Vectorisation

Michael O'Boyle

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School of informatics

Course Structure

- Course work deadline today. New coursework today - see website
- 4/5 lectures on high level restructuring for parallelism and memory
- Dependence Analysis
- Program Transformations
- **Automatic vectorisation** ch2 and 5 of Allen and Kennedy
- Automatic parallelisation
- Speculative Parallelisation

Lecture Overview

- Vector loops - how to write loops in a vector format
- Loop distribution + statement reordering: basic vectorisation
- Dependence condition for vectorisation: Based on loop level
- Kennedy's Vectorisation algorithm based on SCC and hierarchical dependences
- Loop Interchange: Move vector loops innermost
- Scalar Expansion, Renaming and Node splitting. Overcoming cycles

Vector code

- Use Fortran 90 vector notation to express vectorised loops.
- Triple notation used x(start:finish:step) to represent a vector in x
- Vectorisation depends on loop dependence

```
Do i = 1,N
  x(i) = x(i) +c
Enddo
```

No loop carried dependence [0]
Vectorisable

```
x(1:N) = x(1:N) +c
```

```
Do i = 1,N
  x(i+1) = x(i) +c
Enddo
```

Loop carried dependence [1]
Not vectorisable

Vector code: varying vector length

Vector registers are a fixed size. Need to fit code to registers

```
Do i = 1,N
  x(i) = x(i) +c
Enddo
```

Original

```
Do i = 1,N,s
 Do ii = i, i+s-1
  x(ii) = x(ii) +c
 Enddo
Enddo
```

Strip-mine

```
Do i = 1,N,s
 x(i:i+s-1) = x(i:i+s-1) +c
Enddo
```

Vectorise

Loop Distribution + Statement reordering

Standard approach to isolating statements within a loop for later vectorisation

```
Do i = 1,N
  a(i+1) = b(i) +c
  d(i) = a(i) +c
Enddo
```

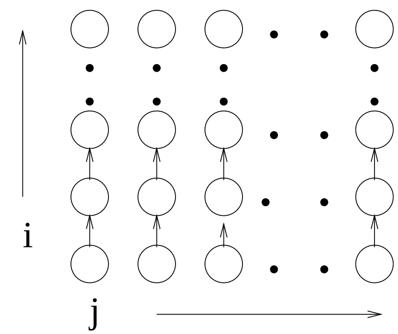
```
Do i = 1,N
  a(i+1) = b(i) +c
Enddo
Do i = 1,N
  d(i) = a(i) +c
Enddo
```

```
  a(2:N+1) = b(1:N) +c
  d(1:N) = a(1:N) +e
```

Cyclic dependence prevent distribution and hence vectorisation. Examine techniques to overcome this.

Inner loop vectorisation

```
Do i = 1,N
 Do j = 1,M
    a(i+1,j) = a(i,j) +c
 Enddo
Enddo
```



Cannot vectorise as dependence (1,0). If outer loop run sequential then can vectorise inner loop with dep (0). Generalises to nested loops.

```
Do i = 1,N
    a(i+1,1:M) = a(i,1:M) +c
Enddo
```

Vectorisation algorithm

- Simple description of Ch2 algorithm. Look at Ch2 for more details
- Form dependence graph
- Strongly Connected Component (SCC) identification (cycles)
- Separate out weakly connected and vectorise using loop distribution and statement reordering
- Strip off outer dependence level (loop will be sequentialised) and repeat

Running Example

```
  Do i = 1,100
s1  x(i) = y(i) +10
    Do j = 1,100
s2    b(j) = a(j,n)
      Do k = 1,100
s3      a(j+1,k) = b(j) +c(j,k)
      Enddo
s4    y(i+j) = a(j+1,n)
   Enddo
```

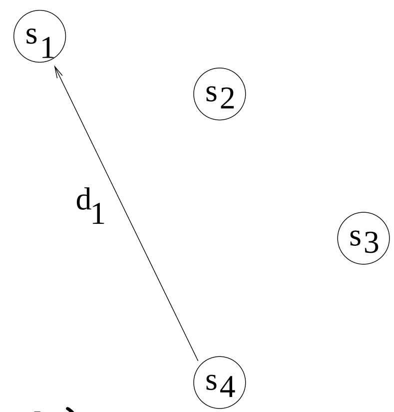
Use d notation where d_x^y is a dependence of type y at loop level x.

Loops numbered from outermost x=1 ... Infinity means within a loop, not loop carried. y=o output, y=-1 anti else flow.

Loop carried flow dependence from s4 to s1 on y. d_1

Running Example with S1 dependences

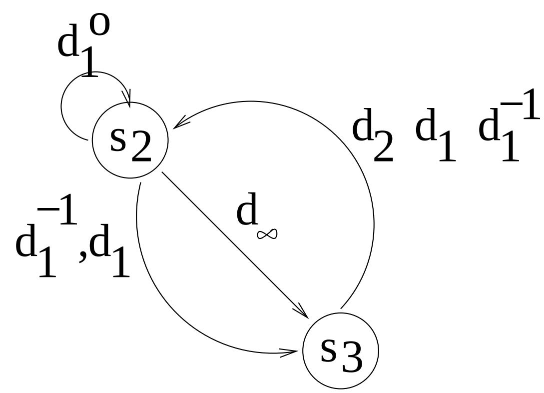
```
  Do i = 1,100
s1  x(i) = y(i) +10
    Do j = 1,100
s2    b(j) = a(j,n)
      Do k = 1,100
s3      a(j+1,k) = b(j) +c(j,k)
      Enddo
s4    y(i+j) = a(j+1,n)
   Enddo
```



Loop carried flow dependence from s4 to s1 on y. d_1

No other dependences reach s1

Running Example S2 dependences

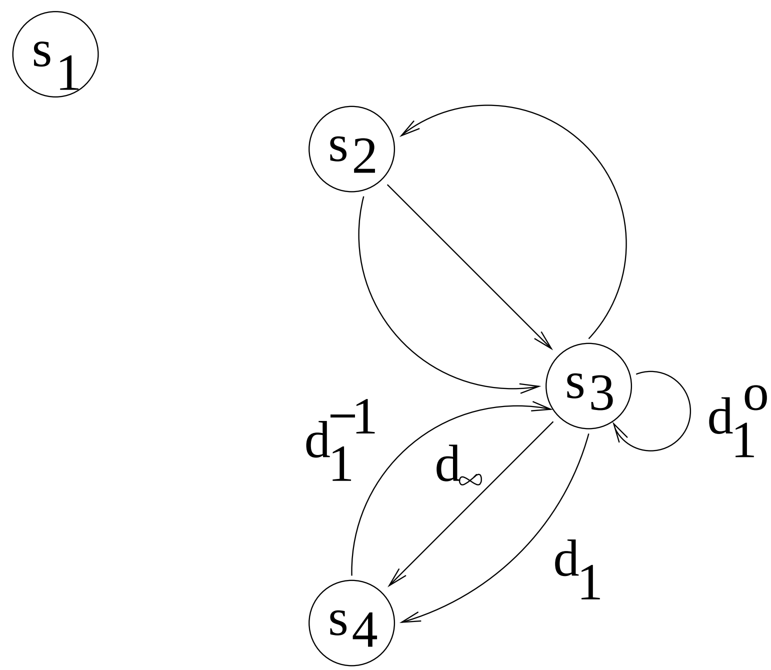


b(j) in s2 has two flow dependences with s3 . Loop carried and loop independent $d_1.d_{\texttt{inf}}$.

Corresponding loop carried anti dep from s3 to s2 d_1^{-1}. Finally loop carried output dependence in s2

a(j+1,k) in s3 has a level one and two flow dep with s2. Corresponding loop carried antidep from s2 to s3.

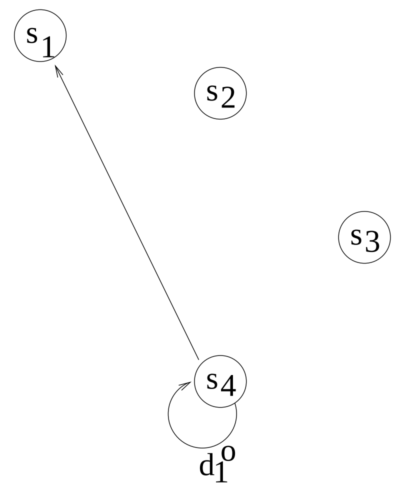
Example S3 dependences



Loop carried and independent flow dependence from a(j+1,k) in s3 to s4

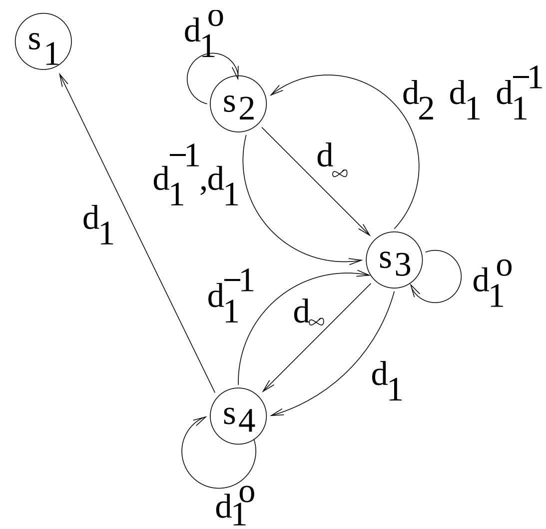
Corresponding loop carried anti-dep from s4. Output dependence

Example S4 dependences



- Trivial loop carried output dependence in s4 at level 1 on write to y(i+j).
- Other dependence with s1 already shown

Putting it all together



- Analysing connected components using Tarjan's algorithm. Two separate groups (s1),(s2,s3,s4).
- Separate out s1 by loop distribution. Statement reordering required here.

Vectorisation algorithm

Apply on outer level vectorise (s1,s2,s3,s4,1).

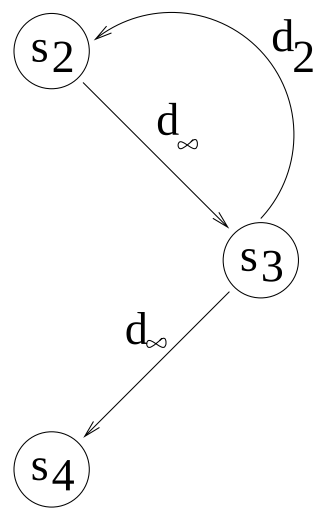
s1 not part of SC. Loop distribution and statement reordering and vectorised gives

```
   Do i = 1,100
    vectorise ({s2,s3,s4},2)
   Enddo
   Do i = 1,100
 s1     x(i) = y(i) +10
   Enddo
```

```
Do i = 1,100
 vectorise ({s2,s3,s4},2)
Enddo
x(1:100) = y(1:100) +10
```

Apply algorithm at next level stripping of level 1 dependences

Vectorise({s2,s3,s4},2) level 1 dependences stripped off



- Analysing connected components using Tarjan's algorithm. Two separate groups (s4),(s2,s3).
- Separate out s4 by loop distribution. No Statement reordering required here.

Vectorisation algorithm

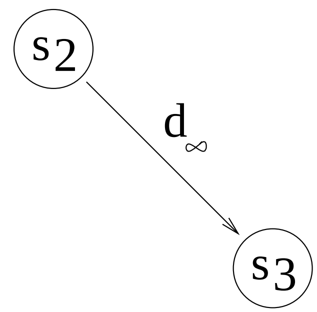
Apply on level2 vectorise (s1,s2,s3,2).

s4 not part of SCC. Isolated, distributed and vectorised giving

```
   Do i = 1,100
    Do j = 1,100
     vectorise({s2,s3},3)
    Enddo
s4  y(i+1:i+100) = a(2:101,N)
   Enddo
s1 x(1:100) = y(1:100) +10
```

- Apply vectorise again striping off next level

Vectorise({s2,s3},3) level 1,2 dependences stripped off



- Analysing connected components using Tarjan's algorithm. Two separate groups (s2),(s3).
- Separate out by loop distribution. No Statement reordering required here.

Vectorisation algorithm : Final code

```
    Do i = 1,100
     Do j = 1,100
s2    b(j) = a(j,n)
s3    a(j+1,1:100) = b(j) +c(j,1:100)
     Enddo
s4   y(i+1:i+100) = a(2:101,N)
    Enddo
s1  x(1:100) = y(1:100) +10
```

As s2 has no loop of depth 3,distribution leaves a single statement.

- What happened if no vectorisable regions found?

- Try transformations

Loop Interchange: move loop carried dependences outermost

```
 Do j = 1,M
   Do i = 1,N
    a(i+1,j) = a(i,j) +c
 Enddo
Enddo
```

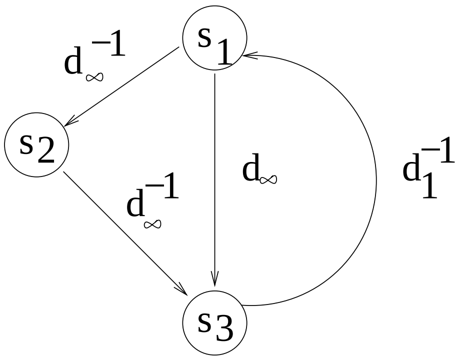
[0,1] even if j run sequentially, loop carried dep - i not vectorisable.

```
 Do i = 1,N
   Do j = 1,M
     a(i+1,j) = a(i,j) +c
  Enddo
 Enddo
```

```
Do i = 1,N
    a(i+1,1:N) = a(i,1:N) +c
Enddo
```

Now [1,0] - inner loop vectorisable

Scalar expansion

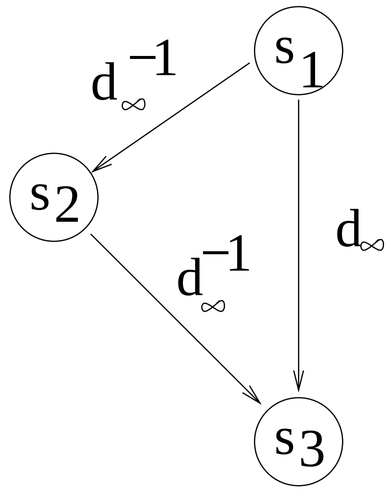



```
Do i = 1,N
 t  = a(i)
 a(i) = b(i)
 b(i) =t
Enddo
```

Cycle in dependence graph prevents distribution and vectorisation (output not shown)

Try to eliminate anti-dependence with scalar expansion

Anti-dependence removed eliminating cycle




```
Do i = 1,N
 tt(i)  = a(i)
 a(i) = b(i)
 b(i) =tt(i)
Enddo
t =tt(N)
```

Can now be easily distributed and vectorised

Scalar expansion :may fail with subsequent uses

```
Do i =1,N
 t= t+a(i) +a(i+1)
 a(i) =t
Enddo
```

```
tt(0) =t
Do i =1,N
 tt(i)= tt(i-1)+a(i) +a(i+1)
 a(i) =tt(i)
Enddo
t= tt(N)
```

- Whether or not scalar expansion can break cycles depends on whether it is a covering definitions

- A covering definition for a use means that there are subsequent later uses.

- In practise recurrence on the scalar is the biggest problem.

Scalar Renaming

- Can be used to eliminate loop independent output and anti-dependences

```
Do i =1,N
 t= t+a(i) +b(i)
 c(i) = t + t
 t = d(i) - b(i)
 a(i+1) = t * t
Enddo
```

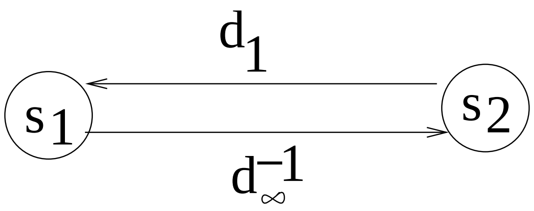
```
Do i =1,N
 t1= t+a(i) +b(i)
 c(i) = t1 + t1
 t2 = d(i) - b(i)
 a(i+1) = t2 * t2
Enddo
```

- Scalar expansion, loop distribution and vectorisation now possible

Node Splitting

- Scalar expansion and renaming cannot eliminate all cycles

```
Do i =1,N
 a(i)  = x(i+1) +x (i)
 x(i+1)= b(i)+ t
Enddo
```

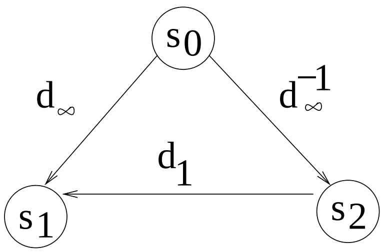


- Renaming does not break cycle. Critical anti-dependence

Node Splitting

- Make copy of node where anti-dep starts

```
Do i =1,N
 xx(i) = x(i+1)
 a(i)  = xx(i) +x (i)
 x(i+1)= b(i)+ t
Enddo
```



- Cycle broken. Vectorisable with statement reordering:s0,s2,s1

- NP-C to find minimal critical deps !

Summary

- Vector loops
- Loop distribution
- Dependence condition for vectorisation
- Vectorisation algorithm based on SCC and hierarchical dependences
- Loop Interchange
- Scalar Expansion, Renaming and Node splitting
- Used in Media SIMD instructions/ GPUs