 11.1–11.2 of Smart (3rd Ed).