A monad on a category \( C \) consists of a functor \( F : C \to C \) together with two natural transformations \( \eta_A : A \to F(A) \) and \( \mu_A : F(F(A)) \to F(A) \) making the following diagrams commute:

\[
\begin{array}{ccc}
F(A) & \xrightarrow{F(\eta_A)} & F^2(A) & \xleftarrow{F(\mu_A)} & F^3(A) \\
\downarrow{\eta_{F(A)}} & & \downarrow{\mu_A} & & \downarrow{\mu_{F(A)}} \\
F^2(A) & \xrightarrow{\mu_A} & F(A) & \xleftarrow{\mu_A} & F^2(A)
\end{array}
\]

1. Given a monad \((F, \mu, \eta)\) on \( C \), show how to make a new category \( C^F \) with the same objects as \( C \), but whose morphisms \( A \to B \) are morphisms \( A \to F(B) \) in \( C \). Find a functor \( C \to C^F \).

2. Show how the following are monads on \( \text{Set} \):
   - exception: \( F(A) = A + 1 \) (where + is disjoint union and 1 is a fixed singleton set)
   - log: \( F(A) = A \times X \) for a fixed set \( X \)
   - input: \( F(A) = A^X \) for a fixed set \( X, A \to A^X \) is the set of functions \( X \to A \)
   - nondeterminism: \( F(A) = \mathcal{P}(A) \), the powerset of \( A \)

Discuss how we might think of morphisms in \( C \) as computations without side-effects, and morphisms in \( C \) as computations that may have side-effects modelled by \( F \), in each of these three cases.

3. If \( C \) is monoidal, we might expect the monad \( F \) to cooperate with the monoidal structure. Look up and discuss the definition of a strong monad: it involves a natural transformation \( \text{st}_{A,B} : F(A) \otimes B \to F(A \otimes B) \).

Show that every monad on \((\text{Set}, \times, 1)\) is strong.

4. If \( C \) is symmetric monoidal, strength induces a similar natural transformation \( A \otimes F(B) \to F(A \otimes B) \). Combining the two strength maps gives two natural transformations \( F(A) \otimes F(B) \to F(A \otimes B) \). The monad is called commutative when these two double strength maps coincide. Which of the above four monads are commutative on \( \text{Set} \)?

5. Consider the nondeterminism monad \( F \) on \( \text{Set} \) as in part (2). Show that the category \( \text{Set}^F \) it induces as in part (1) is equivalent to \( \text{Rel} \).