Program Transformations

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Course Structure

- L1 Introduction and Recap, L2 Course Work
- 4 lectures on high level restructuring for parallelism and memory
- Dependence Analysis
- **Program Transformations - loop and arrays**
- Automatic vectorisation
- Automatic parallelisation

Lecture Overview

- Classification of program transformations - loop and array
- Role of dependence
- Loop restructuring - changing the number/type of loop
- Iteration reordering - reordering the iterations scanned.
- Array transformations - data layout transformation
- Simplified presentation. Large number of technicalities. Applicability. Worth.

References

- Loop Distribution with arbitrary control-flow McKinley and Kennedy Supercomputing 1990
- D.F. Bacon, S.L. Graham, and O.J. Sharp. Compiler Transformations for High-Performance Computing. ACM Computing Surveys, 26(4), 1994.
- A Framework for Unifying Reordering Transformations (1993) TR
- On the Complexity of Loop Fusion Alain Darte, PACT 1999
- L. Lamport. The parallel execution of do loops. Communications of the ACM, pages 83–93, February 1974.

What is a program transformation

- A program transformation is a rewriting of the program such that it has the same semantics
- More conservatively, all data dependences must be preserved
- Previous lectures looked at IR→IR transformations or assembler→assembler transformations
- Focus on transformations in the high level source prog. language: source to source transformations
- Why: Only place where memory reference explicit. Key to restructuring for memory behaviour and large scale parallelism.

Classification

Ongoing open question on a correct taxonomy

- Loop
  - Structure reordering. Change number of loops
  - Iteration reordering. Reorder loop traversal
  - Linear models. Express transformation as unimodular matrices.
- Array
  - Index reordering
  - Duality with loops. Global vs Local.
- All transformations have an associated legality test though some a few are always legal.

Loop Restructuring Index Splitting

Always a legal transformation. No test needed

```
Do i = 1, 100
 a(101 -i) =a(i)
Enddo
```

```
Do i = 1, 50
 a(101 -i) =a(i)
Enddo

Do i = 51, 100
 a(101 -i) =a(i)
Enddo
```

A sequential loop with dependence [*] is transformed into two independent parallel loops. Careful selection of split point.

Neither access in each loop refers to same memory location.

All of first loop must execute before second though - why?

Loop Restructuring: Loop Unrolling

Used for exploiting Instruction Level Parallelism

Always legal - take care of epilogue using index splitting

```
Do i = 1, 100
 a(i) = i
Enddo
```

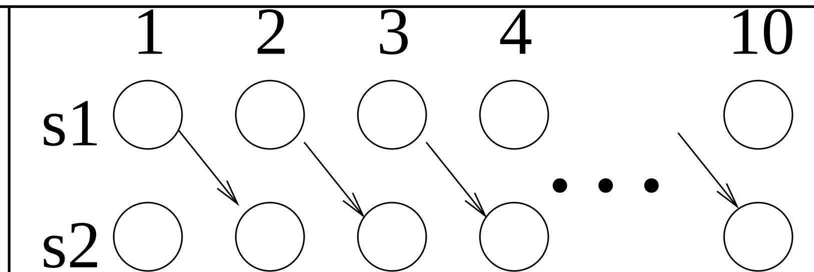
```
Do i = 1, 100, 3
 a(i) = i
 a(i+1) = i+1
 a(i+2) = i+2
Enddo

Do i = 100,100
 a(i) = i
Enddo
```

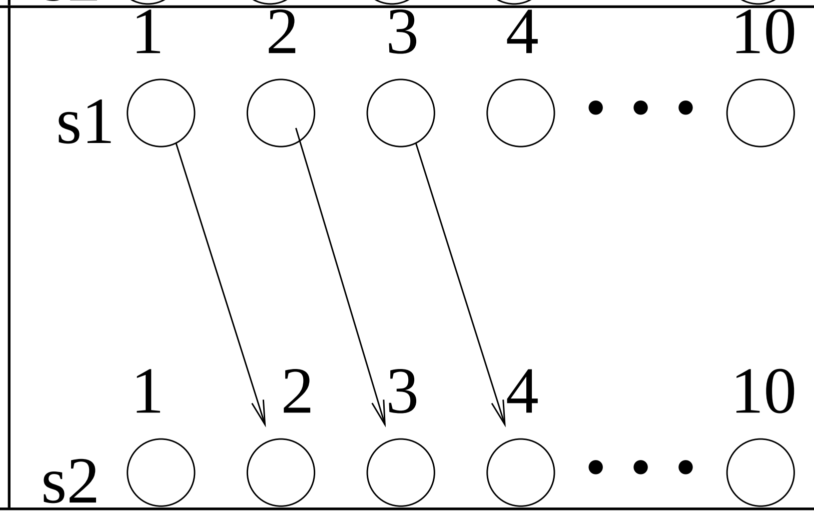
Non-convex iteration space after transformation - steps. Causes difficulties for dependence analysis. Can normalise loop though

Loop Restructuring: Loop Distribution

```
Do i = 1,10
  a(i) =
        =a(i-1)
Enddo
```

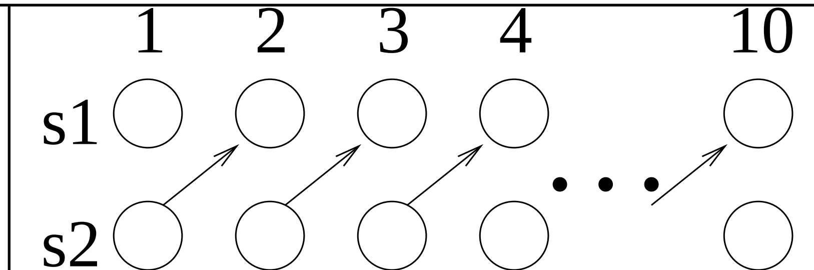



```
Do i = 1,10
  a(i) =
Enddo
Do i = 1,10
        =a(i-1)
Enddo
```

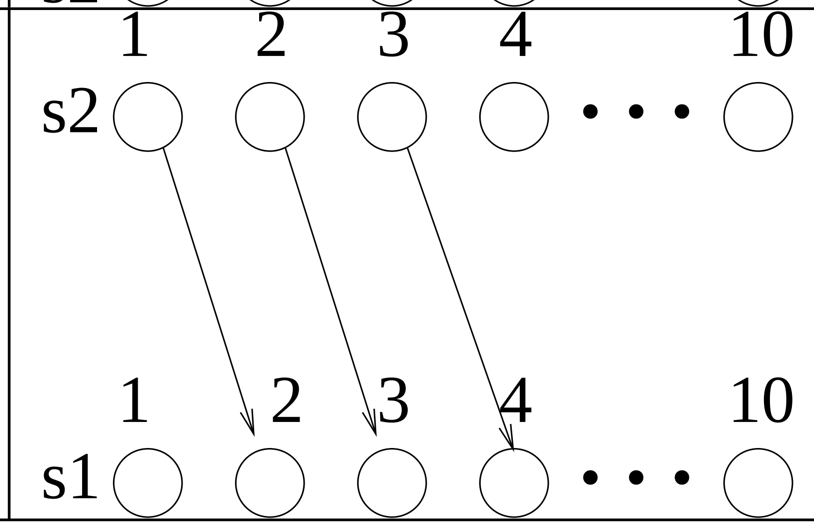


Loop Distribution + Statement Reordering

```
Do i = 1,10
  a(i) =
        =a(i+1)
Enddo
```




```
Do i = 1,100
        =a(i+1)
Enddo
Do i = 1,10
  a(i) =
Enddo
```



Anti-dependences honoured.

Loop Restructuring: Loop Fusion

Inverse of loop distribution - needs conformant loops

`Do i = 1,100`
`   a(i) =`
`Enddo`
`Do j = 1,100`
`  b(j) =`
`Enddo`
	`Do i = 1,100`
	`   a(i) =`
	`   b(i) =`
	`Enddo`

More difficult than distribution. Dependence constrains application.

Used for increasing ILP and improving register use. Also for fork/join based parallelisation.

Loops can be partly fused after pre-distribution

Iteration reordering: Loop interchange

Important widely used transformation

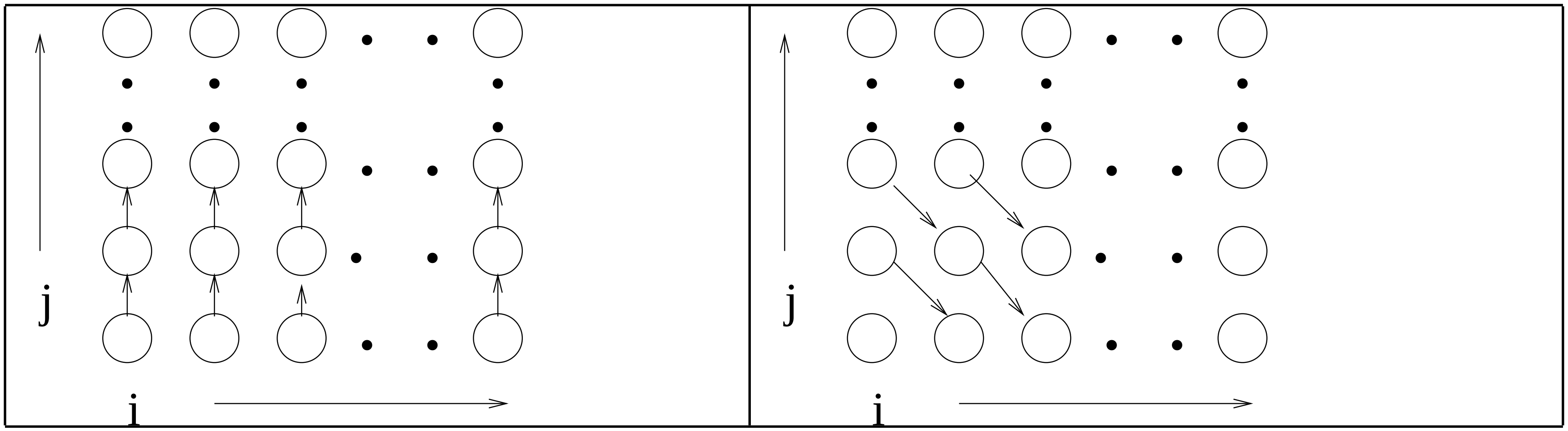
`Do i =1, N`
` Do j = 1,N`
`   a(i,j) = a(i,j-1) +b(i)`
` Enddo`
`Enddo`
	`Do i =1, N`
	`  Do j = 1,N`
	`    a(i,j) = a(i-1,j+1) +b(i)`
	`  Enddo`
	`Enddo`

`Do j =1, N`
`  Do i = 1,N`
`    a(i,j) = a(i,j-1) +b(i)`
`  Enddo`
`Enddo`	

$[i, j] \mapsto [j, i]$

Illegal to interchange [1,-1], ,$[<, >]$ why?

Iteration reordering: Loop interchange



Illegal to interchange [1,-1]: New vector [-1,1]:

Impossible dependence.

Linear models check $TD > 0$

Loop skewing

Always legal used in wavefront parallelisation

<pre>Do i =1, N
  Do j = 1,N
    a(i,j) = a(i,j-1)
           +b(i)
  Enddo
Enddo</pre>
	<pre>Do i =1, N
	  Do j = i+1,i+N
	    a(i,j-i) = a(i,j-i-1)
	             +b(i)
	  Enddo
	Enddo</pre>

- $[i, j] \mapsto [i, j + i]$
- Equivalent to a change of basis.
- Shifting by a constant referred to as loop bumping

Loop reversal

<pre>Do i =1, N
  Do j = 1,N
    a(i,j) = a(i,j-1) +b(i)
  Enddo
Enddo</pre>
	<pre>Do i =N, 1, -1
	  Do j = 1,N
	    a(i,j) = a(i,j-1) +b(i)
	  Enddo
	Enddo</pre>

- $[i, j] \mapsto [-i, j]$
- Rarely used in isolation. In unison with previous two.
- Can combine interchange, shewing and reversal as unimodular transformations/ More on this later.

Tiling = strip-mining plus interchange

```
Do i =1, N
  Do j = 1,N
    a(i,j) = a(i,j) +b(i)
  Enddo
Enddo
```

```
Do i =1, N,s
  Do j = 1,N,s
   Do ii = i+1, i+s-1
     Do jj = j+1,j+s-1
      a(ii,jj) = a(ii,jj) +b(ii)
     Enddo
   Enddo
  Enddo
Enddo
```

```
  Do i =1, N
    Do j = 1,N,s
      Do jj = jj+1,jj+s-1
       a(i,jj) = a(i,jj)+b(i)
     Enddo
    Enddo
Enddo
```

Strip-mine by factor s Non-convex space Interchange placing smaller strip-mine inside

Array layout transformations

- Less extensive literature though perhaps have a more significant impact
- Loop transformations affect all memory references within the loop but not elsewhere. Local in nature
- Array and more generally data transformations have global impact but do not affect other references to other arrays.
- Array layout transformations are used to improve memory access performance
- Also form the basis for data distribution based parallelisation schemes for distributed memory machines.

Global index reordering

Dual of loop interchange. Always legal! $[i_i, i_2] \mapsto [i_2, i_1]$

REAL A[10,20]
Do i =1, 10
 Do j = 1,20
  a(i,j) = a(i+1,j-1) +b(i)
 Enddo
Enddo
a(1,2) =0
	REAL A[20,10]
	Do i =1, 10
	 Do j = 1,20
	   a(j,i) = a(j-1,i+1) +b(i)
	 Enddo
	Enddo
	a(2,1) =0

- Array declaration and subscripts interchanged globally
- Difficulties occur if array reshaped on procedure boundaries

Linearisation/delinearisation

Dual of loop strip-mining/linearisation

`REAL a[10,20]`
`Do i =1, 10`
` Do j = 1,20`
`  a(i,j) = a(i+1,j-1) +b(i)`
` Enddo`
`Enddo`
`a(1,2) =0`
	`REAL a[200]`
	`Do i =1, 10`
	` Do j = 1,20`
	`  a(20*(i-1)+j) = a(20*(i)+j-1)`
	`               +b(i)`
	` Enddo`
	`Enddo`
	`a(2) =0`

Padding

`REAL A[10,20]`
`Do i =1, 10`
` Do j = 1,20`
`   a(i,j) = a(i+1,j-1) +b(i)`
` Enddo`
`Enddo`
`a(1,2) =0`
	`REAL A[17,20]`
	`Do i =1, 10`
	` Do j = 1,20`
	`   a(i,j) = a(i+1,j-1) +b(i)`
	` Enddo`
	`Enddo`
	`a(1,2) =0`

- Frequently used to overcome cache conflicts. Very simple
- Pad factor 7 in first index. Normally prime.

Unification

- Presentation - simplistic conditions of application can be complex for arbitrary programs.
- Little overall structure.
- Unimodular transformation theory based on linear representation
- Extended to non-singular and the Unified Transformation Framework of Bill Pugh.
- Will return to look in more detail at this formulation in later lectures.

Summary

- Large suite of transformations
- Loop restructuring and reordering
- Legality constraints restrict application
- Array based transformations.Always legal but global impact
- Unifying theories provide structured taxonomy.
- Next lecture: Vectorisation