Parallelisation

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Lecture Overview

- Parallelisation for fork/join
- Mapping parallelism to shared memory multi-processors
- Loop distribution and fusion
- Data Partitioning and SPMD parallelism
- Communication, synchronisation and load imbalance.

Approaches to parallelisation

- Two approaches to parallelisation
  - Traditional shared memory. Based on finding parallel loop iterations
  - Distributed memory compilation. Focus on mapping data, computation follows

- Now single address space, physically distributed memory uses a mixture of both.

- Actually, can show equivalence

Loop Parallelisation

- Assume a single address space machine. Each processor sees the same set of addresses. Do not need to know physical location of memory reference.

- Control- orientated approach. Concerned with finding independent iterations of a loop. Then map or schedule these to the processor.

- Aim: find maximum amount of parallelism and minimise synchronisation.

- Secondary aim: improve load imbalance. Inter-processor communication not considered.

- Main memory just part of hierarchy - so use uni-processor approaches.

Loop Parallelisation: Fork/join

- Fork/join assumes that there is a forking of parallel threads at the beginning of a parallel loop
- Each thread executes one or more iterations. Depend on later scheduling policy
- There is a corresponding join or synchronisation at the end
- For this reason loop parallel approaches favour outer loop parallelism
- Can use loop interchange to improve the fork/join overhead.

Parallel Loop : DOALL Implementation

``Do i = 1 , N A(i) = B(i) C(i) = A(i) Enddo```	```p = get_num_proc() fork (x_sub, p) join()```	```SUBROUTINE x_sub() p = get_num_proc() z = my_id() ilo = N/p * (z-1) +1 ihi = min(N, ilo+N/p) Do i = ilo , ihi A(i) = B(i) C(i) = A(i) Enddo END```

- Generate p independent threads of work
  - Each has private local variables, z, ilo, ihi
  - Access shared arrays A,B and C

Loop Parallelisation: Using loop interchange

```
Do i = 1,N
  Do j = 1,M
    a(i+1,j) = a(i,j) +c
  Enddo
Enddo
```

```
Do i = 1,N
  Parallel Do j = 1,M
    a(i+1,j) = a(i,j) +c
  Enddo
Enddo
```

```
Do j = 1,M
  Do i = 1,N
    a(i+1,j) = a(i,j) +c
  Enddo
Enddo
```

```
Parallel Do j = 1,M
  Do i = 1,N
    a(i+1,j) = a(i,j) +c
  Enddo
Enddo
```

Interchange has reduced synchronisation overhead from O(N) to 1.

Parallelisation approach

- Loop distribution eliminates carried dependences and creates opportunity for outer-loop parallelism.
- However increases number of synchronisations needed after each distributed loop.
- Maximal distribution often finds components too small for efficient parallelisation
- Solution: fuse together parallelisable loops.

Loop Fusion

- Fusion is illegal if fusing two loops causes the dependence direction to be changed

```
Do i = 1,N
    a(i) = b(i) +c
Enddo
Do i = 1,N
    d(i) = a(i+1) +e
Enddo
```

```
Do i = 1,N
    a(i) = b(i) +c
    d(i) = a(i+1) +e
Enddo
```

- Profitability: Parallel loops should not generally be merged with sequential loops: Tapered fusion

Data Parallelism

- Alternative approach where we focus on mapping data rather than control flow to the machine
- Data is partitioned/distributed across the processors of the machine
- The computation is then mapped to follow the data - typically such that work writes to local data. Local write/owner computes rule.
- All of this is based on the SPMD computational model. Each processor runs one thread executing the same program, operating on the different data
- This means that loop bounds change from processor to processor.

Data Parallelism: Mapping

- Placement of work and data on processors. Assume parallelism found in a previous stage

- Typically program parallelism $O(n)$ is much greater than machine parallelism $O(p)$, $n >> p$

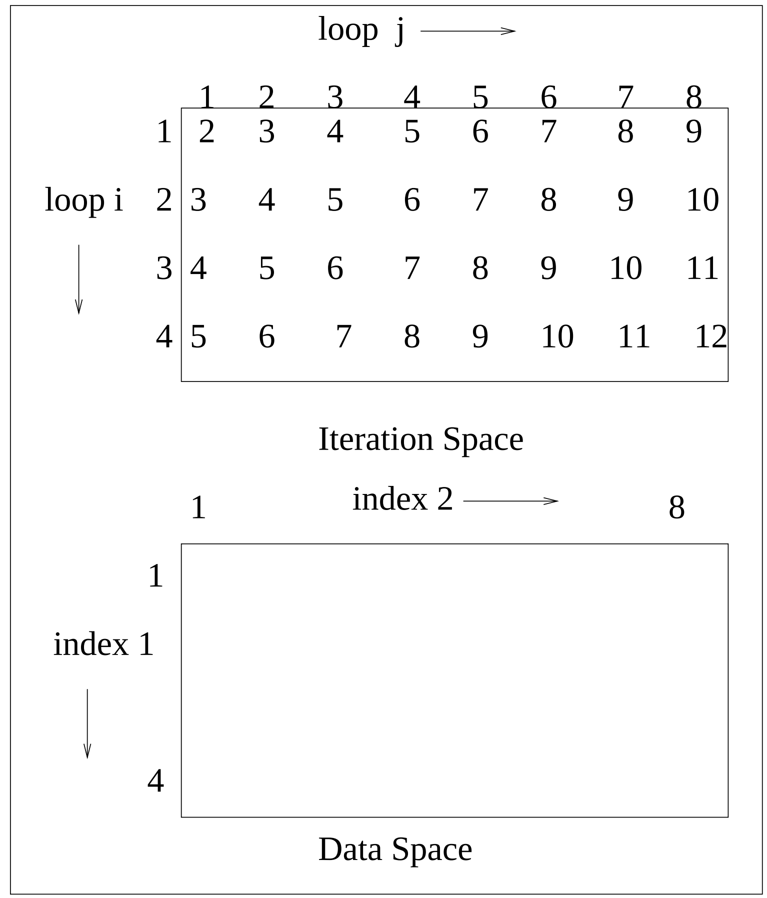
- We have many options as to how to map a parallel program

- Key issue: What is the best mapping that achieves $O(p)$ parallelism but minimises cost

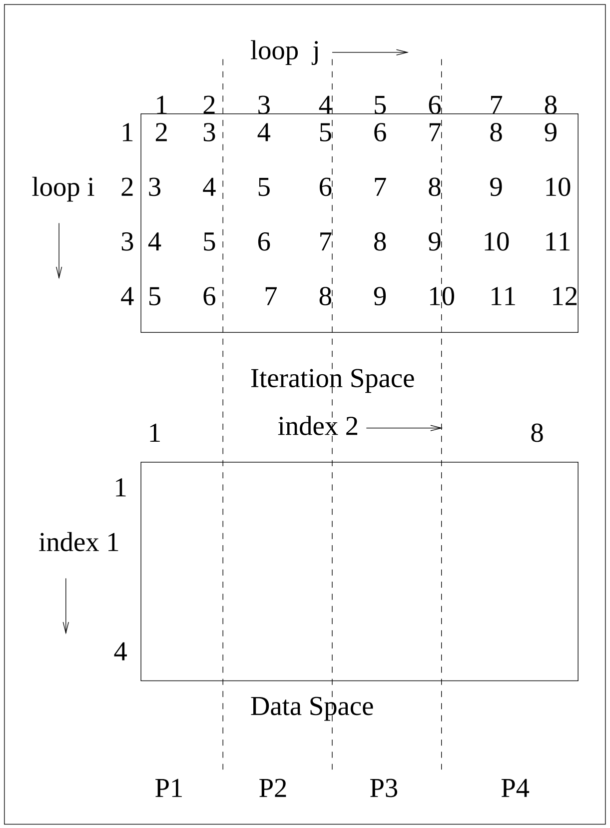
- Costs include communication, load imbalance and synchronisation

Simple Fortran example

```
Dimension Integer a(4,8)
Do i = 1, 4
  Do j = 1,8
    a(i,j) = i + j
  Enddo
Enddo
```

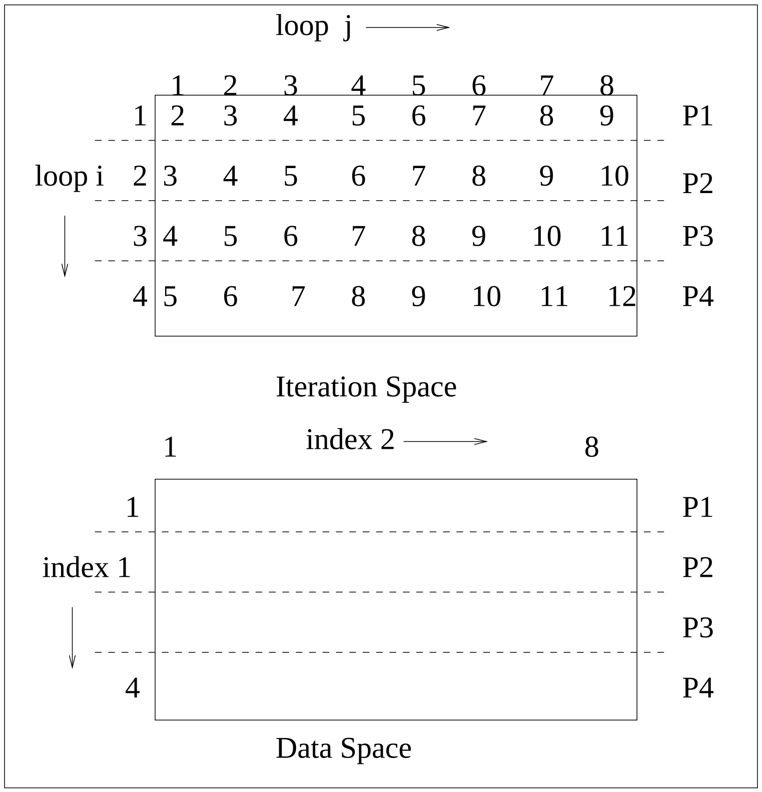


Partitioning by columns of a and hence iterator j : Local writes




```
Dimension Integer a(4,1..2)
Do i = 1, 4            Processor 1
  Do j = 1,2
    a(i,j) = i + j
  Enddo
Enddo
...
Dimension Integer a(4,5..6)
Do i = 1, 4            Processor 3
  Do j = 5,6
    a(i,j) = i + j
  Enddo
Enddo      etc..
```

Partitioning by rows of a and hence iterator i: Local writes




```
Dimension Integer a(1..1,1..8)
Do i = 1, 1          Processor 1
  Do j = 1,8
    a(i,j) = i + j
  Enddo
Enddo
...
Dimension Integer a(3..3,1..8)
Do i = 3, 3          Processor 3
  Do j = 1,8
    a(i,j) = i + j
  Enddo
Enddo      etc..
```

Linear Program representation

```
Do i = 1,16
 Do j = 1,16
  Do k = i,16
   c(i,j) = c(i,j)
           +a(i,k)*b(j,k)
```

$$\left[\begin{array}{ccc} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \left[\begin{array}{c} i \\ j \\ k \end{array}\right] \leq \left[\begin{array}{c} -1 \\ -1 \\ 0 \\ \hline 16 \\ 16 \\ 16 \end{array}\right]$$

Polytope $AJ \leq b$. Access matrices $\mathcal{U}_c\, \mathcal{U}_a\, \mathcal{U}_b$

$$\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right]_c \left[\begin{array}{c} i \\ j \\ k \end{array}\right], \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right]_a \left[\begin{array}{c} i \\ j \\ k \end{array}\right], \left[\begin{array}{ccc} 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]_b \left[\begin{array}{c} i \\ j \\ k \end{array}\right]$$

Can we automatically generate code for each processor given that writes must be local?

Partitioning: Ex. 1st index: 4 procs: c(16,16), a(16,16),b(16,16)

```
Do i = 5,8
 Do j = 1,16
  Do k = i,16
   c(i,j) = c(i,j)
           +a(i,k)*b(j,k)
```

$$\left[\begin{array}{ccc} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \\ \hline 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ \hline -1 & 0 & 0 \\ 1 & 0 & 0 \end{array}\right] \left[\begin{array}{c} i \\ j \\ k \end{array}\right] \leq \left[\begin{array}{c} -1 \\ -1 \\ 0 \\ \hline 16 \\ 16 \\ 16 \\ \hline -5 \\ 8 \end{array}\right]$$

Partitioning: Determine local array bounds λ_z, υ_z for each processor $1 \leq z \leq p$.

$\lambda_1 = 1, \lambda_2 = 5, \lambda_3 = 9, \lambda_4 = 13$ $\upsilon_1 = 4, \upsilon_2 = 8, \upsilon_3 = 12, \upsilon_4 = 16$

Determine local write constraint $\lambda_z \leq \mathcal{U}_c \leq \upsilon_z, 5 \leq i \leq 8$ and add to polytope

Works for arbitrary loop structures and accesses

Load Balance : 4 procs

```
Do i = 1,16
  Do j = 1,16
    Do k = i,16
      c(i,j) = c(i,j) +a(i,k)*b(j,k)
```

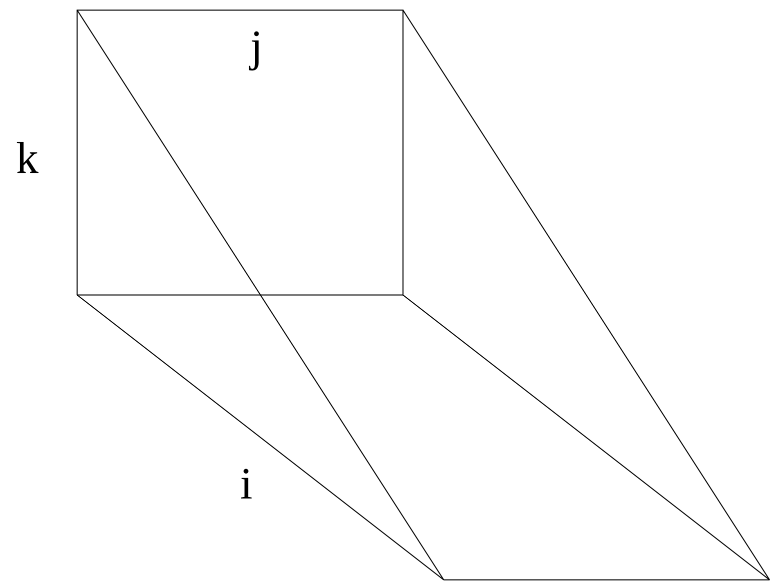
Assuming c, a,b are to be partitioned in a similar manner

How should we partition to minimise load imbalance?

- Row: 928,672,416,160 per processor, load imbalance: 384
- Column: 544 iterations per processor

Why this variation?

Load Balance :



Partition by ""invariant" iterator j.

Can be expressed as a polytope condition

Reducing Communication

We wish to partition work and data to reduce amount of communication or remote accesses

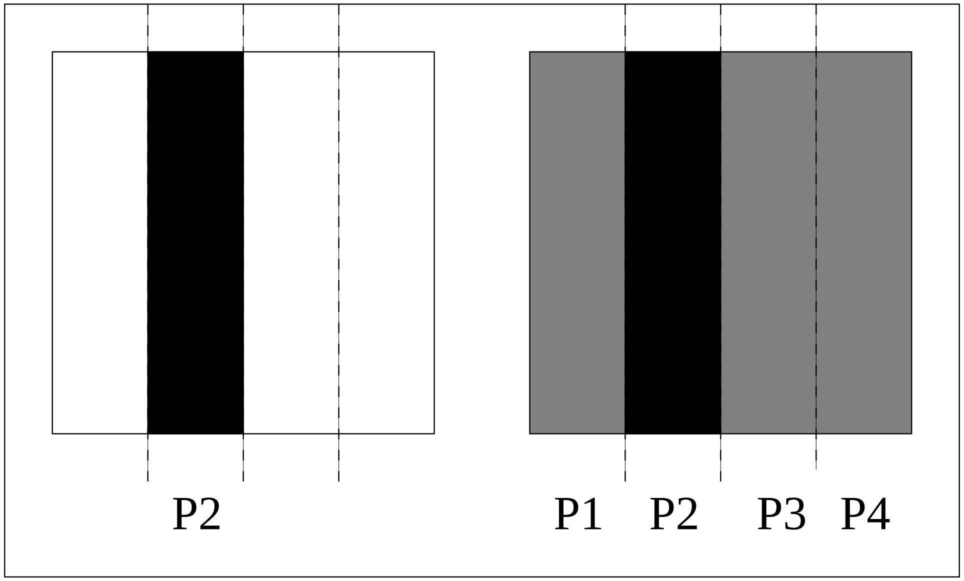
```
Dimension a(n,n) b(n,n)
Do i = 1,n
  Do j = 1,n
    Do k = 1,n
      a(i,j) = b(i,k)
    Enddo
  Enddo
Enddo
```

How should we partition to reduce communication?

Reducing Communication :Column Partitioning

Each processor has columns of a and b allocated to it

Look at access pattern of second processor

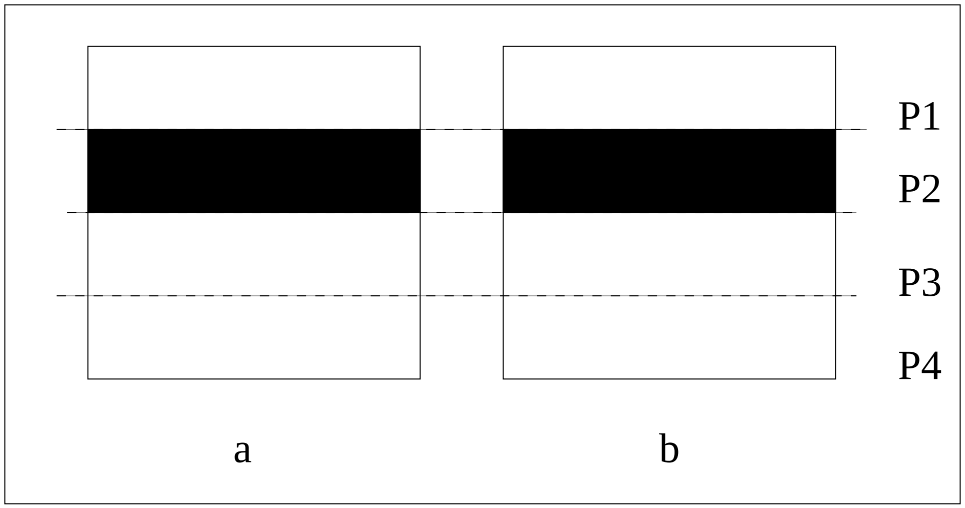


The columns of a scheduled to P2 access all of b $n^2 - \frac{n^2}{p}$ remote access

Reducing Communication :Row Partitioning

Each processor has rows of a and b allocated to it

Look at access pattern of second processor



The rows of a scheduled to P2 access corresponding rows of b.

0 remote accesses.

Alignment

- The first index of a and b have the same subscript a(i,j), b(i,k)
- They are said to be aligned on this index
- Partitioning on an aligned index makes all accesses local to that array reference

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}_a , \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}_b$$

Can transform array layout to make arrays more aligned for partitioning.

Find $\mathcal{A}$ such that $\mathcal{A}\mathcal{U}_x$ is maximally aligned with $\mathcal{U}_y$

Global alignment problem

Synchronisation

- Alignment information can also be used to eliminate synchronisation
- Early work in data parallelisation did not focus on synchronisation
- The placement of message passing synchronous communication between source and sink would (over!) satisfy the synchronisation requirement
- When using data parallel on new single address space machines, have to reconsider this.
- Basic idea, place a barrier synchronisation where there is a cross-processor data dependence.

Synchronisation

```
Do i = 1,16
 a(i) = b(i)
Enddo
Do i = 1,16
 c(i) = a(i)
Enddo
```

```
Do i = 1,16
 a(17-i) = b(i)
Enddo
Do i = 1,16
 c(i) = a(i)
Enddo
```

- Barrier placed between each loop. But are they necessary?
- Data that is written always local. (localwrite rule)
- Data that is aligned on partitioned index is local.
- No need for barriers here

Summary

- VERY brief overview of auto- parallelism
- Parallelisation for fork/join
- Mapping parallelism to shared memory multi-processors
- Data Partitioning and SPMD parallelism
- Multi-core processor are common place
- Sure to be an active area of research for years to come