

Compiler Optimisation

11 – Parallelisation

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Introduction

This lecture:

- Parallelisation for fork/join
- Mapping parallelism to shared memory multi-processors
- Loop distribution and fusion
- Data Partitioning and SPMD parallelism
- Communication, synchronisation and load imbalance.

Introduction

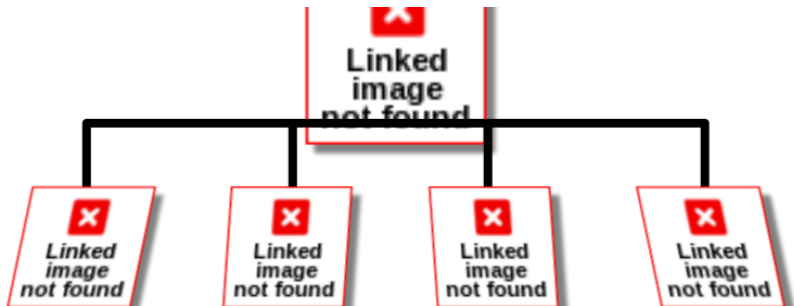
Approaches to parallelisation

- Two approaches to parallelisation
 - Traditional shared memory
 - Single address space
 - Based on finding parallel loop iterations
 - Distributed memory compilation
 - Physically distributed memory uses a mixture of both
 - Focus on mapping data, computation
- Can show equivalence
 - Implement shared memory on distributed
 - Implement distributed memory on shared

Introduction

Approaches to parallelisation

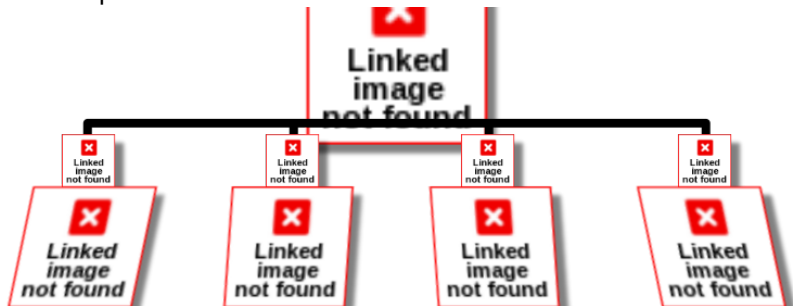
Shared memory - single address space



Introduction

Approaches to parallelisation

Shared memory - probably private caches, but looks like single address space

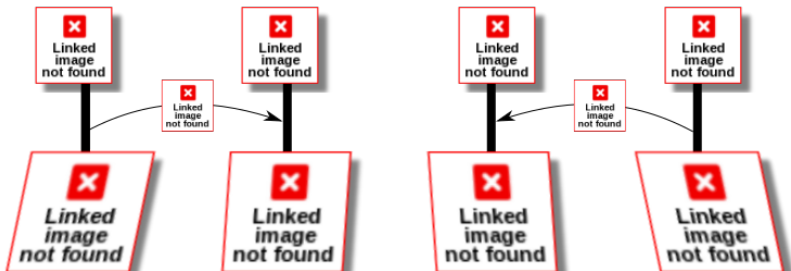


Introduction

Approaches to parallelisation

Distributed memory - each machine has own address space

Use message passing



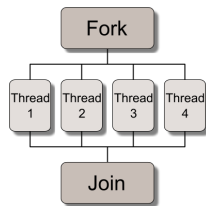
Loop Parallelisation

- Assume a single address space machine. Each processor sees the same set of addresses. Do not need to know physical location of memory reference.
- Control-orientated approach. Concerned with finding independent iterations of a loop. Then map or schedule these to the processor.
- Aim: find maximum amount of parallelism and minimise synchronisation.
- Secondary aim: improve load imbalance. Inter-processor communication not considered.
- Main memory just part of hierarchy - so use uni-processor approaches.

Loop Parallelisation

Fork/join

- Fork (create) threads at beginning of loop
- Thread executes one or more iterations.
Depend on later scheduling policy
- Join (synchronisation/barrier) at end of loop
- Synchronisation expensive
 - Favour outer loop parallelism
 - Loop interchange



Loop Parallelisation

DOALL Implementation

Original

```
Do i = 1, N
  A(i)=B(i)
  C(i)=A(i)
Enddo
```

Driver

```
p=get_num_proc()
fork(x_sub,p)
join()
```

Per thread

```
SUBROUTINE x_sub()
  p = get_num_proc()
  z = my_id()
  ilo = N/p * (z-1) + 1
  ihi = min(N, ilo+N/p)
  Do i = ilo, ihi
    A(i) = B(i)
    C(i) = A(i)
  Enddo
END
```

Generate p independent threads of work

- Each has private local variables, z , ilo , ihi
- Access shared arrays A , B and C

Loop Parallelisation

Using loop interchange

Original

```
Do i = 1, N
  Do j = 1, M
    a(i+1,j) = a(i,j)+c
  Enddo
Enddo
```

$O(n)$ synchronisation points

```
Do i = 1, N
  Parallel Do j = 1, M
    a(i+1,j) = a(i,j)+c
  Enddo
Enddo
```

Interchanged

```
Do j = 1, M
  Do i = 1, N
    a(i+1,j) = a(i,j)+c
  Enddo
Enddo
```

1 synchronisation point

```
Parallel Do j = 1, M
  Do i = 1, N
    a(i+1,j) = a(i,j)+c
  Enddo
Enddo
```

Interchange has reduced synchronisation overhead from $O(N)$ to 1.

Parallelisation approach

- Loop distribution eliminates carried dependences and creates opportunity for outer-loop parallelism.
- However increases number of synchronisations needed after each distributed loop.
- Maximal distribution often finds components too small for efficient parallelisation
- Solution: fuse together parallelisable loops.

Loop Fusion

Fusion illegal if changes the dependence direction

Two loops - same bounds

```
Do i = 1, N
  a(i) = b(i) + c
Enddo
Do i = 1, N
  d(i) = a(i) + e
Enddo
```

Fused

```
Do i = 1, N
  a(i) = b(i) + c
  d(i) = a(i) + e
Enddo
```

Profitability: Parallel and sequential loops should not generally be merged

Loop Fusion

Fusion illegal if changes the dependence direction

Two loops - same bounds

```
Do i = 1, N
  a(i) = b(i) + c
Enddo
Do i = 1, N
  d(i) = a(i+1) + e
Enddo
```

Fused

```
Do i = 1, N
  a(i) = b(i) + c
  d(i) = a(i+1) + e
Enddo
```

Take care that fusing does not prevent parallelisation

Data Parallelism

- Alternative approach where we focus on mapping data rather than control flow to the machine
- Data is partitioned/distributed across the processors of the machine
- The computation is then mapped to follow the data - typically such that work writes to local data. Local write/owner computes rule.
- All of this is based on the SPMD computational model. Each processor runs one thread executing the same program, operating on the different data
- This means that loop bounds change from processor to processor.

Data Parallelism

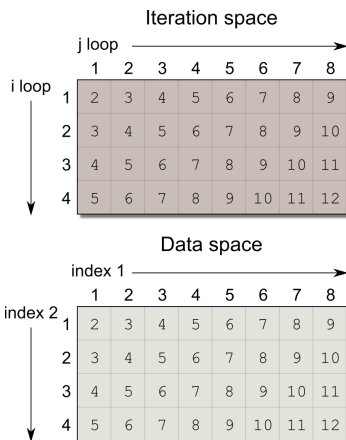
Mapping

- Placement of work and data on processors. Assume parallelism found in a previous stage
- Typically program parallelism $O(n)$ is much greater than machine parallelism $O(p)$, $n \gg p$
- We have many options as to how to map a parallel program
- Key issue: What is the best mapping that achieves $O(p)$ parallelism but minimises cost
- Costs include communication, load imbalance and synchronisation

Data Placement

Simple Fortran example

```
Dimension Integer a(4,8)
Do i = 1, 4
  Do j = 1, 8
    a(i,j) = i + j
  Enddo
Enddo
```



Note that here data and iteration spaces line up. Generally not the case

Data Placement

Simple Fortran example

Partitioning by columns of a and hence iterator j : Local writes

Processor 1

```
Dimension Integer
```

```
a(4,1..2)
```

```
Do i = 1, 4
```

```
  Do j = 1, 2
```

```
    a(i,j) = i + j
```

```
  Enddo
```

```
Enddo
```

...

Processor 3

```
Dimension Integer
```

```
a(4,5..6)
```

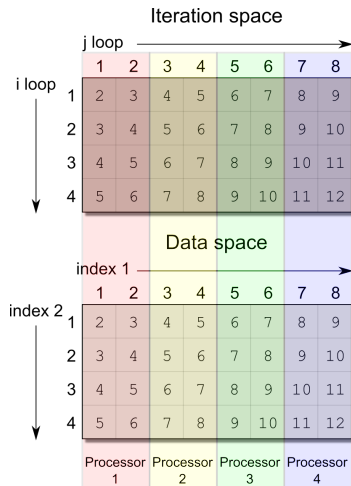
```
Do i = 1, 4
```

```
  Do j = 5, 6
```

```
    a(i,j) = i + j
```

```
  Enddo
```

```
Enddo
```



Data Placement

Simple Fortran example

Partitioning by rows of a and hence iterator i: Local writes

Processor 1

```
Dimension Integer
```

```
a(1..1,1..8)
```

```
Do i = 1, 1
```

```
  Do j = 1, 8
```

```
    a(i,j) = i + j
```

```
  Enddo
```

```
Enddo
```

...

Processor 3

```
Dimension Integer
```

```
a(3..3,1..8)
```

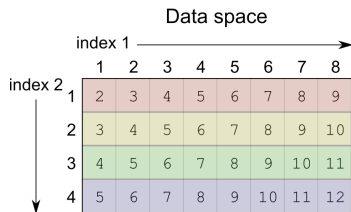
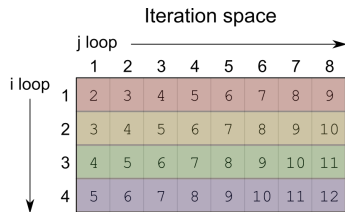
```
Do i = 3, 3
```

```
  Do j = 1, 8
```

```
    a(i,j) = i + j
```

```
  Enddo
```

```
Enddo
```



Linear program representation

- Iteration space defined by loop bound constraints
- Constraints are affine ($\vec{a}i \leq \vec{c}$)
- Matrix standard form ($A\vec{i} \leq \vec{c}$)
- Each constraint defines half space
- Iteration space is intersection of half spaces (polytope)
- Iterations at integer lattice points within iteration space
 - Typically unit lattices
- Array access patterns as affine functions over iteration vectors
($f(\vec{i}) = B\vec{i} + d$)

Linear program representation

Example

Iteration constraints

Do $i = 1, 16$

$$1 \leq i$$

Do $j = 1, 16$

$$1 \leq j$$

Do $k = i, 16$

$$i \leq k$$

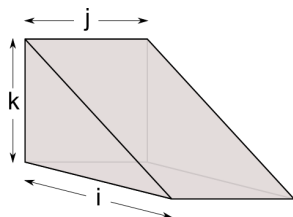
$c(i,j) = c(i,j)$

$$i \leq 16$$

$+a(i,k)*b(j,k)$

$$j \leq 16$$

$$k \leq 16$$



Linear program representation

Example

Make into standard form

Do $i = 1, 16$

Do $j = 1, 16$

Do $k = i, 16$

$c(i,j) = c(i,j)$

$+a(i,k)*b(j,k)$

$$1 - i \leq 0$$

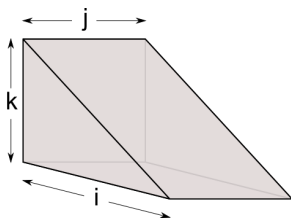
$$1 - j \leq 0$$

$$i - k \leq 0$$

$$i \leq 16$$

$$j \leq 16$$

$$k \leq 16$$



Linear program representation

Example

Do $i = 1, 16$

 Do $j = 1, 16$

 Do $k = i, 16$

$c(i,j) = c(i,j)$

$+a(i,k)*b(j,k)$

Make into standard form

$$-i \leq -1$$

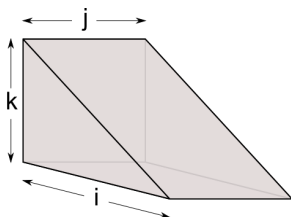
$$-j \leq -1$$

$$i - k \leq 0$$

$$i \leq 16$$

$$j \leq 16$$

$$k \leq 16$$



Linear program representation

Example

```
Do i = 1, 16
  Do j = 1, 16
    Do k = i, 16
      c(i,j) = c(i,j)
        +a(i,k)*b(j,k)
```

Make into standard form

$$-1.i + 0.j + 0.k \leq -1$$

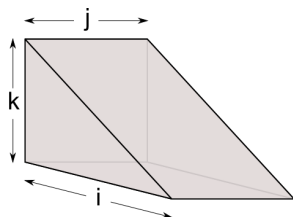
$$0.i + -1.j + 0.k \leq -1$$

$$1.i + 0.j + -1.k \leq 0$$

$$1.i + 0.j + 0.k \leq 16$$

$$0.i + 1.j + 0.k \leq 16$$

$$0.i + 0.j + 1.k \leq 16$$



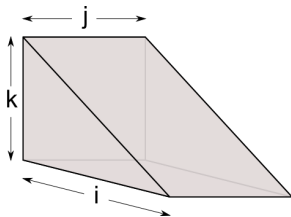
Linear program representation

Example

```
Do i = 1, 16
  Do j = 1, 16
    Do k = i, 16
      c(i,j) = c(i,j)
        +a(i,k)*b(j,k)
```

Make into standard form

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ j \\ k \end{bmatrix} \leq \begin{bmatrix} -1 \\ -1 \\ 0 \\ 16 \\ 16 \\ 16 \end{bmatrix}$$



Linear program representation

Example

```
Do i = 1, 16
  Do j = 1, 16
    Do k = i, 16
      c(i,j) = c(i,j)
        +a(i,k)*b(j,k)
```

Make into standard form

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ j \\ k \end{bmatrix} \leq \begin{bmatrix} -1 \\ -1 \\ 0 \\ 16 \\ 16 \\ 16 \end{bmatrix}$$

Access matrices $U_c U_a U_b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}_c \begin{bmatrix} i \\ j \\ k \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}_a \begin{bmatrix} i \\ j \\ k \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_b \begin{bmatrix} i \\ j \\ k \end{bmatrix}$$

Linear program representation

Transformations

- Many transformations¹ are affine functions over linear program
- **Scanning** then regenerates code
- Partitioning loop for different processors by adding partition constraints

¹Skew, reverse, interchange, etc

Linear program representation

Partitioning example

Split four processors equally along i
Processor 2

Do $i = 5, 8$

Do $j = 1, 16$

Do $k = i, 16$

$c(i, j) = c(i, j)$
 $+ a(i, k) * b(j, k)$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \\ \hline 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ \hline -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i \\ j \\ k \end{bmatrix} \leq \begin{bmatrix} -1 \\ -1 \\ 0 \\ \hline 16 \\ 16 \\ 16 \\ \hline -5 \\ 8 \end{bmatrix}$$

Determine local array bounds λ_z, v_z for each processor $1 \leq z \leq p$.

$$\lambda_1 = 1, \lambda_2 = 5, \lambda_3 = 9, \lambda_4 = 13$$

$$v_1 = 4, v_2 = 8, v_3 = 12, v_4 = 16$$

Determine local write constraint $\lambda_z \leq \mathcal{U}_c \leq v_z, 5 \leq i \leq 8$ and add to polytope

Works for arbitrary loop structures and accesses

Load balancing

- Load describes amount of work each processor must do
- For simple loop bodies is number of iterations assigned to each processor
- All processors wait for slowest at join point
- Want to minimise idle time at join

Load balancing

Example

Do $i = 1, 16$

Do $j = 1, 16$

Do $k = i, 16$

$$c(i,j) = c(i,j) + a(i,k) * b(j,k)$$

Assuming c, a, b are to be partitioned in a similar manner

How should we partition to minimise load imbalance?

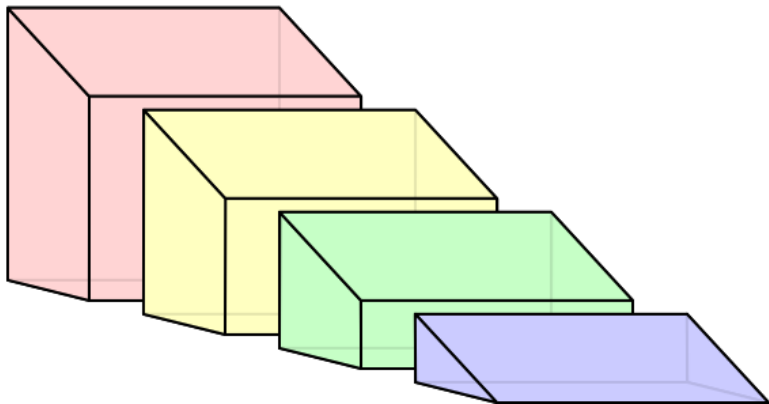
- Row (along i): processor load 928, 672, 416, 160 iterations
- Column (along j): processor load 544, 544, 544, 544 iterations

Why this variation?

Load balance

Example

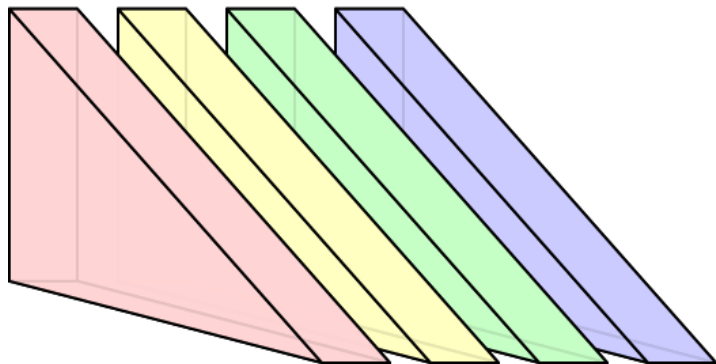
Partition by row (along i)



Load balance

Example

Partition by column (along j)



Partition by “invariant” iterator j .

Load balance

Polytope based

- Generally straightforward to 'read' from polytope
- Iteration variable with zeros elsewhere in rows and columns is 'invariant'
- Partitioning on 'invariant' yields balance

i 'conflicts' with k

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

j 'invariant'

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Reducing Communication

We wish to partition work and data to reduce amount of communication or remote accesses

```
Dimension a(n,n) b(n,n)
Do i = 1, n
  Do j = 1, n
    Do k = 1, n
      a(i,j) = b(i,k)
    Enddo
  Enddo
Enddo
```

How should we partition to reduce communication?

Reducing communication

Each processor has rows of a and b allocated to it
Look at access pattern of second processor

```
Dimension a(n,n) b(n,n)
```

```
Do i = 1, n
```

```
  Do j = 1, n
```

```
    Do k = 1, n
```

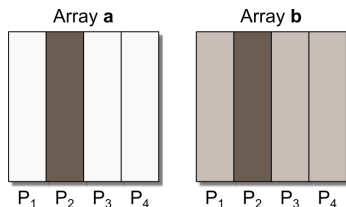
```
      a(i,j) = b(i,k)
```

```
    Enddo
```

```
  Enddo
```

```
Enddo
```

The columns of a scheduled to P2 access all of b $n^2 - \frac{n^2}{p}$ remote access



Reducing communication

Each processor has rows of a and b allocated to it
Look at access pattern of second processor

```
Dimension a(n,n) b(n,n)
```

```
Do i = 1, n
```

```
  Do j = 1, n
```

```
    Do k = 1, n
```

```
      a(i,j) = b(i,k)
```

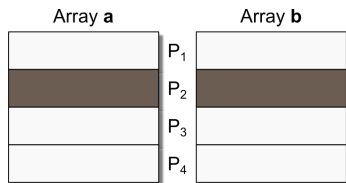
```
    Enddo
```

```
  Enddo
```

```
Enddo
```

The rows of a scheduled to P_2 access corresponding rows of b .

0 remote accesses.



Alignment

- The first index of a and b have the same subscript $a(i,j)$, $b(i,k)$
- They are said to be aligned on this index
- Partitioning on an aligned index makes all accesses local to that array reference

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}_a, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}_b$$

Can transform array layout to make arrays more aligned for partitioning.

Find \mathcal{A} such that $\mathcal{A}U_x$ is maximally aligned with U_y

Global alignment problem

Synchronisation

- Alignment information can also be used to eliminate synchronisation
- Early work in data parallelisation did not focus on synchronisation
- The placement of message passing synchronous communication between source and sink would (over!) satisfy the synchronisation requirement
- When using data parallel on new single address space machines, have to reconsider this.
- Basic idea, place a barrier synchronisation where there is a cross-processor data dependence.

Synchronisation

```
Do i = 1, 16  
  a(i) = b(i)  
Enddo
```

```
Do i = 1, 16  
  c(i) = a(i)  
Enddo
```

```
Do i = 1, 16  
  a(17-i) = b(i)  
Enddo
```

```
Do i = 1, 16  
  c(i) = a(i)  
Enddo
```

- Barrier placed between each loop. But are they necessary?
- Data that is written always local. (local write rule)
- Data that is aligned on partitioned index is local.
- No need for barriers here

Summary

- VERY brief overview of auto- parallelism
- Parallelisation for fork/join
- Mapping parallelism to shared memory multi-processors
- Data Partitioning and SPMD parallelism
- Multi-core processor are common place
- Sure to be an active area of research for years to come

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