This lecture:

- Classification of program transformations - loop and array
- Role of dependence
- Loop restructuring - changing the number/type of loop
- Iteration reordering - reordering the iterations scanned.
- Array transformations - data layout transformation

NB: Simplified presentation.

*Large* number of technicalities.

**Read the book!**
A program transformation is a rewriting of the program such that it has the same semantics.

More conservatively, all data dependences must be preserved.

Previous lectures looked at IR→IR transformations or assembler→assembler transformations.

Now, focus on transformations at higher level: source to source transformations.

Why: Only place where memory reference explicit. Key to restructuring for memory behaviour and large scale parallelism.
Ongoing open question on a correct taxonomy

- Loop
  - Structure reordering. Change number of loops
  - Iteration reordering. Reorder loop traversal
  - Linear models. Express transformation as uni-modular matrices.

- Array
  - Index reordering
  - Duality with loops. Global vs Local.

- All transformations have an associated legality test though some a few are always legal.
Loop restructuring
Transformation: index splitting

- A sequential loop with dependence [*] is transformed into two independent parallel loops. Careful selection of split point.
- Always a legal transformation. No test needed

Original
for(i = 1 to 100)
  a[101 - i] = a[i]

Split at $i = 51$
for(i = 1 to 50)
  a[101 - i] = a[i]
for(i = 51 to 100)
  a[101 - i] = a[i]

Lots of dependences

- Neither access in each loop refers to same memory location.
- All of first loop must execute before second though - why?
Loop restructuring
Transformation: loop unrolling

- Replicate loop body
- Used for exploiting ILP
- Always a legal transformation. No test needed

Original
```
for(i = 1 to 100)
  a[i] = i
```

Unroll 3 times
```
for(i = 1 to 100 step 3)
  a[i] = i
  a[i+1] = i+1
  a[i+2] = i+2

for(i = 100 to 100)
  a[i] = i
```

- Non-convex iteration space after transformation - steps
- Causes difficulties for dependence analysis.
- Can normalise loop though
Loop restructuring
Transformation: loop distribution

- Move loop statements into their own loops

**Original**

```plaintext
for(i = 1 to 10)
a[i] = S_1
   = a[i-1] S_2
```

**Distributed**

```plaintext
for(i = 1 to 10)
a[i] = S_1

for(i = 1 to 10)
   = a[i-1] S_2
```
Loop restructuring
Transformation: loop distribution + statement reordering

- Anti-dependences honoured

Original

for(i = 1 to 10)
  a[i] = \( S_1 \)
  = a[i+1] \( S_2 \)

Distributed

for(i = 1 to 10)
  = a[i+1] \( S_2 \)
  
for(i = 1 to 10)
  a[i] = \( S_1 \)
Loop restructuring
Transformation: loop fusion

- Inverse of loop distribution - needs compatible loops

**Original**

```plaintext
for(i = 1 to 100)
  a[i] =

for(j = 1 to 100)
  b[j] =
```

**Fused**

```plaintext
for(i = 1 to 100)
  a[i] =
  b[i] =
```

- More difficult than distribution. Dependence constrains application.
- Used for increasing ILP and improving register use. Also for fork/join based parallelisation.
- Loops can be partly fused after pre-distribution.
Iteration reordering
Transformation: loop interchange

- Switching the order of nested loops
- Important widely used transformation

**Original**
```
for(i = 1 to N)
    for(j = 1 to N)
        a[i,j]=a[i,j-1]+b[i]
```

**Interchanged**
```
for(j = 1 to N)
    for(i = 1 to N)
        a[i,j]=a[i,j-1]+b[i]
```

- \([i, j] \mapsto [j, i]\)
Iteration reordering
Transformation: loop interchange

- Switching the order of nested loops
- Important widely used transformation

Original

```
for(i = 1 to N)
  for(j = 1 to N)
    a[i,j]=a[i-1,j+1]+b[i]
```

Interchanged

```
for(j = 1 to N)
  for(i = 1 to N)
    a[i,j]=a[i-1,j+1]+b[i]
```

- $[i,j] \mapsto [j,i]$
- Illegal to interchange $[1,-1]$, $[<,>]$ why?
Iteration reordering
Transformation: loop skewing

- Used in wavefront parallelisation

Original

\[
\text{for}(i = 1 \text{ to } N) \\
\quad \text{for}(j = 1 \text{ to } N) \\
\qquad a[i,j] = a[i-1,j] + a[i,j-1]
\]

Skewed

\[
\text{for}(i = 1 \text{ to } N) \\
\quad \text{for}(j = i+1 \text{ to } i+N) \\
\qquad a[i,j-i] = a[i-1,j-i] + a[i,j-i-1]
\]

\([i,j] \mapsto [i,j+i]\)

- Equivalent to a change of basis.
- Shifting by a constant referred to as loop bumping
Iteration reordering
Transformation: loop skewing

- Used in wavefront parallelisation

Original
```
for(i = 1 to N)
  for(j = 1 to N)
    a[i,j] = a[i-1,j]+a[i,j-1]
```

Skewed
```
for(i = 1 to N)
  for(j = i+1 to i+N)
    a[i,j-i] = a[i-1,j-i]+a[i,j-i-1]
```

- $[i,j] \mapsto [i,j+i]$  
- Equivalent to a change of basis.  
- Shifting by a constant referred to as loop bumping
Iteration reordering
Transformation: loop skewing

- Used in wavefront parallelisation

**Original**

```
for(i = 1 to N)
    for(j = 1 to N)
        a[i,j] = a[i-1,j]+a[i,j-1]
```

**Skewed**

```
for(i = 1 to N)
    for(j = i+1 to i+N)
        a[i,j-i] = a[i-1,j-i]+a[i,j-i-1]
```

\[ [i, j] \mapsto [i, j + i] \]

- Equivalent to a change of basis.
- Shifting by a constant referred to as loop bumping
Iteration reordering
Transformation: loop skewing
- Used in wavefront parallelisation

Original
```
for(i = 1 to N)
    for(j = 1 to N)
        a[i,j] = a[i-1,j]+a[i,j-1]
```

Skewed
```
for(i = 1 to N)
    for(j = i+1 to i+N)
        a[i,j-i] = a[i-1,j-i]+a[i,j-i-1]
```

- \([i,j] \mapsto [i,j + i]\)
- Equivalent to a change of basis.
- Shifting by a constant referred to as loop bumping
Iteration reordering

Transformation: loop skewing

- Used in wavefront parallelisation

Original

for (i = 1 to N)
  for (j = 1 to N)
    a[i,j] = a[i-1,j] + a[i,j-1]

Skewed

for (i = 1 to N)
  for (j = i+1 to i+N)
    a[i,j-i] = a[i-1,j-i] + a[i,j-i-1]

\[ [i, j] \mapsto [i, j + i] \]

- Equivalent to a change of basis.
- Shifting by a constant referred to as loop bumping
Iteration reordering
Transformation: loop skewing

- Used in wavefront parallelisation

**Original**

```
for(i = 1 to N)
    for(j = 1 to N)
        a[i,j] = a[i-1,j]+a[i,j-1]
```

**Skewed**

```
for(i = 1 to N)
    for(j = i+1 to i+N)
        a[i,j-i] = a[i-1,j-i]+a[i,j-i-1]
```

- $[i,j] \mapsto [i,j + i]$
- Equivalent to a change of basis.
- Shifting by a constant referred to as loop bumping
Iteration reordering
Transformation: loop skewing

- Used in wavefront parallelisation

<table>
<thead>
<tr>
<th>Original</th>
<th>Skewed</th>
</tr>
</thead>
</table>
| for(i = 1 to N)  
  for(j = 1 to N)  
    a[i,j] = a[i-1,j]+a[i,j-1] | for(i = 1 to N)  
  for(j = i+1 to i+N)  
    a[i,j-i] = a[i-1,j-i]+a[i,j-i-1] |

- $[i,j] \mapsto [i,j+i]$  
- Equivalent to a change of basis.  
- Shifting by a constant referred to as loop bumping
Iteration reordering

Transformation: loop skewing

- Used in wavefront parallelisation

**Original**

```plaintext
for(i = 1 to N)
  for(j = 1 to N)
    a[i,j] = a[i-1,j]+a[i,j-1]
```

**Skewed**

```plaintext
for(i = 1 to N)
  for(j = i+1 to i+N)
    a[i,j-i] = a[i-1,j-i]+a[i,j-i-1]
```

- \([i,j] \mapsto [i,j+i]\)
- Equivalent to a change of basis.
- Shifting by a constant referred to as loop bumping.
Iteration reordering
Transformation: loop skewing

- Used in wavefront parallelisation

**Original**

```plaintext
for(i = 1 to N)
    for(j = 1 to N)
        a[i,j] = a[i-1,j]+a[i,j-1]
```

**Skewed**

```plaintext
for(i = 1 to N)
    for(j = i+1 to i+N)
        a[i,j-i] = a[i-1,j-i]+a[i,j-i-1]
```

- $[i,j] \mapsto [i,j+i]$
- Equivalent to a change of basis.
- Shifting by a constant referred to as loop bumping
Iteration reordering

Transformation: loop skewing

- Used in wavefront parallelisation

Original

\[
\text{for}(i = 1\ \text{to}\ N) \\
\quad \text{for}(j = 1\ \text{to}\ N) \\
\quad \quad a[i,j] = a[i-1,j] + \\
\quad \quad a[i,j-1]
\]

Skewed

\[
\text{for}(i = 1\ \text{to}\ N) \\
\quad \text{for}(j = i+1\ \text{to}\ i+N) \\
\quad \quad a[i,j-i] = a[i-1,j-i] + \\
\quad \quad a[i,j-i-1]
\]

- \([i,j] \mapsto [i,j + i]\)
- Equivalent to a change of basis.
- Shifting by a constant referred to as loop bumping
Iteration reordering
Transformation: loop skewing

- Used in wavefront parallelisation

**Original**

```plaintext
for(i = 1 to N)
    for(j = 1 to N)
        a[i,j] = a[i-1,j] + a[i,j-1]
```

**Skewed**

```plaintext
for(i = 1 to N)
    for(j = i+1 to i+N)
        a[i,j-i] = a[i-1,j-i] + a[i,j-i-1]
```

- \([i,j] \mapsto [i,j+i]\)
- Equivalent to a change of basis.
- Shifting by a constant referred to as loop bumping
## Iteration reordering

**Transformation: loop reversal**

- Reverse loop direction

<table>
<thead>
<tr>
<th>Original</th>
<th>Fused</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>for(i = 1 to N)</td>
<td></td>
</tr>
<tr>
<td>for(j = 1 to M)</td>
<td></td>
</tr>
<tr>
<td>a[i,j] = a[i,j-1]+b[i]</td>
<td></td>
</tr>
<tr>
<td>for(i = N to 1 step -1)</td>
<td></td>
</tr>
<tr>
<td>for(j = 1 to M)</td>
<td></td>
</tr>
<tr>
<td>a[i,j] = a[i,j-1]+b[i]</td>
<td></td>
</tr>
</tbody>
</table>

- \([i,j] \mapsto [-i,j]\)
- Rarely used in isolation. In unison with previous two.
- Can combine interchange, skewing and reversal as uni-modular transformations.
Iteration reordering
Transformation: loop tiling/blocking

- Break loop into rectangular tiles
- May increase locality (reduce cache misses)

**Original**

```plaintext
definition = 1 to N
    for(j = 1 to M)
        a[i,j] = a[i,j]+b[i]
```

**Tiled**

```plaintext
definition = 1 to N step si
    for(j = 1 to M step sj)
        for(ii = i to i+si-1)
            for(jj = j to j+sj-1)
                a[ii,jj] = a[ii,jj]+b[ii]
```

- Non-convex space
- Interchange placing smaller strip-mine inside
Array layout transformations

- Less extensive literature though perhaps have a more significant impact
- Loop transformations affect all memory references within the loop but not elsewhere. Local in nature
- Array and more generally data transformations have global impact but do not affect other references to other arrays.
- Array layout transformations are used to improve memory access performance
- Also form the basis for data distribution based parallelisation schemes for distributed memory machines.
Array layout transformations
Transformation: global index reordering

- Swap indices (transpose)
- Dual of loop interchange
- \([i, j] \mapsto [j, i]\)

Original

```c
int a[10,20]
for (i = 1 to 9)
    for (j = 2 to 20)
        a[i,j] = a[i+1,j-1]+b[i]
a[1,2] = 0
```

Indices reordered

```c
int a[20,10]
for (i = 1 to 9)
    for (j = 2 to 20)
        a[j,i] = a[j-1,i+1]+b[i]
a[2,1] = 0
```

- Array declaration and subscripts interchanged globally
- Difficulties occur if array reshaped on procedure boundaries
Array layout transformations
Transformation: linearisation

- Map multidimensional array to fewer dimensions (mostly one)
- Dual of loop linearisation

**Original**

```c
int a[10,20]
for(i = 1 to 9)
    for(j = 2 to 20)
        a[i,j] = a[i+1,j-1]+b[i]
    a[1,2] = 0
```

**Linearised**

```c
int a[200]
for(i = 1 to 9)
    for(j = 2 to 20)
        a[20*(i-1)+j]=a[20*i+j-i]+b[i]
    a[2] = 0
```
Array layout transformations
Transformation: padding

- Increase one or more dimensions with redundant values

**Original**
```c
int a[10,20]
for(i = 1 to 9)
    for(j = 2 to 20)
        a[i,j] = a[i+1,j-1]+b[i]
a[1,2] = 0
```

**Padded by 7**
```c
int a[17,20]
for(i = 1 to 9)
    for(j = 2 to 20)
        a[i,j] = a[i+1,j-1]+b[i]
a[1,2] = 0
```

- Frequently used to overcome cache conflicts. Very simple
- Pad factor 7 in first index. Normally prime.
Unification

- Presentation - simplistic conditions of application can be complex for arbitrary programs.
- Little overall structure.
- Uni-modular transformation theory based on linear representation
- Extended to non-singular and the Unified Transformation Framework of Bill Pugh.
- Will return to look in more detail at this formulation in later lectures.
Summary

- Classification of program transformations - loop and array
- Role of dependence
- Loop restructuring - changing the number/type of loop
- Iteration reordering - reordering the iterations scanned.
- Array transformations - data layout transformation
The biggest revolution in the technological landscape for fifty years

Now accepting applications!
Find out more and apply at: pervasiveparallelism.inf.ed.ac.uk

- 4-year programme: MSc by Research + PhD

- Research-focused: Work on your thesis topic from the start

- Collaboration between:
  - University of Edinburgh’s School of Informatics
    - Ranked top in the UK by 2014 REF
  - Edinburgh Parallel Computing Centre
    - UK’s largest supercomputing centre

- Research topics in software, hardware, theory and application of:
  - Parallelism
  - Concurrency
  - Distribution

- Full funding available

- Industrial engagement programme includes internships at leading companies