Today’s lecture

- Solving quadratic with if-statements
- General form of the if-statement
- Boolean tests (using relational operators)
- What about degenerate quadratics?
- Refining quadratic.c

Choice of variables for quadratic.c

- We will need to compute the square-root of $b^2 - 4ac$.
- The sqrt function available in the math library for C is of the type double sqrt(double x);
- For this simple reason we use double variables for our real roots.
- Precision is not really important to us, at least not now.

Last Lecture

- Arithmetic
- Quadratic equation problem: $ax^2 + bx + c = 0$
- Floating point data

Choice of variables for quadratic.c

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- The sqrt function available in the math library for C is of the type double sqrt(double x);
- For this simple reason we use double variables for our real roots.
- Precision is not really important to us, at least not now.
C program to Solve Quadratic Equations

\[ x = -b \pm \sqrt{b^2 - 4ac} \]
\[ 2a \]

Steps of our program:

▶ Take in the inputs \(a\), \(b\) and \(c\) from the user (\texttt{scanf}).

Need three int variables to store these values: \(a\), \(b\), \(c\) say;

▶ test whether \(b^2 - 4ac\) is non-negative.

What we do will depend on the result of the test

▶ If negative, output a message about “No real roots”.
▶ If exactly 0, a repeated root.
▶ Otherwise, two differing roots as per formula.

▶ Get the square root of \(b^2 - 4ac\) (if non-negative).

▶ Output both roots (or one if repeated).

▶ return \texttt{EXIT\_SUCCESS};

Running \texttt{quadratic.c}

\texttt{quadratic.c} (and the refinements of this program) uses the \texttt{sqrt} function from the \texttt{math} library.

\textbf{note:} Not \texttt{enough} to include \texttt{<math.h>} in the code.

▶ \texttt{<math.h>} is just a header file for the \texttt{math} library (it explains the “shape” of the \texttt{sqrt} function, and other \texttt{math} functions).

▶ To run our program, we need to \texttt{link} to \texttt{executable} code for the \texttt{math} functions.

▶ Link by adding \texttt{-lm} to \texttt{gcc} command when compiling:

\texttt{gcc -Wall quadratic.c -lm}

▶ \texttt{-lm} is ‘minus ell m’, NOT ‘minus one m’.

Assumptions :-(

We made some assumptions for \texttt{quadratic.c}

▶ By solving a quadratic, we (implicitly) assumed \(a\) is non-zero.
▶ Same for \(b\).
▶ We might have had a linear or constant equation.

\textbf{SOLUTION} - use the (general) if statement.

body of \texttt{quadratic.c}

\begin{verbatim}
int s = (b*b - 4*a*c);

if (s < 0) {
    printf("No real roots to this quadratic.\n");
} else if (s == 0) {
    printf("Eq. has the repeated root %.1f.\n", -(double)b/(2.0*a));
} else {
    x1 = (-(double)b - sqrt(s))/(2.0*a);
    x2 = (-(double)b + sqrt(s))/(2.0*a);

    printf("The sols to \%dx^2 +\%dx +\%d = 0 are ", a, b, c);
    printf("%.1f and %.1f.\n", x1, x2 );
}
\end{verbatim}

Note: \texttt{sqrt()} takes a double argument, so the int expression \(s\) is automatically promoted to a double in \texttt{sqrt(s)}.

\textbf{Question:} is the cast \texttt{(double)b} necessary?
If `condition_1`, ..., `condition_{n-1}` are all boolean expressions: either true or false.

`statement-sequence_1`, ..., `statement-sequence_n` are all sequences of C-programming statements.

Note that it is possible to use `if` alone without any `else` branch.

Warning about `if`

If you look in the textbooks, you will see that when the `statement-sequence` has only one statement, you can miss out the curly brackets round it:

```c
if ( t > 0 )
  x = x + 1;
else
  x = x - 1;
```

We recommend that you don't do this (at least until you're experienced enough to understand when you can ignore our advice!).

You will see it in other people's programs though.

Actually, we lied about the general form of the `if` statement. In truth, the general form is

```c
if ( condition ) statement_1 else statement_2
```

and a `statement` can have the form `{ statement-sequence }`

or can be an `if`-statement itself. However, the form on the previous slide is the way we typically use it, so best to 'learn' that.

Relational operators

What kind of conditions can we use in `if` statements?

Assume `e_1` and `e_2` are (usually arithmetic) expressions . . .

We can apply relational operators to form a boolean expression.

```c
e_1 == e_2          e_1 equal to e_2
e_1 != e_2          e_1 not equal to e_2
e_1 < e_2          e_1 less than e_2
e_1 <= e_2          e_1 less than or equal to e_2
e_1 > e_2          e_1 greater than e_2
e_1 >= e_2          e_1 greater than or equal to e_2.
```

Never write `e_1 < e_2 < e_3` it is legal C, but it doesn't mean anything like what you think it means – if you remember to `-Wall`, the compiler will warn you if you do this!

Note: We can compare `float` and `double` expressions in this way - but only int comparisons are fully reliable. Why is this?

More complicated Boolean expressions

Assume `e_1` and `e_2` are boolean expressions . . .

Can build more complicated boolean expressions iteratively, using boolean operators.

```c
0 false (always)

non-zero true (always)

!e_1 true if `e_1` is false

e_1 && e_2 true if (`e_1` is true and `e_2` is true)

e_1 || e_2 true if (`e_1` is true or `e_2` is true)
```

For example, can write `(e_1 < e_2) && (e_2 < e_3)`.

The expressions `e_1`, `e_2` are (formally) integer expressions.

We define boolean variables as `int`.

Think of integers as (informally) acting as boolean 'type'.
Boolean expressions in if-else statements

We have seen lots of simple and complex boolean expressions: whenever any of these are used as tests in an if-else statement, they must be enclosed in parentheses.

For example

- The simple Boolean expression \( x < 5+z \), when being used as a test for an else if-branch of an if-else statement, would appear as
  
  ```
  else if (x < 5+z) { ... }
  ```

- The complex Boolean expression \((a \neq 0) \land (b* b > 4*a*c)\), when being used as the test for the if branch of an if-else statement, would appear as
  
  ```
  if ( (a \neq 0) \land (b*b > 4*a*c) ) { ... }
  ```

Nested if-statements

- The statement-sequence place-holder in the general if-statement allows other if-statements to be part of the program fragment.
- This is a 'nested' use of the if-statement.
- Example – refine the quadratic.c program further to include a solution for the \( a = 0 \) case (given a linear equation).

Improved quadratic.c

Issues for improving quadratic:

(i) make our quadratic solver cope with \( a = 0 \) (quadratic2.c) ... and with \( a = 0, b = 0 \).

(ii) complex roots (homework).

quadratic equations – what if \( a = 0 \)

If \( ax^2 + bx + c = 0 \) is a quadratic, and \( a \) is 0, then we have a linear equation:

\[
bx + c = 0
\]

This has ...

- Exactly one root of value \(-c/b\), if \( b \neq 0 \).
- No root at all, if \( b = 0 \) and \( c \neq 0 \).
- Everything is a root, if \( b = c = 0 \).

Can incorporate this case into our code:
Start off as before ...

```c
#include <stdio.h>
#include <stdlib.h>
#include <math.h> // Need to include math.h to use sqrt.

int main(void) {
    int a, b, c, s;
    double x1, x2;

    printf("Input the x^2 co-efficient a: ");
    scanf("%d", &a);
    printf("Input the x co-efficient b: ");
    scanf("%d", &b);
    printf("Input the constant term c: ");
    scanf("%d", &c);

    s = b*b - 4*a*c;

    if (a != 0) {
        if (s < 0) {
            /* THIS (a != 0) */
            ... /* BRANCH IS */
        } /* EXACTLY WHAT */
        else if (s == 0) {
            /* WE PUT FOR THE */
            ... /* BODY OF */
        } /* quadratic.c */
        else {
            ... /* quadratic.c */
        }
    } /* a != 0 */
    else if (b != 0) { /* a==0 WITH b NON-ZERO */
        x1 = -(double)c/((double)b);
        printf("1 sol to %dx^2 +%dx +%d = 0.
", a, b, c);
        printf("It is %lf.
", x1);
    } /* a==0 WITH b NON-ZERO */
    else if (c != 0) { /* a AND b BOTH ZERO, c NON-ZERO */
        printf("No sols to %dx^2 + %dx +%d = 0.
", a, b, c);
    } /* a AND b BOTH ZERO, c NON-ZERO */
    else { /* a, b, c ALL ZERO */
        printf("Degenerate equation - everything is a solution!
");
    } /* a, b, c ALL ZERO */
    return EXIT_SUCCESS;
}
```

We need to complete the else (a being 0) branch.

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### quadratic2.c – a != 0 versus a == 0

```c
if (a != 0) {
    if (s < 0) {
        /* THIS (a != 0) */
        ... /* BRANCH IS */
    } /* EXACTLY WHAT */
    else if (s == 0) {
        /* WE PUT FOR THE */
        ... /* BODY OF */
    } /* quadratic.c */
    else {
        ... /* quadratic.c */
    }
} /* a != 0 */
else if (b != 0) { /* a==0 WITH b NON-ZERO */
    x1 = -(double)c/((double)b);
    printf("1 sol to %dx^2 +%dx +%d = 0.
", a, b, c);
    printf("It is %lf.
", x1);
} /* a==0 WITH b NON-ZERO */
else if (c != 0) { /* a AND b BOTH ZERO, c NON-ZERO */
    printf("No sols to %dx^2 + %dx +%d = 0.
", a, b, c);
} /* a AND b BOTH ZERO, c NON-ZERO */
else { /* a, b, c ALL ZERO */
    printf("Degenerate equation - everything is a solution!
");
} /* a, b, c ALL ZERO */
return EXIT_SUCCESS;
```

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### Reading and Working

Relevant sections of "A book on C" are Sections 4.1, 4.2, 4.3, 4.4 (on Boolean expressions, Relational operators, etc) and Section 4.7 (on the if and the if-then-else statements).

You already have the week 3 Tutorial sheet.

Please attempt all Questions before your tutorial group.

Also please think of one question about the CP material so far, and bring that question to the tutorial.

How about coding up quadratic2.c?

Could make quadratic3.c by also doing complex roots.