Introduction

This lecture:
- Parallelisation for fork/join
- Mapping parallelism to shared memory multi-processors
- Loop distribution and fusion
- Data Partitioning and SPMD parallelism
- Communication, synchronisation and load imbalance.
Introduction
Approaches to parallelisation

- Two approaches to parallelisation
  - Traditional shared memory
    Single address space
    Based on finding parallel loop iterations
  - Distributed memory compilation
    Physically distributed memory uses a mixture of both
    Focus on mapping data, computation

- Can show equivalence
  Implement shared memory on distributed
  Implement distributed memory on shared
Introduction
Approaches to parallelisation

Shared memory - single address space
Introduction
Approaches to parallelisation

Shared memory - probably private caches, but looks like single address space
Introduction
Approaches to parallelisation

Distributed memory - each machine has own address space
Use message passing
Loop Parallelisation

- Assume a single address space machine. Each processor sees the same set of addresses. Do not need to know physical location of memory reference.

- Control-orientated approach. Concerned with finding independent iterations of a loop. Then map or schedule these to the processor.

- Aim: find maximum amount of parallelism and minimise synchronisation.

- Secondary aim: improve load imbalance. Inter-processor communication not considered.

- Main memory just part of hierarchy - so use uni-processor approaches.
Loop Parallelisation
Fork/join

- Fork (create) threads at beginning of loop
- Thread executes one or more iterations. Depend on later scheduling policy
- Join (synchronisation/barrier) at end of loop
- Synchronisation expensive
  - Favour outer loop parallelism
  - Loop interchange
Loop Parallelisation

DOALL Implementation

Original

\[
\begin{align*}
\text{Do } & i = 1, N \\
\text{A}(i) &= B(i) \\
\text{C}(i) &= A(i) \\
\text{Enddo}
\end{align*}
\]

Driver

\[
\begin{align*}
\text{p} &= \text{get_num_proc()} \\
\text{fork(x_sub,p)} \\
\text{join()}
\end{align*}
\]

Per thread

\[
\begin{align*}
\text{SUBROUTINE x_sub()}
\end{align*}
\]

\[
\begin{align*}
p &= \text{get_num_proc()} \\
z &= \text{my_id()} \\
ilo &= N/p \ast (z-1) + 1 \\
ihi &= \text{min}(N, ilo+N/p) \\
\text{Do } i &= ilo, ihi \\
\text{A}(i) &= B(i) \\
\text{C}(i) &= A(i) \\
\text{Enddo}
\end{align*}
\]

\[
\text{END}
\]

Generate \(p\) independent threads of work

- Each has private local variables, \(z\), \(ilo\), \(ihi\)
- Access shared arrays \(A\), \(B\) and \(C\)
Loop Parallelisation
Using loop interchange

Original
Do i = 1, N
   Do j = 1, M
      a(i+1,j) = a(i,j)+c
   Enddo
Enddo

Interchanged
Do j = 1, M
   Do i = 1, N
      a(i+1,j) = a(i,j)+c
   Enddo
Enddo

$O(n)$ synchronisation points
Do i = 1, N
   Parallel Do j = 1, M
      a(i+1,j) = a(i,j)+c
   Enddo
Enddo

1 synchronisation point
Parallel Do j = 1, M
   Do i = 1, N
      a(i+1,j) = a(i,j)+c
   Enddo
Enddo

Interchange has reduced synchronisation overhead from $O(N)$ to 1.
Parallelisation approach

- Loop distribution eliminates carried dependences and creates opportunity for outer-loop parallelism.
- However increases number of synchronisations needed after each distributed loop.
- Maximal distribution often finds components too small for efficient parallelisation.
- Solution: fuse together parallelisable loops.
Fusion illegal if changes the dependence direction

Two loops - same bounds

```
Do i = 1, N
  a(i) = b(i) + c
Enddo
Do i = 1, N
  d(i) = a(i) + e
Enddo
```

Fused

```
Do i = 1, N
  a(i) = b(i) + c
  d(i) = a(i) + e
Enddo
```

Profitability: Parallel and sequential loops should not generally be merged
Loop Fusion

Fusion illegal if changes the dependence direction

### Two loops - same bounds

<table>
<thead>
<tr>
<th>Do i = 1, N</th>
</tr>
</thead>
<tbody>
<tr>
<td>a(i) = b(i) + c</td>
</tr>
<tr>
<td>Enddo</td>
</tr>
<tr>
<td>Do i = 1, N</td>
</tr>
<tr>
<td>d(i) = a(i+1) + e</td>
</tr>
<tr>
<td>Enddo</td>
</tr>
</tbody>
</table>

### Fused

<table>
<thead>
<tr>
<th>Do i = 1, N</th>
</tr>
</thead>
<tbody>
<tr>
<td>a(i) = b(i) + c</td>
</tr>
<tr>
<td>d(i) = a(i+1) + e</td>
</tr>
<tr>
<td>Enddo</td>
</tr>
</tbody>
</table>

Take care that fusing does not prevent parallelisation
Data Parallelism

- Alternative approach where we focus on mapping data rather than control flow to the machine
- Data is partitioned/distributed across the processors of the machine
- The computation is then mapped to follow the data - typically such that work writes to local data. Local write/owner computes rule.
- All of this is based on the SPMD computational model. Each processor runs one thread executing the same program, operating on the different data
- This means that loop bounds change from processor to processor.
Data Parallelism
Mapping

- Placement of work and data on processors. Assume parallelism found in a previous stage
- Typically program parallelism $O(n)$ is much greater than machine parallelism $O(p)$, $n >> p$
- We have many options as to how to map a parallel program
- Key issue: What is the best mapping that achieves $O(p)$ parallelism but minimises cost
- Costs include communication, load imbalance and synchronisation
Data Placement
Simple Fortran example

Dimension Integer a(4,8)
Do i = 1, 4
  Do j = 1, 8
    a(i,j) = i + j
  Enddo
Enddo

Note that here data and iteration spaces line up. Generally not the case.
**Data Placement**

**Simple Fortran example**

Partitioning by columns of `a` and hence iterator `j` : Local writes

**Processor 1**
Dimension Integer
`a(4,1..2)`
Do `i = 1, 4`
  Do `j = 1 ,2`
    `a(i,j) = i + j`
  Enddo
Enddo
...
**Processor 3**
Dimension Integer
`a(4,5..6)`
Do `i = 1, 4`
  Do `j = 5, 6`
    `a(i,j) = i + j`
  Enddo
Enddo
Partitioning by rows of a and hence iterator i: Local writes

**Processor 1**
Dimension Integer
a(1..1,1..8)
Do i = 1, 1
  Do j = 1, 8
    a(i,j) = i + j
  Enddo
Enddo
...

**Processor 3**
Dimension Integer
a(3..3,1..8)
Do i = 3, 3
  Do j = 1, 8
    a(i,j) = i + j
  Enddo
Enddo
Linear program representation

- Iteration space defined by loop bound constraints
- Constraints are affine ($\vec{a}_i \leq \vec{c}$)
- Matrix standard form ($A\vec{i} \leq \vec{c}$)
- Each constraint defines half space
- Iteration space is intersection of half spaces (polytope)
- Iterations at integer lattice points within iteration space
  - Typically unit lattices
- Array access patterns as affine functions over iteration vectors
  ($f(\vec{i}) = B\vec{i} + d$)
Linear program representation

Example

Iteration constraints

\begin{align*}
\text{Do } i &= 1, 16 \\
\text{Do } j &= 1, 16 \\
\text{Do } k &= i, 16 \\
\quad c(i,j) &= c(i,j) \\
&\quad +a(i,k)*b(j,k) \\
1 &\leq i \\
1 &\leq j \\
i &\leq k \\
i &\leq 16 \\
j &\leq 16 \\
k &\leq 16
\end{align*}
Linear program representation

Example

Make into standard form

\[
\begin{align*}
\text{Do } i &= 1, 16 \\
\text{Do } j &= 1, 16 \\
\text{Do } k &= i, 16 \\
&
c(i,j) = c(i,j) + a(i,k) \cdot b(j,k) \\
1 - i &\leq 0 \\
1 - j &\leq 0 \\
i - k &\leq 0 \\
i &\leq 16 \\
j &\leq 16 \\
k &\leq 16
\end{align*}
\]
Linear program representation

Example

\[
\begin{aligned}
&\text{Do } i = 1, 16 \\
&\quad \text{Do } j = 1, 16 \\
&\quad \quad \text{Do } k = i, 16 \\
&\quad \quad \quad c(i,j) = c(i,j) + a(i,k) \times b(j,k)
\end{aligned}
\]

Make into standard form

\[
\begin{aligned}
&-i \leq -1 \\
&-j \leq -1 \\
&i - k \leq 0 \\
&i \leq 16 \\
&j \leq 16 \\
&k \leq 16
\end{aligned}
\]
Linear program representation
Example

Do i = 1, 16
  Do j = 1, 16
    Do k = i, 16
      c(i,j) = c(i,j) + a(i,k)*b(j,k)

Make into standard form

-1.0.i + 0.0.j + 0.0.k ≤ -1
0.0.i + -1.0.j + 0.0.k ≤ -1
1.0.i + 0.0.j -1.0.k ≤ 0
1.0.i + 0.0.j + 0.0.k ≤ 16
0.0.i + 1.0.j + 0.0.k ≤ 16
0.0.i + 0.0.j + 1.0.k ≤ 16
Linear program representation

Example

\[
\begin{align*}
\text{Do } i &= 1, 16 \\
\text{Do } j &= 1, 16 \\
\text{Do } k &= i, 16 \\
&\quad \begin{align*}
&c(i,j) = c(i,j) \\
&\quad + a(i,k) \cdot b(j,k)
\end{align*}
\end{align*}
\]

Make into standard form

\[
\begin{bmatrix}
-1 & 0 & 0 \\
0 & -1 & 0 \\
1 & 0 & -1 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
i \\
j \\
k
\end{bmatrix}
\leq
\begin{bmatrix}
-1 \\
-1 \\
0 \\
16 \\
16 \\
16
\end{bmatrix}
\]
Linear program representation

Example

Do $i = 1, 16$
  Do $j = 1, 16$
    Do $k = i, 16$
      $c(i,j) = c(i,j) + a(i,k) \cdot b(j,k)$

Make into standard form

$$\begin{bmatrix}
-1 & 0 & 0 \\
0 & -1 & 0 \\
1 & 0 & -1 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
i \\
j \\
k
\end{bmatrix}
\leq
\begin{bmatrix}
-1 \\
-1 \\
16 \\
16 \\
16 \\
16
\end{bmatrix}$$

Access matrices $U_c \ U_a \ U_b$

$$\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 1 & 0
\end{bmatrix}_c
\begin{bmatrix}
i \\
j \\
k
\end{bmatrix},
\begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 1
\end{bmatrix}_a
\begin{bmatrix}
i \\
j \\
k
\end{bmatrix},
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 1
\end{bmatrix}_b
\begin{bmatrix}
i \\
j \\
k
\end{bmatrix}$$
Many transformations\(^1\) are affine functions over linear program

**Scanning** then regenerates code

Partitioning loop for different processors by adding partition constraints

\(^1\)Skew, reverse, interchange, etc
Linear program representation
Partitioning example

Split four processors equally along $i$
Processor 2

Do $i = 5,8$
  Do $j = 1,16$
    Do $k = i,16$
      $c(i,j) = c(i,j)$
      $+ a(i,k) * b(j,k)$

Determine local array bounds $\lambda_z, \upsilon_z$ for each processor $1 \leq z \leq p$.
$
\lambda_1 = 1, \lambda_2 = 5, \lambda_3 = 9, \lambda_4 = 13
\upsilon_1 = 4, \upsilon_2 = 8, \upsilon_3 = 12, \upsilon_4 = 16$

Determine local write constraint $\lambda_z \leq U_c \leq \upsilon_z, 5 \leq i \leq 8$ and add to polytope

Works for arbitrary loop structures and accesses

\[
\begin{bmatrix}
-1 & 0 & 0 \\
0 & -1 & 0 \\
1 & 0 & -1 \\
-1 & 0 & 0 \\
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1 \\
-1 & 0 & 0 \\
1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
i \\
j \\
k
\end{bmatrix}
\leq
\begin{bmatrix}
16 \\
16 \\
16 \\
-5 \\
8
\end{bmatrix}
\]
Load balancing

- Load describes amount of work each processor must do.
- For simple loop bodies is number of iterations assigned to each processor.
- All processors wait for slowest at join point.
- Want to minimise idle time at join.
Do $i = 1, 16$
  Do $j = 1, 16$
    Do $k = i, 16$
      \[ c(i,j) = c(i,j) + a(i,k) \times b(j,k) \]

Assuming $c, a, b$ are to be partitioned in a similar manner
How should we partition to minimise load imbalance?

- **Row (along $i$):** processor load 928, 672, 416, 160 iterations
- **Column (along $j$):** processor load 544, 544, 544, 544 iterations

Why this variation?
Load balance
Example

Partition by row (along $i$)
Load balance

Example

Partition by column (along \( j \))

Partition by “invariant” iterator \( j \).
Load balance
Polytope based

- Generally straightforward to ‘read’ from polytope
- Iteration variable with zeros elsewhere in rows and columns is ‘invariant’
- Partitioning on ‘invariant’ yields balance

\[
\begin{bmatrix}
-1 & 0 & 0 \\
0 & -1 & 0 \\
1 & 0 & -1 \\
\hline
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
-1 & 0 & 0 \\
0 & -1 & 0 \\
1 & 0 & -1 \\
\hline
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
Reducing Communication

We wish to partition work and data to reduce amount of communication or remote accesses

Dimension a(n,n) b(n,n)
Do i = 1, n
    Do j = 1, n
        Do k = 1, n
            a(i,j) = b(i,k)
        Enddo
    Enddo
Enddo

How should we partition to reduce communication?
Reducing communication

Each processor has rows of $a$ and $b$ allocated to it
Look at access pattern of second processor

Dimension $a(n,n)$ $b(n,n)$
Do $i = 1$, $n$
  Do $j = 1$, $n$
    Do $k = 1$, $n$
      $a(i,j) = b(i,k)$
    Enddo
  Enddo
Enddo

The columns of $a$ scheduled to P2 access all of $b$ $n^2 - \frac{n^2}{p}$ remote access
Reducing communication

Each processor has rows of $a$ and $b$ allocated to it.
Look at access pattern of second processor.

Dimension $a(n,n)$ $b(n,n)$
Do $i = 1, n$
  Do $j = 1, n$
    Do $k = 1, n$
      $a(i,j) = b(i,k)$
    Enddo
  Enddo
Enddo
The rows of $a$ scheduled to P2 access corresponding rows of $b$.
0 remote accesses.
The first index of a and b have the same subscript \( a(i,j), b(i,k) \)

They are said to be aligned on this index.

Partitioning on an aligned index makes all accesses local to that array reference.

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
\end{bmatrix}_a \quad \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}_b
\]

Can transform array layout to make arrays more aligned for partitioning.

Find \( \mathcal{A} \) such that \( \mathcal{A} U_x \) is maximally aligned with \( U_y \)

Global alignment problem
Synchronisation

- Alignment information can also be used to eliminate synchronisation.
- Early work in data parallelisation did not focus on synchronisation.
- The placement of message passing synchronous communication between source and sink would (over!) satisfy the synchronisation requirement.
- When using data parallel on new single address space machines, have to reconsider this.
- Basic idea, place a barrier synchronisation where there is a cross-processor data dependence.
Synchronisation

Do i = 1, 16
  a(i) = b(i)
Enddo

Do i = 1, 16
  c(i) = a(i)
Enddo

Do i = 1, 16
  a(17-i) = b(i)
Enddo

Do i = 1, 16
  c(i) = a(i)
Enddo

- Barrier placed between each loop. But are they necessary?
- Data that is written always local. (local write rule)
- Data that is aligned on partitioned index is local.
- No need for barriers here
Summary

- VERY brief overview of auto-parallelism
- Parallelisation for fork/join
- Mapping parallelism to shared memory multi-processors
- Data Partitioning and SPMD parallelism
- Multi-core processor are common place
- Sure to be an active area of research for years to come
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