Communication and Concurrency
Lecture 4

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Renaming and linking

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- renames a_i to b_i (and $\overline{a_i}$ to $\overline{b_i}$)
- and leaves any other action c unchanged

Transition rule

Associated with f is the renaming operator $[f]$

$$\mathrm{R}([f]) \quad \frac{E[f] \xrightarrow{b} F[f]}{E \xrightarrow{a} F} \; b = f(a)$$

Example: $\mathtt{Cop}$ is $\mathtt{B}[\mathtt{in}/\mathtt{i}, \mathtt{out}/\mathtt{o}]$

Assuming e.g $\mathtt{in}/\mathtt{i}$ maps each action $\mathtt{i}(v)$ to $\mathtt{in}(v)$

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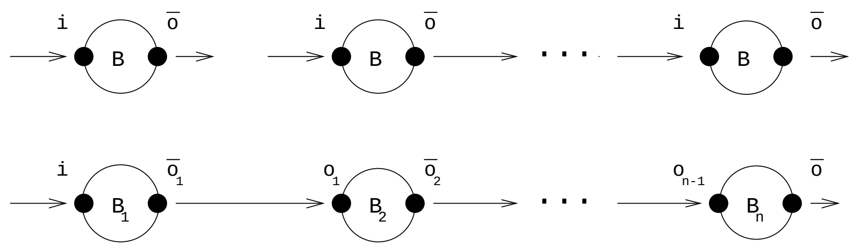

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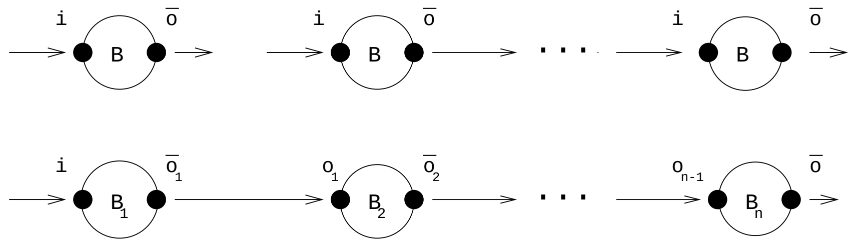

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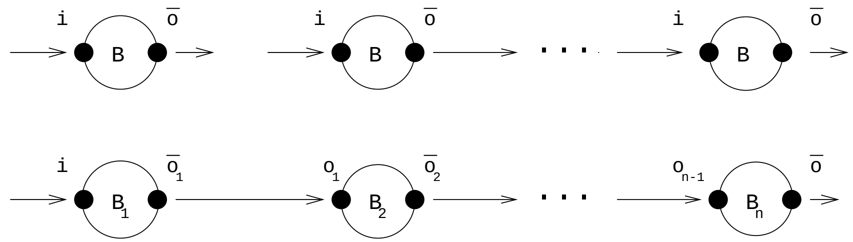

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$$B(n) \equiv (B_1 \mid \ldots \mid B_n)\backslash\{\mathtt{o}_1, \ldots, \mathtt{o}_{n-1}\}$$

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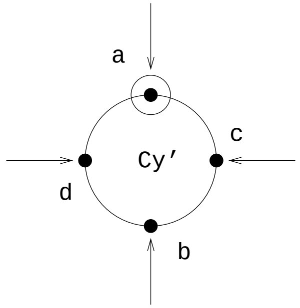
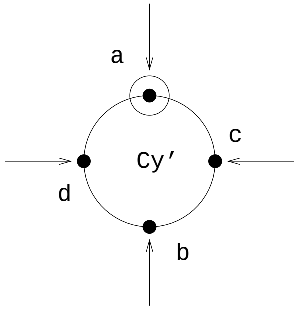


Solution using n simple cyclers ?



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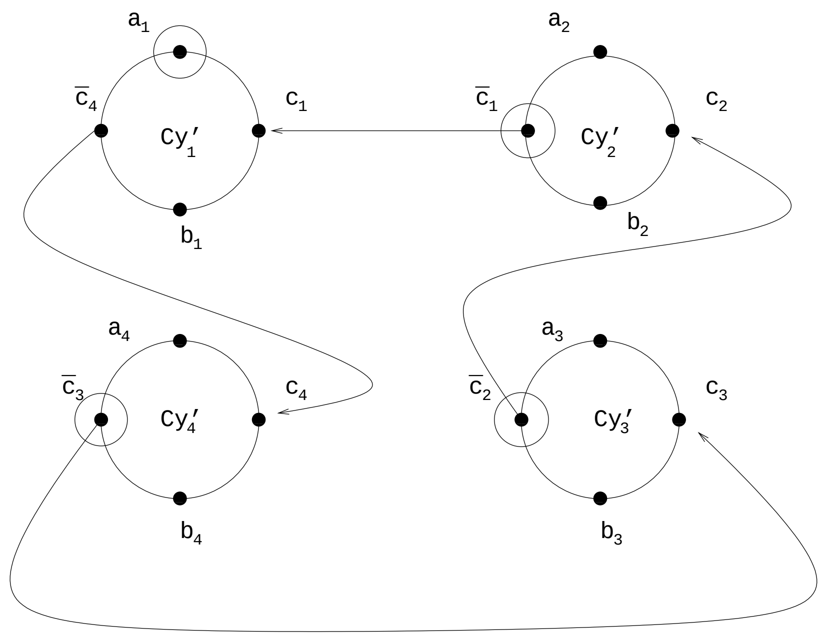
When $n = 4$. What is wrong ?



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How do we know it is right?

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Reading: Chapters 1 and 2, Robin Milner *Communication and Concurrency*, Prentice-Hall, 1989