Searching an array

typedef enum {FALSE, TRUE} Bool_t;

Bool_t LinearSearch(int n, int a[], int sKey)
/* Returns TRUE iff (if and only if) sKey is contained in
* the array, i.e., there exists an index i with 0 <= i < n
* such that a[i] == sKey.
*/
{
    int i;
    for (i = 0; i < n; ++i) {
        if (a[i] == sKey) return TRUE;
    }
    return FALSE;
}

variant:
- Could use return type int with #DEFINE for TRUE, FALSE (see BinarySearch)

Binary search

Sometimes we quickly want to find an entry in an array.
It helps if the array is sorted.

How do you search for a name in a telephone book?
Computers aren’t so clever, so we do a simplified version:
Repeatedly chop the array in half to close in on where the element must be.
E.g., to search for 17 in:
2 3 5 7 11 13 17 19 23 29
Find the mid-point: 17 > 11, so narrow to right half:
Find the mid-point: 17 <= 19, so narrow to left half:
Find the mid-point: 17 <= 17, so narrow to left half:
(yes, we could stop here because we’ve found it . . .)
Find the mid-point: 17 > 13, so narrow to right half:
Now we’re left with an array of size 1, so either its element is 17 and
we’ve found it, or 17 isn’t there.

int BinarySearch(int n, int a[], int sKey)
/* Assumes: elements of array a are in ascending order.
* Returns TRUE iff sKey is contained in the array, i.e.,
* there exists an index i with 0 <= i < n and a[i] == sKey.
*/
{
    /* Precondition: (n > 0)
        AND a[0] <= a[1] <= ... <= a[n-1] */
    int i, j, m;
    /* i will be the start of the sub-array
    * we’re currently chopping;
    * j will be the end of it (its last element);
    * m will be the mid-point of it.
    */
    i = 0;
    j = n - 1;
/* Invariant just before (re-)entering loop: i <= j AND * if sKey is in a[0:n-1] then sKey is in a[i:j] */
while (i < j) {
    m = (i + j)/2;
    if (sKey <= a[m]) {
        j = m;
    } else {
        i = m + 1;
    }
}
/* After exiting loop: * (i >= j), by i, j updates, means (i == j). * now EITHER a[i] == sKey OR sKey is not in a[0:n-1] */
return a[i] == sKey;

Note how we return true/false ... 

Running time
The (worst-case) running time of a function (or algorithm) is defined to be the maximum number of steps that might be performed by the program as a function of the input size.

- For functions which take an array (of some basic type) as the input, the length of the array (n in lots of our examples) is usually taken to represent size.
- The running time of Linear Search proportional to n (i.e., around $c \cdot n$ for some constant c), and the running time of Binary Search is proportional to $\log(n)$.

Measuring running time on a machine
#include <time.h>
Bool_t flag = FALSE;
int a[24000000];
clock_t start, stop;
double t;
...
start = clock();
flag = LinearSearch(a, 24000000, -5);
stop = clock();
t = ((double)(stop-start))/CLOCKS_PER_SEC;
printf("Time spent by Linear Search was %lf seconds.\n", t);
...
On my laptop:
Time spent by Linear Search was 0.069064 seconds.
Time spent by BinarySearch was 0.000001 seconds.

Sorting
Given an array of integers (or any comparable type), re-arrange the array so that the items appear in increasing order.
Bubble sort

'Pseudo-code'

```c
for (i = n - 1; i >= 1; i--) {
    /* Rearrange the contents of
     * array elements a[0], ..., a[i],
     * so that the largest value appears
     * in element a[i].
     */
}

'Method':
▶ Find the largest item, and move it to the end;
▶ repeat for 2nd largest item, and so on...
```

Bubble sort (cont'd)

The task of rearranging the contents of array elements `a[0]`, `a[1]`, ..., `a[i]` so that the largest value appears in element `a[i]`, may be handled by the following simple loop:

```c
for (j = 0; j < i; j++) {
    if (a[j] > a[j+1]) {
        swap(&a[j], &a[j+1]);
    }
}
```

(The largest value supposedly ‘bubbles’ up the array into its appropriate position.)

Running time of Bubble Sort

The (worst case) running time of Bubble Sort is proportional to $n^2$. why?

There are better sorting algorithms . . . for example MergeSort or HeapSort run in time proportional to $n \log(n)$.

For general purpose sorting, often use QuickSort, which runs in time around $n \log n$ in most cases, though in bad cases (which?) it can take $n^2$. Standard C systems provide QuickSort as `qsort`. Occasionally you might know that BubbleSort would be quicker in your application, and want to program it. Anything else is probably specialist.

More about Bubble-Sort can be found in Section 6.7 of ‘A Book on C’.

Bubble sort code

```c
/* Sorts a[0], a[1], ..., a[n-1] into ascending order. */
void BubbleSort(int a[], int n) {
    int i, j;
    for (i = n - 1; i >= 1; i--) {
        /* Invariant: The values in locations to the right of
         * a[i] are in their correct resting places: they are
         * the (n - i - 1)-largest elements arranged in
         * positions (i+1), ..., (n-1), in non-descending order.
        */
        for (j = 0; j < i; j++) {
            if (a[j] > a[j+1]) {
                swap(&a[j], &a[j+1]);
            }
        }
    }
}
```

The swap function used above is the (correct) one from lab 5.
Understanding your loops
These slides are logically small and green: for the mathematically and logically inclined only!

▶ How can you show that a program is correct?
▶ One way is to show that certain statements are true at all times in the program (invariants)
▶ In particular, to understand a complex while/for-loop, it’s useful to know what remains true every time you go through it.
▶ For functions (or other blocks of code) we have preconditions (things assumed be true before) and postconditions (things which will be true afterwards given the preconditions).

We’ll do a simple example now; then look (in your own time) at the comments in the searching and sorting code, and try to understand what they’re saying about invariants.

Example: \( n = 3, \ k = 4 \). The answer should be \( 3^4 = 81 \).
The computation progresses as follows. Initially, \( i = k \) and \( p = 1 \). Note that \( p \times n^i \) is invariant!

\[
\text{Power of a number}
\]

\[
\text{int Power(int n, int k)}
\]

\[
/* \text{Pre-condition: } k \geq 0. */
/* \text{On-exit: returns } n^k \text{ (n raised to the power } k). */
\]

\[
\{ \text{int p = 1, i = k;}
/* \text{Invariant before (re-)entering:}
* \text{i } \geq 0 \ \text{AND } p \times n^i = n^k */
\]

\[
\text{while } (i > 0) \{ \text{p } *= n;
-\text{i;}
\}
/* \text{After exiting loop: } i \leq 0 \ \text{AND } p = n^k */
\]

\[
\text{return p;}
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{i} & \text{p} & \text{p } \times n^i \\
\hline
\text{Initial} & 4 & 1 \times 3^4 = 81 \\
\hline
\text{Iteration 1} & 3 & 3 \times 3^3 = 81 \\
\hline
\text{Iteration 2} & 2 & 9 \times 3^2 = 81 \\
\hline
\text{Iteration 3} & 1 & 27 \times 3^1 = 81 \\
\hline
\text{Iteration 4} & 0 & 81 \times 3^0 = 81 \\
\hline
\end{array}
\]

Warning: \( n^k \) in the comments is maths notation, not C notation. In C, the ^ symbol is the bitwise exclusive-or operator, something entirely different!

Reading material
Sections of 'A Book on C' that are relevant are:

▶ A good idea to refresh your memory of arrays (early sections of Chapter 6).
▶ Section 6.7 has a discussion of BubbleSort.