Computer Programming: Skills & Concepts (CP)
Searching and sorting

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Monday 13 November 2017
Searching an array

typedef enum {FALSE, TRUE} Bool_t;

Bool_t LinearSearch(int n, int a[], int sKey)
/* Returns TRUE iff (if and only if) sKey is contained in
 * the array, i.e., there exists an index i with 0 <= i < n
 * such that a[i] == sKey.
 */
{
    int i;
    for (i = 0; i < n; ++i) {
        if (a[i] == sKey) return TRUE;
    }
    return FALSE;
}

variant:
▶ Could use return type int with #DEFINE for TRUE, FALSE (see
BinarySearch)
Binary search

Sometimes we quickly want to find an entry in an array. It helps if the array is sorted.
Binary search

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Repeatedly chop the array in half to close in on where the element must be. E.g., to search for 17 in:
2 3 5 7 11 13 17 19 23 29
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Find the mid-point: 17 ≤ 19, so narrow to left half:
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(yes, we could stop here because we’ve found it...)
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Computers aren't so clever, so we do a simplified version: Repeatedly chop the array in half to close in on where the element must be. E.g., to search for 17 in:

2 3 5 7 11 13 17 19 23 29  

Find the mid-point: $17 > 11$, so narrow to right half:

Find the mid-point: $17 \leq 19$, so narrow to left half:

Find the mid-point: $17 \leq 17$, so narrow to left half:  
(yes, we could stop here because we’ve found it…)

Find the mid-point: $17 > 13$, so narrow to right half:

Now we’re left with an array of size 1, so either its element is 17 and we’ve found it, or 17 isn’t there.
Binary search

```c
int BinarySearch(int n, int a[], int sKey)
/* Assumes: elements of array a are in ascending order.
   * Returns TRUE iff sKey is contained in the array, i.e.,
   * there exists an index i with 0 <= i < n and a[i] == sKey.
   */
{
    /* Precondition: (n > 0)
       AND a[0] <= a[1] <= ... <= a[n-1] */
    int i, j, m;
    /* i will be the start of the sub-array
       * we’re currently chopping;
       * j will be the end of it (its last element);
       * m will be the mid-point of it.
       */
    i = 0;
    j = n - 1;
```
/* Invariant just before (re-)entering loop: $i \leq j$ AND
   * if sKey is in $a[0:n-1]$ then sKey is in $a[i:j]$ */

while ($i < j$) {
    $m = \frac{(i + j)}{2}$;
    if (sKey $\leq a[m]$) {
        $j = m$;
    } else {
        $i = m + 1$;
    }
}

/* After exiting loop:
   * ($i \geq j$), by $i$, $j$ updates, means ($i == j$).
   * now EITHER $a[i] == sKey$ OR sKey is not in $a[0:n-1]$ */
return $a[i] == sKey$;

Note how we return true/false ...
Running time

The (worst-case) running time of a function (or algorithm) is defined to be the maximum number of steps that might be performed by the program as a function of the input size.

- For functions which take an array (of some basic type) as the input, the length of the array (n in lots of our examples) is usually taken to represent size.
- The running time of Linear Search proportional to n (i.e., around $c \cdot n$ for some constant $c$), and the running time of Binary Search is proportional to $\lg(n)$. 
Measuring running time on a machine

```c
#include <time.h>

Bool_t flag = FALSE;
int a[24000000];
clock_t start, stop;
double t;
...
start = clock();
flag = LinearSearch(a, 24000000, -5);
stop = clock();
t = ((double)(stop-start))/CLOCKS_PER_SEC;
printf("Time spent by Linear Search was %lf seconds.\n", t);
...
```

On my laptop:

Time spent by LinearSearch was 0.069064 seconds.
Time spent by BinarySearch was 0.000001 seconds.
Measuring running time on a machine

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#include <time.h>
Bool_t flag = FALSE;
int a[24000000];
clock_t start, stop;
double t;
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start = clock();
flag = LinearSearch(a, 24000000, -5);
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t = ((double)(stop-start))/CLOCKS_PER_SEC;
printf("Time spent by Linear Search was %lf seconds.\n", t);
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On my laptop:

Time spent by LinearSearch was 0.069064 seconds.
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```
Sorting

Given an array of integers (or any comparable type), re-arrange the array so that the items appear in increasing order.
Bubble sort

‘Pseudo-code’

```cpp
for (i = n - 1; i >= 1; i--) {
    /* Rearrange the contents of 
        * array elements a[0], ..., a[i], 
        * so that the largest value appears 
        * in element a[i]. 
    */
}
```
Bubble sort

‘Pseudo-code’

```c
for (i = n - 1; i >= 1; i--)
{
    /* Rearrange the contents of
    * array elements a[0], ..., a[i],
    * so that the largest value appears
    * in element a[i].
    */
}
```

‘Method’:

- Find the largest item, and move it to the end;
- `repeat` for 2nd largest item, and so on . . .
Bubble sort (cont’d)

The task of rearranging the contents of array elements $a[0]$, $a[1]$, ..., $a[i]$ so that the largest value appears in element $a[i]$, may be handled by the following simple loop:

```c
for (j = 0; j < i; j++) {
    if (a[j] > a[j+1]) {
        swap(&a[j], &a[j+1]);
    }
}
```

(The largest value supposedly ‘bubbles’ up the array into its appropriate position.)
Bubble sort code

/* Sorts $a[0]$, $a[1]$, ..., $a[n-1]$ into ascending order. */
void BubbleSort(int a[], int n) {
    int i, j;
    for (i = n - 1; i >= 1; i--) {
        /* Invariant: The values in locations to the right of
         * $a[i]$ are in their correct resting places: they are
         * the $(n - i - 1)$-largest elements arranged in
         * positions $(i+1)$, ..., $(n-1)$, in non-descending order.
         */
        for (j = 0; j < i; j++) {
            if (a[j] > a[j+1]) {
                swap(&a[j], &a[j+1]);
            }
        }
    }
}

The swap function used above is the (correct) one from lab 5.

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Running time of Bubble Sort

The (worst case) running time of Bubble Sort is proportional to $n^2$. Why?

There are better sorting algorithms . . . for example, *MergeSort* or *HeapSort* run in time proportional to $n \lg(n)$.

For general purpose sorting, often use *QuickSort*, which runs in time around $n \lg n$ in most cases, though in bad cases (which?) it can take $n^2$. Standard C systems provide QuickSort as *qsort*. Occasionally you might know that BubbleSort would be quicker in your application, and want to program it. Anything else is probably specialist.

*More about Bubble-Sort can be found in Section 6.7 of ‘A Book on C’.*
Understanding your loops

These slides are logically small and green: for the mathematically and logically inclined only!

- How can you show that a program is correct?
- One way is to show that certain statements are true at all times in the program (*invariants*)
- In particular, to understand a complex *while*/*for*-loop, it’s useful to know what remains true every time you go through it.
- For functions (or other blocks of code) we have *preconditions* (things *assumed* be true before) and *postconditions* (things which *will* be true afterwards given the preconditions).

We’ll do a simple example now; then look (in your own time) at the comments in the searching and sorting code, and try to understand what they’re saying about invariants.
Power of a number

int Power(int n, int k)
/* Pre-condition: k >= 0. */
/* On-exit: returns $n^k$ (n raised to the power k). */
{
  int p = 1, i = k;
  /* Invariant before (re-)entering:
   * i >= 0 AND p * n^i == n^k */
  while (i > 0) {
    p *= n;
    --i;
  }
  /* After exiting loop: i <= 0 AND p = n^k */
  return p;
}

Warning: $n^k$ in the comments is maths notation, not C notation. In C, the ^ symbol is the bitwise exclusive-or operator, something entirely different!
Example: $n = 3$, $k = 4$. The answer should be $3^4 = 81$. The computation progresses as follows. Initially, $i = k$ and $p = 1$. Note that $p \times n^i$ is invariant!

```c
/* Invariant before (re-)entering: 
i >= 0 AND p * n^i == n^k */
while (i > 0) {
    p *= n;
    --i;
}

/* After exiting loop: i <= 0 AND p = n^k */
return p;
```

<table>
<thead>
<tr>
<th></th>
<th>$i$</th>
<th>$p$</th>
<th>$p \times n^i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>4</td>
<td>1</td>
<td>$1 \times 3^4 = 81$</td>
</tr>
<tr>
<td>Iteration 1</td>
<td>3</td>
<td>3</td>
<td>$3 \times 3^3 = 81$</td>
</tr>
<tr>
<td>Iteration 2</td>
<td>2</td>
<td>9</td>
<td>$9 \times 3^2 = 81$</td>
</tr>
<tr>
<td>Iteration 3</td>
<td>1</td>
<td>27</td>
<td>$27 \times 3^1 = 81$</td>
</tr>
<tr>
<td>Iteration 4</td>
<td>0</td>
<td>81</td>
<td>$81 \times 3^0 = 81$</td>
</tr>
</tbody>
</table>

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Reading material

Sections of ‘A Book on C’ that are relevant are:

- A good idea to refresh your memory of arrays (early sections of Chapter 6).
- Section 6.7 has a discussion of BubbleSort.