Communication and Concurrency
Lecture 5

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Modal (Hennessy-Milner) logic: syntax

$$\Phi ::= \mathtt{tt} \mid \mathtt{ff} \mid \Phi_1 \wedge \Phi_2 \mid \Phi_1 \vee \Phi_2 \mid [K]\Phi \mid \langle K \rangle \Phi$$

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- a formula $\langle K \rangle \Phi$, where K is any set of actions, read as "diamond K Φ", or "for some K-derivative Φ."

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We define when a process E satisfies a formula Φ. Either E satisfies Φ, denoted by $E \models \Phi$, or it doesn't, denoted by $E \not\models \Phi$.

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- A process E satisfies $\langle K \rangle \Phi$ if E can become a process that satisfies Φ by carrying out an action in K

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  E cannot do a tick
- $E \models \langle \mathtt{tick} \rangle \mathtt{ff}$
  This is equivalent to ff!
- $E \models [\mathtt{tick}] \mathtt{tt}$
  This is equivalent to true!

Checking satisfaction

$$\mathtt{Cl} \stackrel{\text{def}}{=} \mathtt{tick}.\mathtt{Cl}$$

Does $\mathtt{Cl}$ have the property: $[\mathtt{tick}](\langle\mathtt{tick}\rangle\mathtt{tt} \wedge [\mathtt{tock}]\mathtt{ff})$?

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- iff $\mathtt{Cl} \models \langle\mathtt{tick}\rangle\mathtt{tt} \wedge [\mathtt{tock}]\mathtt{ff}$
- iff $\mathtt{Cl} \models \langle\mathtt{tick}\rangle\mathtt{tt}$ and $\mathtt{Cl} \models [\mathtt{tock}]\mathtt{ff}$

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We write

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- $-a_1, \ldots, a_n$ for $A - \{a_1, \ldots, a_n\}$

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- Φ holds after one step
- $E \models \langle - \rangle \mathtt{tt} \wedge [-](\langle - \rangle \mathtt{tt} \wedge [-](\langle - \rangle \mathtt{tt} \wedge [-a]\mathtt{ff}))$

Exercise

Process	Formula	Y/N
$\mathtt{a}.0 + \mathtt{a}.\mathtt{b}.0$	$\langle \mathtt{a} \rangle \langle \mathtt{b} \rangle \mathtt{tt}$	
	$\langle \mathtt{a} \rangle [\mathtt{b}] \mathtt{ff}$	
	$[\mathtt{a}] \langle \mathtt{b} \rangle \mathtt{tt}$	
	$[\mathtt{a}][\mathtt{b}] \mathtt{ff}$	
$(\mathtt{a}.0 \mid \overline{\mathtt{a}}.0)$	$\langle \mathtt{a} \rangle \mathtt{tt}$	
	$\langle \tau \rangle \mathtt{tt}$	
	$\langle \mathtt{a} \rangle \langle \tau \rangle \mathtt{tt}$	
$(\mathtt{a}.0 \mid \overline{\mathtt{a}}.0) \backslash \mathtt{a}$	$\langle \mathtt{a} \rangle \mathtt{tt}$	
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	$[\mathtt{a}] \langle \mathtt{b} \rangle \mathtt{tt}$	N
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	$\langle \tau \rangle \mathtt{tt}$	Y
	$\langle \mathtt{a} \rangle \langle \tau \rangle \mathtt{tt}$	N

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Φ^c is inductively defined as follows:

$$\begin{aligned}
\mathtt{tt}^c &= \mathtt{ff} \\
\mathtt{ff}^c &= \mathtt{tt} \\
(\Phi_1 \wedge \Phi_2)^c &= \Phi_1^c \vee \Phi_2^c \\
(\Phi_1 \vee \Phi_2)^c &= \Phi_1^c \wedge \Phi_2^c \\
([K]\Phi)^c &= \langle K \rangle \Phi^c \\
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\end{aligned}$$

Proposition: For every process F and HML-formula Φ:

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Basis: $\Phi = \mathtt{tt}$ and $\Phi = \mathtt{ff}$. Trivial.

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Induction step:

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Induction step:

Case $\Phi = \Phi_1 \wedge \Phi_2$

$$\begin{array}{lll}
 & F \models (\Phi_1 \wedge \Phi_2)^c & \\
\text{iff} & F \models \Phi_1^c \vee \Phi_2^c & \\
\text{iff} & F \models \Phi_1^c \text{ or } F \models \Phi_2^c & \text{(by clause for } \vee) \\
\text{iff} & F \not\models \Phi_1 \text{ or } F \not\models \Phi_2 & \text{(by i.h.)} \\
\text{iff} & F \not\models \Phi_1 \wedge \Phi_2 & \text{(by clause for } \wedge).
\end{array}$$

Case $\Phi = [K]\Phi_1$.

$$
\begin{array}{ll}
 & F \models ([K]\Phi_1)^c \\
\text{iff} & F \models \langle K \rangle \Phi_1^c \\
\text{iff} & \exists G.\, \exists a \in K.\, F \xrightarrow{a} G \text{ and } G \models \Phi_1^c \\
\text{iff} & \exists G.\, \exists a \in K.\, F \xrightarrow{a} G \text{ and } G \not\models \Phi_1 \qquad \text{(by i.h.)} \\
\text{iff} & F \not\models [K]\Phi_1
\end{array}
$$

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- A formula is satisfiable (or realisable) if some process satisfies it.
- A formula is unsatisfiable if no process satisfies it.

Realisability, validity, equivalence

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- A formula is satisfiable (or realisable) if some process satisfies it.
- A formula is unsatisfiable if no process satisfies it.
- A formula is valid if all processes satisfy it.
- Two formulas are equivalent if they are satisfied by exactly the same processes

Exercise

Are the following statements true?

				Y/N
If	Φ valid	then	Φ satisfiable	
If	Φ satisfiable	then	Φ^c unsatisfiable	
If	Φ valid	then	Φ^c unsatisfiable	
If	Φ unsatisfiable	then	Φ^c valid	

Exercise

Are the following statements true?

				Y/N
If	Φ valid	then	Φ satisfiable	Y
If	Φ satisfiable	then	Φ^c unsatisfiable	N
If	Φ valid	then	Φ^c unsatisfiable	Y
If	Φ unsatisfiable	then	Φ^c valid	Y

Exercise

Let $\rightarrow$ be the implies connective whose definition is

$$\Phi \rightarrow \Psi \stackrel{\text{def}}{=} \Phi^c \vee \Psi.$$

Are the following statements true?

		Y/N
If $(\Phi \rightarrow \Psi)$ valid and Φ valid	then Ψ valid	
If $(\Phi \rightarrow \Psi)$ satisfiable and Φ satisfiable	then Ψ satisfiable	
If $(\Phi \rightarrow \Psi)$ valid and Φ satisfiable	then Ψ satisfiable	

Exercise

Let $\rightarrow$ be the implies connective whose definition is

$$\Phi \rightarrow \Psi \stackrel{\text{def}}{=} \Phi^c \vee \Psi.$$

Are the following statements true?

		Y/N
If $(\Phi \rightarrow \Psi)$ valid and Φ valid	then Ψ valid	Y
If $(\Phi \rightarrow \Psi)$ satisfiable and Φ satisfiable	then Ψ satisfiable	N
If $(\Phi \rightarrow \Psi)$ valid and Φ satisfiable	then Ψ satisfiable	Y

Exercise: valid V, unsatisfiable U, or neither *N* ?

	V	U	N
$\Phi \rightarrow \neg\Phi$			
$\Phi \rightarrow (\Psi \rightarrow \Phi)$			
$\Phi \rightarrow (\Phi \rightarrow \Psi)$			
$\langle a\rangle \mathtt{tt} \wedge [a]\mathtt{ff}$			
$\langle a\rangle[b](\langle a\rangle \mathtt{tt} \wedge [a]\mathtt{ff})$			
$\langle a\rangle[b](\langle a\rangle \mathtt{tt} \wedge [a]\mathtt{ff}) \wedge [-]\langle b\rangle \mathtt{tt}$			
$\langle a\rangle[b](\langle a\rangle \mathtt{tt} \wedge [a]\mathtt{ff}) \wedge [-]\langle -\rangle \mathtt{tt}$			
$\langle a\rangle(\Phi \vee \Psi) \rightarrow (\langle a\rangle\Phi \vee \langle a\rangle\Psi)$			
$(\langle a\rangle\Phi \wedge \langle a\rangle\Psi) \rightarrow \langle a\rangle(\Phi \wedge \Psi)$			
$[a](\Phi \rightarrow \Psi) \rightarrow ([a]\Phi \rightarrow [a]\Psi)$			
$([a]\Phi \rightarrow [a]\Psi) \rightarrow [a](\Phi \rightarrow \Psi)$			

Exercise: valid V, unsatisfiable U, or neither *N* ?

	V	U	N
$\Phi \rightarrow \neg\Phi$			$\surd$
$\Phi \rightarrow (\Psi \rightarrow \Phi)$	$\surd$		
$\Phi \rightarrow (\Phi \rightarrow \Psi)$			$\surd$
$\langle a\rangle \mathtt{tt} \wedge [a]\mathtt{ff}$		$\surd$	
$\langle a\rangle[b](\langle a\rangle \mathtt{tt} \wedge [a]\mathtt{ff})$			$\surd$
$\langle a\rangle[b](\langle a\rangle \mathtt{tt} \wedge [a]\mathtt{ff}) \wedge [-]\langle b\rangle \mathtt{tt}$		$\surd$	
$\langle a\rangle[b](\langle a\rangle \mathtt{tt} \wedge [a]\mathtt{ff}) \wedge [-]\langle -\rangle \mathtt{tt}$			$\surd$
$\langle a\rangle(\Phi \vee \Psi) \rightarrow (\langle a\rangle\Phi \vee \langle a\rangle\Psi)$	$\surd$		
$(\langle a\rangle\Phi \wedge \langle a\rangle\Psi) \rightarrow \langle a\rangle(\Phi \wedge \Psi)$			$\surd$
$[a](\Phi \rightarrow \Psi) \rightarrow ([a]\Phi \rightarrow [a]\Psi)$	$\surd$		
$([a]\Phi \rightarrow [a]\Psi) \rightarrow [a](\Phi \rightarrow \Psi)$			$\surd$