Cryptography V: Digital Signatures

Computer Security Lecture 10

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Outline

- Basics
- Constructing signature schemes
- Security of signature schemes
- ElGamal
- DSA
- Summary

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Security of signature schemes

ElGamal

DSA

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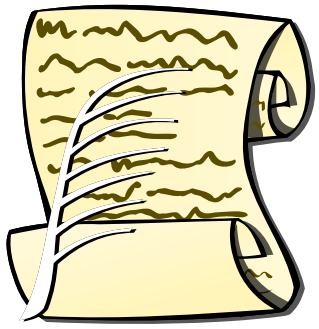
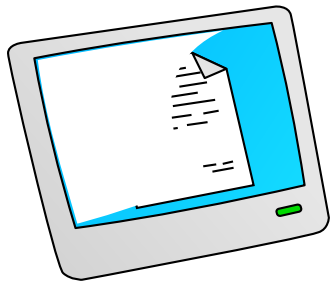
- Digital signatures are the asymmetric analogue of MACs, with a crucial difference. MACs can't disinguish which of A or B provided integrity to a message (so no non-repudiation or independent verifiability).
- NB: **electronic signature** is a more general notion.

Handwritten versus Digital Signatures





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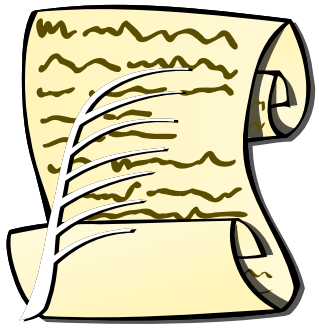

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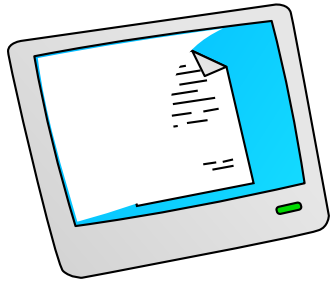

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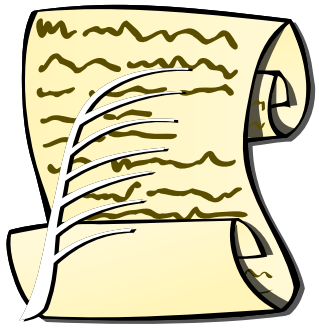

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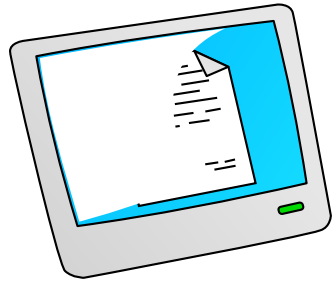

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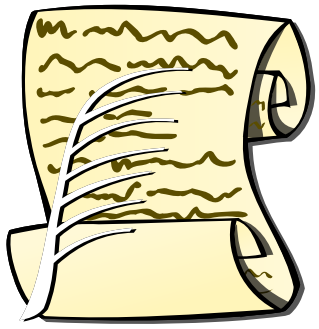

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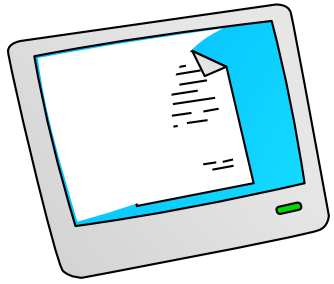
signatures always same



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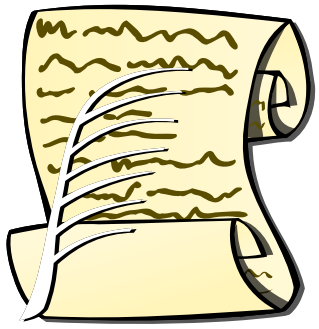

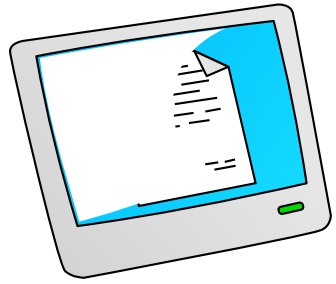

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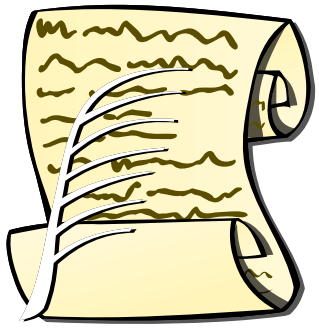

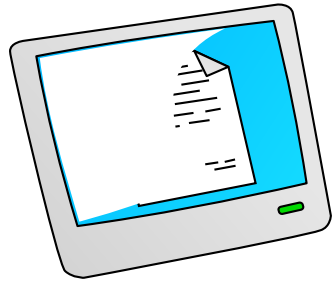

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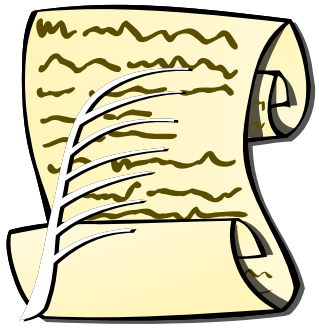

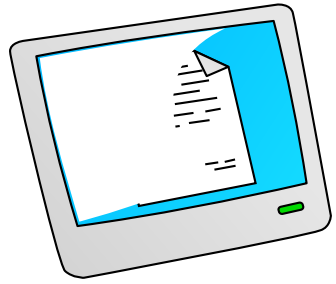

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have legal impact	may have legal impact

Signature mechanism

A signature mechanism for principal A is given by:

- A message space $\mathcal{M}$ of messages for signing
- A set $\mathcal{S}$ of *signatures* (e.g. strings $\{0,1\}^n$)
- A secret **signing function** $S_A : \mathcal{M} \to \mathcal{S}$
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1. $V_A(m, s) = \text{true}$ if and only if $S_A(m) = s$.
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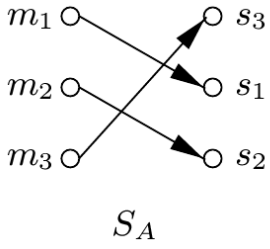
Remark: nobody has proved a signature mechanism satisfying 2 exists, although there are good candidates.

Using a signature scheme



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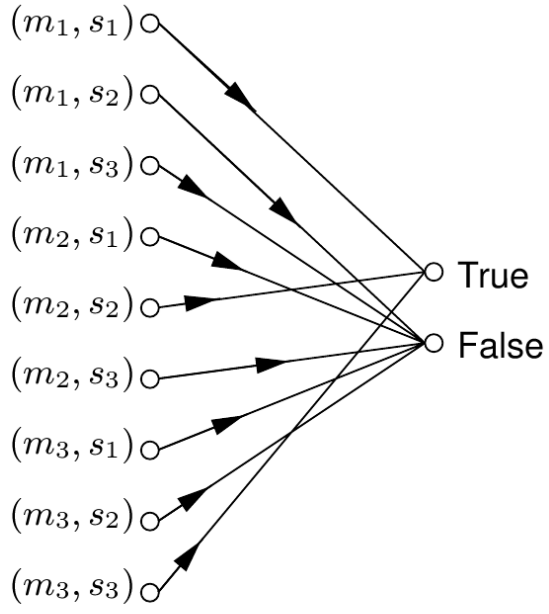

V_A

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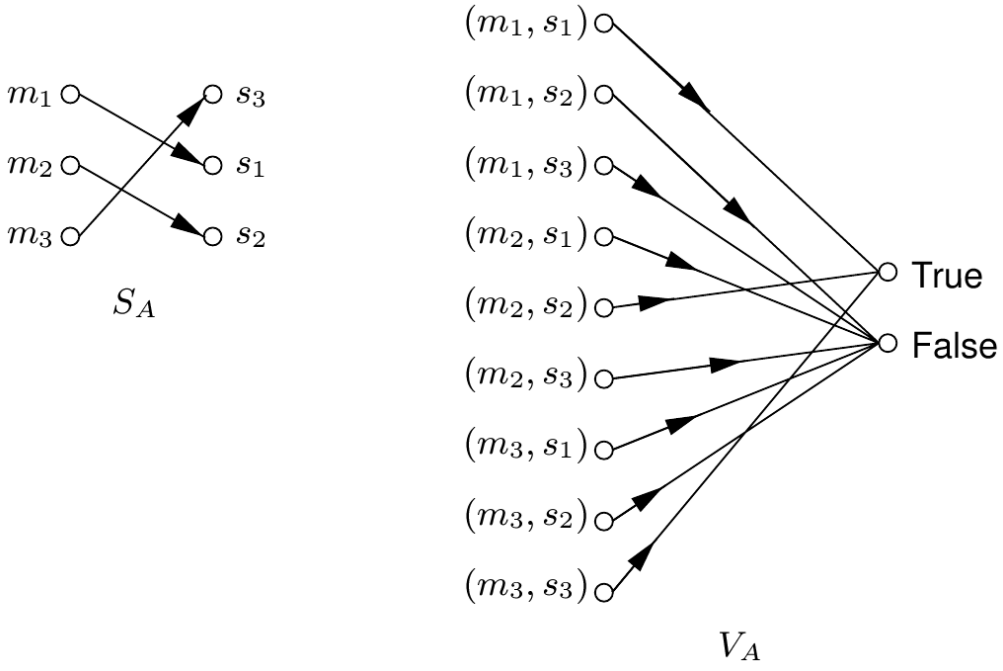

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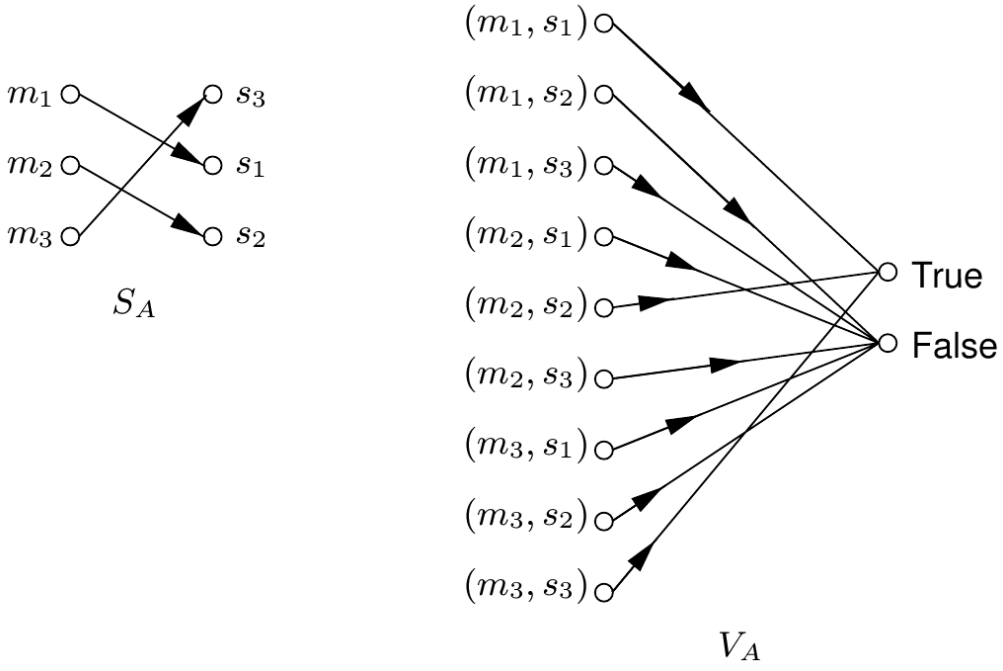

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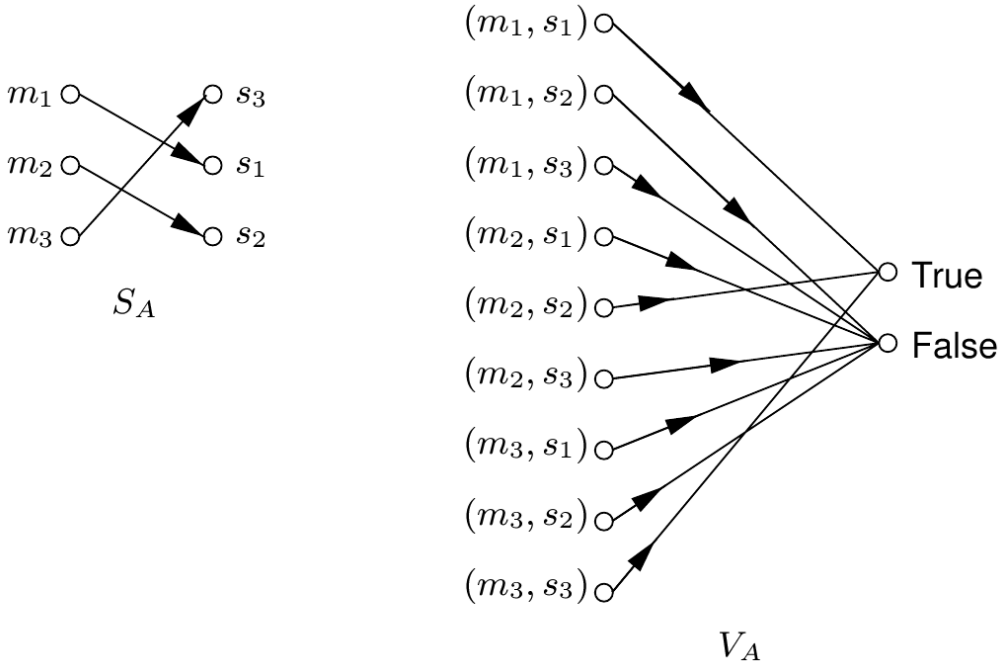

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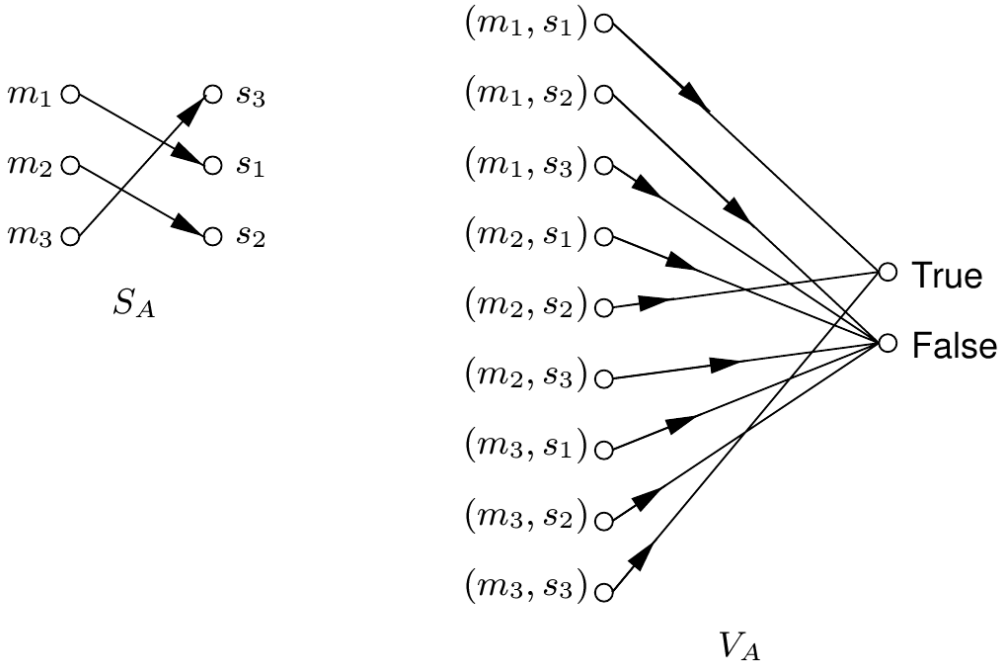

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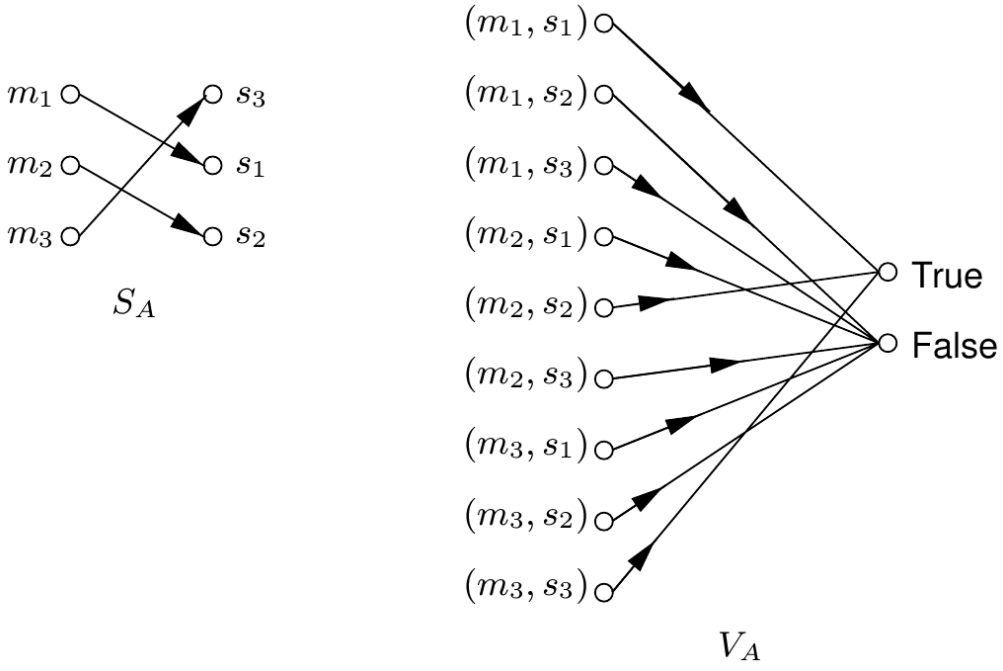

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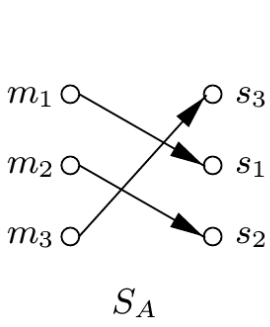

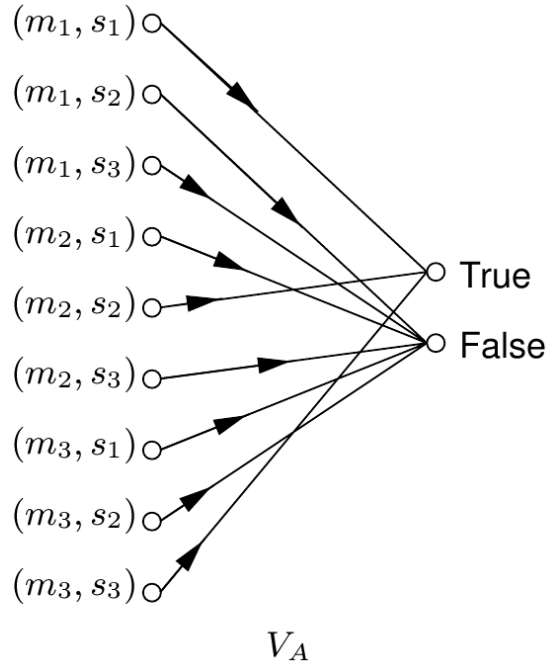

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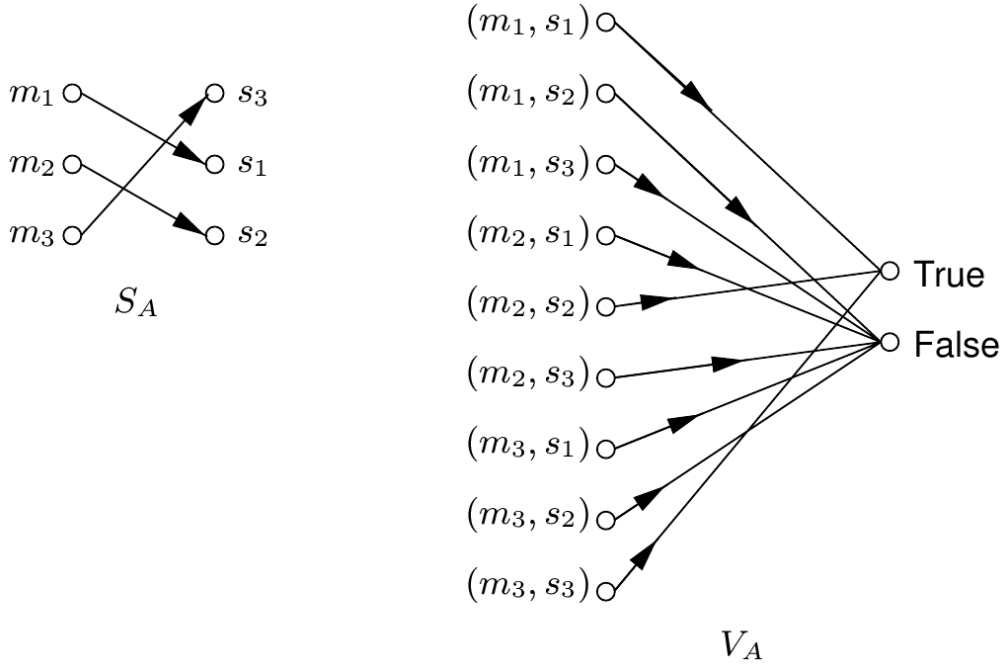

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  3. **Accepts** the signature if $u = \text{true}$, **Rejects** it if $u = \text{false}$.

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- If A and B disagree about a signature, a judge Judy can verify the contracts also using S:

$$\begin{array}{lll} \text{Message 1.} & J \rightarrow S: & \{M\}_{K_{as}}, \{M\}_{K_{bs}} \\ \text{Message 2.} & S \rightarrow J: & \{\textit{yes or no}\}_{K_{js}} \end{array}$$

Digital signatures from PK encryption

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  - the signing function $S_A = D_d$
  - the verification function V_A is defined by

$$V_A(m, s) = \begin{cases} \text{true} & \text{if } E_e(s) = m, \\ \text{false} & \text{otherwise.} \end{cases}$$

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  2. **Known-message attack**: adversary has signatures for some known (not chosen) messages.
  3. **Chosen-message attack**: adversary can obtain signatures for messages of his choosing. Messages may be determined in advance or in **adaptive** way, using signer as oracle.

Existential forgery

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- But this ability violates property 2 given earlier.

Signatures with redundancy

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- Existential forgery is now less likely.

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- This is called a **signature scheme with appendix**.

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- To verify the signature, upon receipt of (m, s), compute $s^e \bmod n$ and verify whether it equals $h(m)$

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- Signatures can optionally be *distributed* so that each of t users contributes to the signature. A trusted party T computes t *shares* such that

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- *Secret sharing* can also be used so that $l < t$ users could be used to construct a signature.

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- Verification works because for a correct signature,

$$(g^d)^e e^s \equiv g^{de+rs} \equiv g^m \pmod{p}.$$

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- **Security** of both ElGamal and DSA schemes relies on the intractability of the DLP.
- Comparison with RSA signature scheme: key generation is faster; signature generation is about the same; DSA verification is slower. Verification is the most common operation in general.

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Summary: Digital Signature Schemes

- **RSA**, **ElGamal**, **DSA** already described. There are several variants of ElGamal, including schemes with message recovery.
- Notice difference between *randomized* and *deterministic* schemes.
- Schemes for **one-time signatures** (e.g., Rabin, Merkle), require a fresh public key for each use.
  - Typically more efficient than RSA/ElGamal methods.
  - But tedious for multiple documents
- E-cash protocols use **blind signature** schemes that prevent the signer (e.g., a bank) linking a signed message (e.g., the cash) with the user.
- For real world security guarantees:
  - **obtaining correct public key** is vital;
  - non-repudiation supposes that **private key has not been stolen**;
  - we may require **secure time stamps**.

References

Alfred J. Menezes, Paul C. Van Oorschot, and Scott A. Vanstone, editors. *Handbook of Applied Cryptography*.

CRC Press Series on Discrete Mathematics and Its Applications. CRC Press, 1997.

Online version at `http://www.cacr.math.uwaterloo.ca/hac`.

Digital signatures covered in Section 1.6 and Chapter 11.

Nigel Smart. *Cryptography: An Introduction*.

McGraw-Hill, 2003. Third edition online: `http://www.cs.bris.ac.uk/~nigel/Crypto_Book/`

Recommended Reading

Chapter 14 (14.2–14.4, 14.7) of Smart (3rd Ed).