Compiler Optimisation
7 – Register Allocation

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Introduction

This lecture:

- Local Allocation - spill code
- Global Allocation based on graph colouring
- Techniques to reduce spill code
Register allocation

- Physical machines have limited number of registers
- Scheduling and selection typically assume infinite registers
- Register allocation and assignment $\infty \to k$ registers

**Requirements**

- Produce correct code that uses $k$ (or fewer) registers
- Minimise added loads and stores
- Minimise space used to hold spilled values
- Operate efficiently
  - $O(n)$, $O(n \log_2 n)$, maybe $O(n^2)$, but not $O(2^n)$
Register allocation

Definitions

Allocation versus assignment

- *Allocation* is deciding which values to keep in registers
- *Assignment* is choosing specific registers for values

Interference

Two values\(^a\) cannot be mapped to the same register wherever they are both *live*\(^b\)

Such values are said to **interfere**

\(^a\)A value is stored in a variable
\(^b\)A value is live from its definition to its last use

Live range

The live range of a value is the set of statements at which it is live

May be conservatively overestimated (e.g. just begin \(\rightarrow\) end)
### Register allocation

#### Definitions

<table>
<thead>
<tr>
<th>Spilling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spilling saves a value from a register to memory</td>
</tr>
<tr>
<td>That register is then free – Another value often loaded</td>
</tr>
<tr>
<td>Requires $\mathcal{F}$ registers to be reserved</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Clean and dirty values</th>
</tr>
</thead>
<tbody>
<tr>
<td>A previously spilled value is <strong>clean</strong> if not changed since last spill</td>
</tr>
<tr>
<td>Otherwise it is dirty</td>
</tr>
<tr>
<td>A clean value can be spilled without a new store instruction</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spilling in ILOC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{F}$ is 0 (assuming $r_{arp}$ already reserved)</td>
</tr>
<tr>
<td><strong>Dirty value</strong></td>
</tr>
<tr>
<td>storeAI $r_x \rightarrow r_{arp}, @x$</td>
</tr>
<tr>
<td>loadAI $r_{arp}, @y \Rightarrow r_y$</td>
</tr>
<tr>
<td><strong>Clean value</strong></td>
</tr>
<tr>
<td>loadAI $r_{arp}, @y \Rightarrow r_y$</td>
</tr>
</tbody>
</table>
Local register allocation

Register allocation only on basic block

**MAXLIVE**

Let MAXLIVE be the maximum, over each instruction $i$ in the block, of the number of values (pseudo-registers) live at $i$.

- If $\text{MAXLIVE} \leq k$, allocation should be easy
- If $\text{MAXLIVE} \leq k$, no need to reserve $F$ registers for spilling
- If $\text{MAXLIVE} > k$, some values must be spilled to memory
- If $\text{MAXLIVE} > k$, need to reserve $F$ registers for spilling

Two main forms:
- Top down
- Bottom up
Local register allocation
MAXLIVE

Example MAXLIVE computation

Some simple code with virtual registers

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Register</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>loadI 1028</td>
<td>ra</td>
<td>ra</td>
</tr>
<tr>
<td>load ra</td>
<td>rb</td>
<td>MEM(ra)</td>
</tr>
<tr>
<td>mult ra, rb</td>
<td>rc</td>
<td>1028 * y</td>
</tr>
<tr>
<td>load x</td>
<td>rd</td>
<td>x</td>
</tr>
<tr>
<td>sub rd, rb</td>
<td>re</td>
<td>x - y</td>
</tr>
<tr>
<td>load z</td>
<td>rf</td>
<td>z</td>
</tr>
<tr>
<td>mult re, rf</td>
<td>rg</td>
<td>z * (x - y)</td>
</tr>
<tr>
<td>sub rg, rc</td>
<td>rh</td>
<td>z * (x - y) - 1028 * y</td>
</tr>
<tr>
<td>store rh</td>
<td>ra</td>
<td>MEM(ra) = z * (x - y) - 1028 * y</td>
</tr>
</tbody>
</table>
Local register allocation
MAXLIVE

Example MAXLIVE computation

Live registers

```
loadI 1028 \rightarrow r_a  // r_a
load r_a \rightarrow r_b  // r_a r_b
mult r_a, r_b \rightarrow r_c  // r_a r_b r_c
load x \rightarrow r_d  // r_a r_b r_c r_d
sub r_d, r_b \rightarrow r_e  // r_a r_c r_e
load z \rightarrow r_f  // r_a r_c r_e r_f
mult r_e, r_f \rightarrow r_g  // r_a r_c r_g
sub r_g, r_c \rightarrow r_h  // r_a r_g r_h
store r_h \rightarrow r_a  //
```
Local register allocation

MAXLIVE

Example MAXLIVE computation

MAXLIVE is 4

```
load 1 1028 \rightarrow r_a  \quad // \quad r_a
load r_a \rightarrow r_b  \quad // \quad r_a \ r_b
mult r_a, r_b \rightarrow r_c  \quad // \quad r_a \ r_b \ r_c
load x \rightarrow r_d  \quad // \quad r_a \ r_b \ r_c \ r_d
sub r_d, r_b \rightarrow r_e  \quad // \quad r_a \ r_c \ r_e
load z \rightarrow r_f  \quad // \quad r_a \ r_c \ r_e \ r_f
mult r_e, r_f \rightarrow r_g  \quad // \quad r_a \ r_c \ r_g
sub r_g, r_c \rightarrow r_h  \quad // \quad r_a \ r_c \ r_h
store r_h \rightarrow r_a  \quad //
```
Local register allocation
Top down

Algorithm:
- If number of values $> k$
  - Rank values by occurrences
  - Allocate first $k - \mathcal{F}$ values to registers
  - Spill other values
Local register allocation

Top down

Example top down

Usage counts

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Destination</th>
<th>Source 1</th>
<th>Source 2</th>
<th>Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td>loadI 1028</td>
<td>$r_a$</td>
<td></td>
<td></td>
<td>$r_a=4$</td>
</tr>
<tr>
<td>load $r_a$</td>
<td>$r_b$</td>
<td>$r_a$</td>
<td></td>
<td>$r_b=3$</td>
</tr>
<tr>
<td>mult $r_a$, $r_b$</td>
<td>$r_c$</td>
<td>$r_a$, $r_b$</td>
<td></td>
<td>$r_c=2$</td>
</tr>
<tr>
<td>load $x$</td>
<td>$r_d$</td>
<td>$r_a$, $r_b$</td>
<td>$r_c$</td>
<td>$r_d=2$</td>
</tr>
<tr>
<td>sub $r_d$, $r_b$</td>
<td>$r_e$</td>
<td>$r_a$, $r_c$</td>
<td>$r_e$</td>
<td>$r_e=2$</td>
</tr>
<tr>
<td>load $z$</td>
<td>$r_f$</td>
<td>$r_a$, $r_c$, $r_e$</td>
<td>$r_f$</td>
<td>$r_f=2$</td>
</tr>
<tr>
<td>mult $r_e$, $r_f$</td>
<td>$r_g$</td>
<td>$r_a$, $r_c$, $r_e$, $r_f$</td>
<td></td>
<td>$r_g=2$</td>
</tr>
<tr>
<td>sub $r_g$, $r_c$</td>
<td>$r_h$</td>
<td>$r_a$</td>
<td></td>
<td>$r_h=2$</td>
</tr>
<tr>
<td>store $r_h$</td>
<td>$r_a$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Local register allocation

Top down

Example top down

Spill $r_c$. Now only 3 values live at once

```
loadI 1028  $r_a
load  $r_a  $r_b
mult $r_a, $r_b  $r_c
load  $x  $r_d
sub $r_d, $r_b  $r_e
load  $z  $r_f
mult $r_e, $r_f  $r_g
sub $r_g, $r_c  $r_h
store $r_h  $r_a
```

Must have $r_d$

Counts

<table>
<thead>
<tr>
<th>Register</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_a$</td>
<td>4</td>
</tr>
<tr>
<td>$r_b$</td>
<td>3</td>
</tr>
<tr>
<td>$r_c$</td>
<td>2</td>
</tr>
<tr>
<td>$r_d$</td>
<td>2</td>
</tr>
<tr>
<td>$r_e$</td>
<td>2</td>
</tr>
<tr>
<td>$r_f$</td>
<td>2</td>
</tr>
<tr>
<td>$r_g$</td>
<td>2</td>
</tr>
</tbody>
</table>

Spill $r_c$

Restore $r_c, r_g, r_h$
Local register allocation
Top down

Example top down

Spill code inserted

```
loadI 1028  \( \rightarrow r_a \)
load  \( r_a \)  \( \rightarrow r_b \)
mult  \( r_a, r_b \)  \( \Rightarrow r_c \)
store  \( r_c \)  \( \rightarrow r_{arp}, spill_c \)
load  \( x \)  \( \rightarrow r_d \)
sub  \( r_d, r_b \)  \( \rightarrow r_e \)
load  \( z \)  \( \rightarrow r_f \)
mult  \( r_e, r_f \)  \( \rightarrow r_g \)
load  \( r_{arp}, spill_c \)  \( \rightarrow r_c \)
sub  \( r_f, r_c \)  \( \rightarrow r_h \)
store  \( r_h \)  \( \rightarrow r_a \)
```
Local register allocation
Top down

Example top down

Register assignment straightforward

```
loadI 1028                      r1
load  r1                        r2
mult  r1, r2                    r3
store r3                        r_{arp}, spill_c
load  x                         r3
sub   r3, r2                    r2
load  z                         r3
mult  r2, r3                    r2
load  r_{arp}, spill_c           r3
sub   r2, r3                    r2
store r2                        r1
```
Local register allocation
Bottom up

Algorithm:
- Start with empty register set
- Load on demand
- When no register is available, free one

Replacement:
- Spill the value whose next use is farthest in the future
- Prefer clean value to dirty value
Local register allocation

Top down

Example bottom down

Spill \( r_a \). Now only 3 values live at once

```
loadI 1028 \rightarrow r_a // r_a
load \ r_a \rightarrow r_b // r_a \ r_b
mult \ r_a, \ r_b \rightarrow r_c // r_a \ r_b \ r_c
load \ x \rightarrow r_d // r_a \ r_b \ r_c \ r_d
sub \ r_d, \ r_b \rightarrow r_e // r_a \ r_c \ r_e
load \ z \rightarrow r_f // r_a \ r_c \ r_e \ r_f
mult \ r_e, \ r_f \rightarrow r_g // r_a \ r_c \ r_e \ r_g
sub \ r_g, \ r_c \rightarrow r_h // r_a \ r_c \ r_e \ r_g \ r_h
store \ r_h \rightarrow r_a // r_a
```

Spill \( r_a \)

Restore \( r_a \)
Local register allocation

Top down

Example bottom down

Spill code inserted

\[
\begin{align*}
\text{loadI} & 1028 & r_a \\
\text{load} & r_a & r_b \\
\text{mult} & r_a, r_b & \Rightarrow r_c \\
\text{store} & r_a & \Rightarrow r_{\text{arp}}, \text{spilla} \\
\text{load} & x & r_d \\
\text{sub} & r_d, r_b & r_e \\
\text{load} & z & r_f \\
\text{mult} & r_e, r_f & r_g \\
\text{sub} & r_f, r_c & r_h \\
\text{load} & r_{\text{arp}}, \text{spilla} & r_a \\
\text{store} & r_h & \Rightarrow r_a
\end{align*}
\]
Global register allocation

Local allocation does not capture reuse of values across multiple blocks.
Most modern, global allocators use a graph-colouring paradigm.
- Build a “conflict graph” or “interference graph”
  - Data flow based liveness analysis for interference
- Find a k-colouring for the graph, or change the code to a nearby problem that it can k-colour
- NP-complete under nearly all assumptions

---

1Local allocation is NP-complete with dirty vs clean.
Global register allocation
Algorithm sketch

- From live ranges construct an interference graph
- Colour interference graph so that no two neighbouring nodes have same colour
- If graph needs more than k colours - transform code
  - Coalesce merge-able copies
  - Split live ranges
  - Spill
- Colouring is NP-complete so we will need heuristics
- Map colours onto physical registers
Global register allocation

Graph colouring

**Definition**

A graph $G$ is said to be **k-colourable** iff the nodes can be labeled with integers $1 \ldots k$ so that no edge in $G$ connects two nodes with the same label.

**Examples**

- 2-colourable
- 3-colourable
The interference graph, $G_I = (N_I, E_I)$
- Nodes in $G_I$ represent values, or live ranges
- Edges in $G_I$ represent individual interferences
- $\forall x, y \in N_I, x \to y \in E_I$ iff $x$ and $y$ interfere

A $k$-colouring of $G_I$ can be mapped into an allocation to $k$ registers

---

Two values interfere wherever they are both live
Two live ranges interfere if their values interfere at any point
Global register allocation
Colouring the interference graph

- Degree\(^3\) of a node \((n^\circ)\) is a loose upper bound on colourability
- Any node, \(n\), such that \(n^\circ < k\) is always trivially \(k\)-colourable
  - Trivially colourable nodes cannot adversely affect the colourability of neighbours\(^4\)
  - Can remove them from graph
  - Reduces degree of neighbours - may be trivially colourable
- If left with any nodes such that \(n^\circ \geq k\) spill one
  - Reduces degree of neighbours - may be trivially colourable

---

\(^3\)Degree is number of neighbours

\(^4\)Proof as exercise
Global register allocation
Chaitin’s algorithm

1. While ∃ vertices with < k neighbours in $G_\mathcal{I}$
   - Pick any vertex $n$ such that $n^\circ < k$ and put it on the stack
   - Remove $n$ and all edges incident to it from $G_\mathcal{I}$

2. If $G_\mathcal{I}$ is non-empty ($n^\circ \geq k, \forall n \in G_\mathcal{I}$) then:
   - Pick vertex $n$ (heuristic), spill live range of $n$
   - Remove vertex $n$ and edges from $G_\mathcal{I}$, put $n$ on “spill list”
   - Goto step 1

3. If the spill list is not empty, insert spill code, then rebuild the interference graph and try to allocate, again

4. Otherwise, successively pop vertices off the stack and colour them in the lowest colour not used by some neighbour
Global register allocation
Chaitin’s algorithm

Example: colouring with Chaitin’s algorithm

Colour with \( k = 3 \) colours

\[ G_I \]

- \( a \)
- \( b \)
- \( c \)
- \( d \)
- \( e \)

Stack

- \( r_1 \)
- \( r_2 \)
- \( r_3 \)

Colours
Global register allocation
Chaitin’s algorithm

Example: colouring with Chaitin’s algorithm

\[ a^o = 2 < k \]

Choose a

\[ \text{Stack} \]

\[ \text{Colours} \]

\[ r_1 \]

\[ r_2 \]

\[ r_3 \]
Global register allocation
Chaitin’s algorithm

Example: colouring with Chaitin’s algorithm

Push a and remove from graph

Diagram showing a graph with vertices labeled a, b, c, d, e connected in a specific pattern. To the right of the graph is a stack labeled 'Stack' with a vertex 'a' at the bottom. Below the graph is G_i, and to the right of the stack is a section labeled 'Colours' with vertices colored in red, green, and blue, labeled r_1, r_2, and r_3, respectively.
Global register allocation
Chaitin’s algorithm

Example: colouring with Chaitin’s algorithm

\[ b^\circ = 2 < k \text{ and } c^\circ = 2 < k \text{ Choose } b \]
Global register allocation
Chaitin’s algorithm

Example: colouring with Chaitin’s algorithm

Push b and remove from graph

G_i

Stack

Colours
Global register allocation
Chaitin’s algorithm

Example: colouring with Chaitin’s algorithm

\[ c^\circ = 2 < k, \quad d^\circ = 2 < k, \quad \text{and} \quad e^\circ = 2 < k \]

Choose \( c \)
Global register allocation
Chaitin’s algorithm

Example: colouring with Chaitin’s algorithm

Push c and remove from graph

\[ G_I \]

Stack

Colours
Global register allocation
Chaitin’s algorithm

Example: colouring with Chaitin’s algorithm

\[ d^\circ = 1 < k \quad \text{and} \quad e^\circ = 1 < k \quad \text{Choose } d \]

\[ G_I \]

Stack

Colours

\[ r_1 \]

\[ r_2 \]

\[ r_3 \]
Global register allocation
Chaitin’s algorithm

Example: colouring with Chaitin’s algorithm

Push $d$ and remove from graph

$G_I$  Stack  Colours

$e$  $d$  $c$  $b$  $a$

$r_1$  $r_2$  $r_3$
Global register allocation
Chaitin’s algorithm

Example: colouring with Chaitin’s algorithm

$e^\circ = 0 < k$ Choose $e$

$G_I$  Stack  Colours

$e$  $d$  $a$  $r_1$

$c$  $b$  $r_2$

$a$  $r_3$
Global register allocation
Chaitin’s algorithm

Example: colouring with Chaitin’s algorithm

Push e and remove from graph

$G_I$  Stack  Colours

r_1  r_2  r_3
Global register allocation
Chaitin’s algorithm

Example: colouring with Chaitin’s algorithm

Pop $e$, neighbours use no colours, choose red

$G_I$

Stack

Colours

$r_1$

$r_2$

$r_3$
Global register allocation
Chaitin’s algorithm

Example: colouring with Chaitin’s algorithm

Pop d, neighbours use red, choose green

G_I
Stack
Colours

r_1
r_2
r_3
Global register allocation
Chaitin’s algorithm

Example: colouring with Chaitin’s algorithm

Pop c, neighbours use red and green choose blue
Example: colouring with Chaitin’s algorithm

Pop \( b \), neighbours use red and green choose blue
Global register allocation
Chaitin’s algorithm

Example: colouring with Chaitin’s algorithm

Pop a, neighbours use blue choose red

Graph $G_1$ with nodes a, b, c, d, e and edges connecting them. The nodes are coloured with red and blue. The stack and colours are also shown, with red, blue, and green colours assigned to registers $r_1$, $r_2$, and $r_3$ respectively.
Global register allocation

Optimistic colouring

- If Chaitin's algorithm reaches a state where every node has $k$ or more neighbours, it chooses a node to spill.

**Example of Chaitin overzealous spilling**

$k = 2$

Graph is 2-colourable
Chaitin must immediately spill one of these nodes

- Briggs said, take that same node and push it on the stack
  - When you pop it off, a colour might be available for it!
- Chaitin-Briggs algorithm uses this to colour that graph
Global register allocation
Chaitin-Briggs algorithm

1. While $\exists$ vertices with $< k$ neighbours in $G_I$
   - Pick any vertex $n$ such that $n^\circ < k$ and put it on the stack
   - Remove $n$ and all edges incident to it from $G_I$

2. If $G_I$ is non-empty ($n^\circ \geq k, \forall n \in G_I$) then:
   - Pick vertex $n$ (heuristic) (Do not spill)
   - Remove vertex $n$ from $G_I$, put $n$ on stack (Not spill list)
   - Goto step 1

3. Otherwise, successively pop vertices off the stack and colour them in the lowest colour not used by some neighbour
   - If some vertex cannot be coloured, then pick an uncoloured vertex to spill, spill it, and restart at step 1

Step 3 is also different
Global register allocation
Chaitin-Briggs algorithm

Example: colouring with Chaitin-Briggs algorithm

Colour with \( k = 2 \) colours

\[ G_I \]

\[ \begin{array}{cccc}
& b & & \\
& & & \\
c & & a & d \\
& & & \\
& & & \\
\end{array} \]

Stack

\[ r_1 \]

\[ r_2 \]

Colours
Global register allocation
Chaitin-Briggs algorithm

Example: colouring with Chaitin-Briggs algorithm

$a^\circ = 2 \geq k$  Don’t Spill! Choose a

$G_I$  Stack  Colours

r1  r2
Global register allocation
Chaitin-Briggs algorithm

Example: colouring with Chaitin-Briggs algorithm

Push $a$ and remove from graph

$G_I$  
\[ b \]
\[ d \]
\[ c \]

Stack 
\[ a \]

Colours 
\[ r_1 \]
\[ r_2 \]
Global register allocation
Chaitin-Briggs algorithm

Example: colouring with Chaitin-Briggs algorithm

\[ b^\circ = 1 < k \text{ and } c^\circ = 1 < k \]

Choose \( b \)
Global register allocation
Chaitin-Briggs algorithm

Example: colouring with Chaitin-Briggs algorithm

Push \( b \) and remove from graph

\[ G_I \]

\[ \text{Stack} \]

\[ \text{Colours} \]
Global register allocation
Chaitin-Briggs algorithm

Example: colouring with Chaitin-Briggs algorithm

$c^\circ = 1 < k$, and $d^\circ = 1 < k$ Choose $c$
Global register allocation
Chaitin-Briggs algorithm

Example: colouring with Chaitin-Briggs algorithm
Push $c$ and remove from graph

$G_I$  Stack  Colours

$d$  $c$  $b$  $a$  $r_1$  $r_2$
Global register allocation
Chaitin-Briggs algorithm

Example: colouring with Chaitin-Briggs algorithm

\[ d^\circ = 1 < k \]

Choose \( d \)

\[ G_I \]

Stack

Colours

\[ r_1 \]

\[ r_2 \]
Global register allocation
Chaitin-Briggs algorithm

Example: colouring with Chaitin-Briggs algorithm

Push \( d \) and remove from graph

\[
\begin{align*}
G_I & \\
\text{Stack} & \\
\text{Colours} &
\end{align*}
\]
Global register allocation

Chaitin-Briggs algorithm

Example: colouring with Chaitin-Briggs algorithm

Pop d, neighbours use no colours, choose red

\[ G_I \]

Stack

Colours
Global register allocation
Chaitin-Briggs algorithm

Example: colouring with Chaitin-Briggs algorithm

Pop c, neighbours use red choose green
Global register allocation
Chaitin-Briggs algorithm

Example: colouring with Chaitin-Briggs algorithm
Pop $b$, neighbours use red choose green
Global register allocation
Chaitin-Briggs algorithm

Example: colouring with Chaitin-Briggs algorithm

Pop $a$, neighbours use green choose red

$G_I$

Stack

Colours

$r_1$

$r_2$
Global register allocation

Spill candidates

- Minimise spill cost/ degree
- Spill cost is the loads and stores needed. Weighted by scope - i.e. avoid inner loops
- The higher the degree of a node to spill the greater the chance that it will help colouring
- Negative spill cost load and store to same memory location with no other uses
- Infinite cost - definition immediately followed by use. Spilling does not decrease live range
Global register allocation
Alternative spilling

- Splitting live ranges
- Coalesce
A whole live range may have many interferences, but perhaps not all at the same time

Split live range into two variables connected by copy

Can reduce degree of interference graph

Smart splitting allows spilling to occur in “cheap” regions
Global register allocation
Live ranges splitting

Splitting example

Non contiguous live ranges - cannot be 2 coloured

Live ranges

Interference Graph
Global register allocation
Live ranges splitting

Splitting example

Split live ranges - can be 2 coloured

Live ranges

Interference Graph
Global register allocation

Coalescing

If two ranges don't interfere and are connected by a copy coalesce into one – opposite of splitting
Reduces degree of nodes that interfered with both

If \( x \assign y \) and \( x \rightarrow y \in G_I \) then can combine \( LR_x \) and \( LR_y \)
- Eliminates the copy operation
- Reduces degree of LRs that interfere with both \( x \) and \( y \)
- If a node interfered with both both before, coalescing helps
- As it reduces degree, often applied before colouring takes place
Global register allocation
Coalescing

Coalescing can make the graph harder to color

- Typically, $LR_{xy} > \max(LR_{x}, LR_{y})$
- If $\max(LR_{x}, LR_{y}) < k$ and $k < LR_{xy}$ then $LR_{xy}$ might spill, while $LR_{x}$ and $LR_{y}$ would not spill
Global register allocation
Coalescing

Observation led to conservative coalescing

- Conceptually, coalesce $x$ and $y$ iff $x \rightarrow y \in G_I$ and $LR_{xy} \circ < k$
- We can do better
  - Coalesce $LR_x$ and $LR_y$ iff $LR_{xy}$ has $< k$ neighbours with degree $> k$
  - Only neighbours of “significant degree” can force $LR_{xy}$ to spill
- Always safe to perform that coalesce
  - Cannot introduce a node of non-trivial degree
  - Cannot introduce a new spill
Global register allocation

Other approaches

- Top-down uses high level priorities to decide on colouring
- Hierarchical approaches - use control flow structure to guide allocation
- Exhaustive allocation - go through combinatorial options - very expensive but occasional improvement
- Re-materialisation - if easy to recreate a value do so rather than spill
- Passive splitting using a containment graph to make spills effective
- Linear scan - fast but weak; useful for JITs
Global register allocation
Ongoing work

- Eisenbeis et al examining optimality of combined reg alloc and scheduling. Difficulty with general control-flow
- Partitioned register sets complicate matters. Allocation can require insertion of code which in turn affects allocation. Leupers investigated use of genetic algs for TM series partitioned reg sets.
- New work by Fabrice Rastello and others. Chordal graphs reduce complexity
- As latency increases see work in combined code generation, instruction scheduling and register allocation
Summary

- Local Allocation - spill code
- Global Allocation based on graph colouring
- Techniques to reduce spill code
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