Process equivalence: motivation

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- More natural way of specifying this:
  When all actions but $a_1, \ldots, a_n$ are restricted, the system should “behave like” the process $P$, defined by

$$P \overset{\text{def}}{=} a_1 \cdot a_2 \cdot \ldots \cdot a_n \cdot P$$
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Generally: many systems are informally specified by “behave like” statements.
Example: when using telnet our machine should “behave like” the remote machine (abstracting from delays).
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- Generally: many systems are informally specified by “behave like” statements.
  Example: when using `telnet` our machine should “behave like” the remote machine (abstracting from delays).
- But how to formalise “behavioural equivalence”?
Wish list

1. Behavioural equivalence should be an equivalence relation, reflexive, symmetric and transitive.
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2. Processes that may terminate (deadlock) should not be equivalent to processes that may not terminate (deadlock).
3. Congruence: if a component $Q$ of $P$ is replaced by an equivalent component $Q'$ yielding $P'$, then $P$ and $P'$ should also be equivalent.
4. Two processes should be equivalent iff they satisfy exactly the same properties (such as expressible in modal or temporal logic)
5. It should abstract from silent actions.

We deal first with conditions 1−4
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A first candidate: trace equivalence

- A trace of a process $E$ is a sequence $w$ of actions such that $E \xrightarrow{w} F$ for some process $F$. 
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- Counterexample. $Cl$, $Cl'$ trace equivalent

\[
\begin{align*}
Cl & \overset{\text{def}}{=} \text{tick}.Cl \\
Cl' & \overset{\text{def}}{=} \text{tick}.Cl' + \text{tick}.0
\end{align*}
\]
A second candidate: completed trace equivalence

- A completed trace of $E$ is a sequence $w$ of actions such that $E \xrightarrow{w} F$ for some process $F$ that cannot execute any action.
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- This notion satisfies 1 and 2, but not 3.

$$\text{Ven}_1 \overset{\text{def}}{=} 1p.1p.\left(\text{tea.Ven}_1 + \text{coffee.Ven}_1\right)$$

$$\text{Ven}_2 \overset{\text{def}}{=} 1p.\left(1p.\text{tea.Ven}_2 + 1p.\text{coffee.Ven}_2\right)$$

$$\text{Use} \overset{\text{def}}{=} 1p.1p.\text{tea.ok.0}$$
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\begin{align*}
\text{Ven}_1 & \overset{\text{def}}{=} 1p.1p.(\text{tea.Ven}_1 + \text{coffee.Ven}_1) \\
\text{Ven}_2 & \overset{\text{def}}{=} 1p.(1p.\text{tea.Ven}_2 + 1p.\text{coffee.Ven}_2) \\
\text{Use} & \overset{\text{def}}{=} 1p.1p.\text{tea.ok}.0
\end{align*}
\]

- $\text{Ven}_1$ and $\text{Ven}_2$ are completed-trace equivalent, but $(\text{Ven}_1 \mid \text{Use})\setminus K$ and $(\text{Ven}_2 \mid \text{Use})\setminus K$, where $K = \{1p, \text{tea, coffee}\}$, are not.