A monad on a category $C$ consists of a functor $F: C \to C$ together with two natural transformations $\eta_A: A \to F(A)$ and $\mu_A: F(F(A)) \to F(A)$ making the following diagrams commute:

$$
\begin{align*}
F(A) & \xrightarrow{F(\eta_A)} F^2(A) \xleftarrow{F(\mu_A)} F^3(A) \\
\downarrow {\eta_F(A)} & \quad \downarrow {\text{id}_{F(A)}} & \quad \downarrow {\mu_A} \\
F^2(A) & \xrightarrow{\mu_A} F(A) \xleftarrow{\mu_A} F^2(A)
\end{align*}
$$

1. Given a monad $(F, \mu, \eta)$ on $C$, show how to make a new category $C_F$ with the same objects as $C$, but whose morphisms $A \to B$ are morphisms $A \to F(B)$ in $C$. Find a functor $C \to C_F$.

2. Show how the following are monads on $\text{Set}$:
   - exception: $F(A) = A + 1$ (where $+$ is disjoint union and $1$ is a fixed singleton set)
   - log: $F(A) = A \times X$ for a fixed set $X$
   - input: $F(A) = A^X$ for a fixed set $X$ (where $A^X$ is the set of functions $X \to A$)
   - nondeterminism: $F(A) = \mathcal{P}(A)$, the powerset of $A$

Discuss how we might think of morphisms in $C$ as computations without side-effects, and morphisms in $C$ as computations that may have side-effects modelled by $F$, in each of these four cases.

3. If $C$ is monoidal, we might expect the monad $F$ to cooperate with the monoidal structure. Look up and discuss the definition of a strong monad: it involves a natural transformation $\text{st}_{A,B}: F(A) \otimes B \to F(A \otimes B)$.

Show that every monad on $(\text{Set}, \times, 1)$ is strong.

4. If $C$ is symmetric monoidal, strength induces a similar natural transformation $A \otimes F(B) \to F(A \otimes B)$. Combining the two strength maps gives two natural transformations $F(A) \otimes F(B) \to F(A \otimes B)$. The monad is called commutative when these two double strength maps coincide. Which of the above four monads are commutative on $\text{Set}$?

5. Consider the nondeterminism monad $F$ on $\text{Set}$ as in part (2). Show that the category $\text{Set}_F$ it induces as in part (1) is equivalent to $\text{Rel}$.