

Communication and Concurrency

Lecture 2

Colin Stirling (cps)

School of Informatics

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Concurrent composition: $E \mid F$

$$\text{R}(\mid \text{com}) \quad \frac{E \mid F \xrightarrow{\tau} E' \mid F'}{E \xrightarrow{a} E' \quad F \xrightarrow{\bar{a}} F'}$$

$$\text{R}(\mid) \quad \frac{E \mid F \xrightarrow{a} E' \mid F}{E \xrightarrow{a} E'} \quad \frac{E \mid F \xrightarrow{a} E \mid F'}{F \xrightarrow{a} F'}$$

Concurrent composition: $E \mid F$

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Example: user of a copier

$$\begin{array}{l} \text{Cop} \quad \stackrel{\text{def}}{=} \quad \text{in}(x).\overline{\text{out}}(x).\text{Cop} \\ \text{User} \quad \stackrel{\text{def}}{=} \quad \text{write}(x).\text{User}_x \\ \text{User}_v \quad \stackrel{\text{def}}{=} \quad \overline{\text{in}}(v).\text{User} \end{array}$$

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 \end{array}$$

$$\frac{\text{Cop} \mid \text{User}_v \xrightarrow{\tau} \overline{\text{out}}(v).\text{Cop} \mid \text{User}}{\frac{\text{Cop} \xrightarrow{\text{in}(v)} \overline{\text{out}}(v).\text{Cop} \quad \text{User}_v \xrightarrow{\overline{\text{in}}(v)} \text{User}}{\text{in}(x).\overline{\text{out}}(x).\text{Cop} \xrightarrow{\text{in}(v)} \overline{\text{out}}(v).\text{Cop} \quad \overline{\text{in}}(v).\text{User} \xrightarrow{\overline{\text{in}}(v)} \text{User}}}$$

More users

$$\begin{aligned}\text{Cop} &\stackrel{\text{def}}{=} \text{in}(x).\overline{\text{out}}(x).\text{Cop} \\ \text{User} &\stackrel{\text{def}}{=} \text{write}(x).\text{User}_x \\ \text{User}_v &\stackrel{\text{def}}{=} \overline{\text{in}}(v).\text{User}\end{aligned}$$

$$\frac{\text{Cop} \mid (\text{User}_{v_1} \mid \text{User}_{v_2}) \xrightarrow{\tau} \overline{\text{out}}(v_1).\text{Cop} \mid (\text{User} \mid \text{User}_{v_2})}{\frac{\text{Cop} \xrightarrow{\text{in}(v_1)} \overline{\text{out}}(v_1).\text{Cop} \quad \text{User}_{v_1} \mid \text{User}_{v_2} \xrightarrow{\overline{\text{in}}(v_1)} \text{User} \mid \text{User}_{v_2}}{\text{in}(x).\overline{\text{out}}(x).\text{Cop} \xrightarrow{\text{in}(v_1)} \overline{\text{out}}(v_1).\text{Cop} \quad \text{User}_{v_1} \xrightarrow{\overline{\text{in}}(v_1)} \text{User}}{\overline{\text{in}}(v_1).\text{User} \xrightarrow{\overline{\text{in}}(v_1)} \text{User}}}$$

Exercise

1. What are the possible initial transitions of

$\text{Cop} \mid (\text{User}_{v1} \mid \text{User}_{v2})$

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2. Draw the transition graph of Cnt

$$\text{Cnt} \stackrel{\text{def}}{=} \text{up}.\text{Cnt} \mid \text{down}.0$$

And compare it with Ct_0

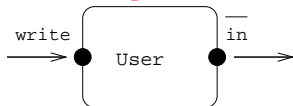
$$\text{Ct}_0 \stackrel{\text{def}}{=} \text{up}.\text{Ct}_1 + \text{round}.\text{Ct}_0$$

$$\text{Ct}_{i+1} \stackrel{\text{def}}{=} \text{up}.\text{Ct}_{i+2} + \text{down}.\text{Ct}_i$$

Flow graphs

Summarizes potential movement of information flowing into and out of ports.

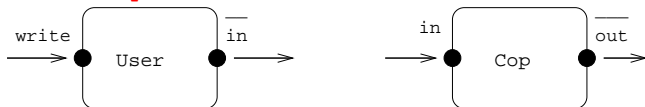
► User and Cop



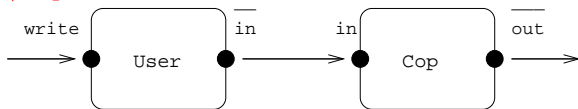
Flow graphs

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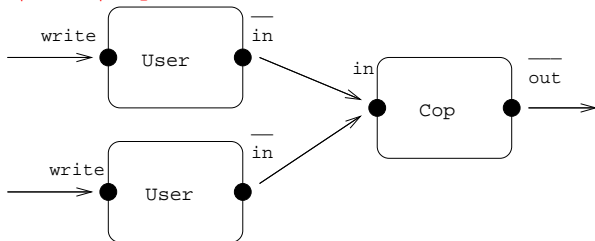
► User and Cop



► User | Cop



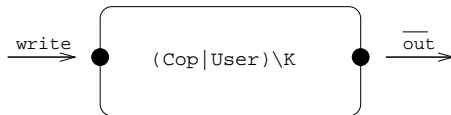
► User | User | Cop



► And so on with more users

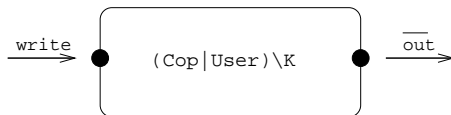
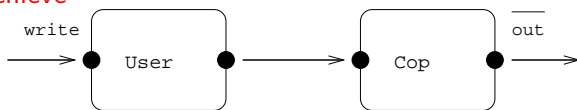
A private copier?

- ▶ Like to achieve



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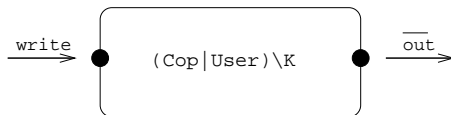
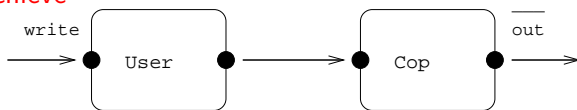


- ▶ Operation $\backslash K$: Restriction

$K = \{\text{in}(v) : v \in D\}$ abbreviate to in

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- ▶ Operation $\setminus K$: Restriction

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- ▶ $(\text{User} | \text{Cop}) \setminus \text{in}$

Transition rule for $\setminus J$

Assume $\tau \notin J$ and \bar{J} is $\{\bar{a} : a \in J\}$

$$\frac{E \setminus J \xrightarrow{a} F \setminus J}{E \xrightarrow{a} F} \quad a \notin J \cup \bar{J}$$

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Example

$$\frac{\frac{\frac{(\text{Cop} \mid \text{User}_v) \setminus \text{in} \xrightarrow{\tau} (\overline{\text{out}(v)}. \text{Cop} \mid \text{User}) \setminus \text{in}}{\text{Cop} \mid \text{User}_v \xrightarrow{\tau} \overline{\text{out}(v)}. \text{Cop} \mid \text{User}}}{\text{Cop} \xrightarrow{\text{in}(v)} \overline{\text{out}(v)}. \text{Cop}} \quad \text{User}_v \xrightarrow{\bar{\text{in}}(v)} \text{User}}{\text{in}(x). \overline{\text{out}(x)}. \text{Cop} \xrightarrow{\text{in}(v)} \overline{\text{out}(v)}. \text{Cop}} \quad \bar{\text{in}}(v). \text{User} \xrightarrow{\bar{\text{in}}(v)} \text{User}}$$

Abbreviations

Process descriptions can become large, especially when they consist of multiple components.

So $P \equiv F$ means that P abbreviates F

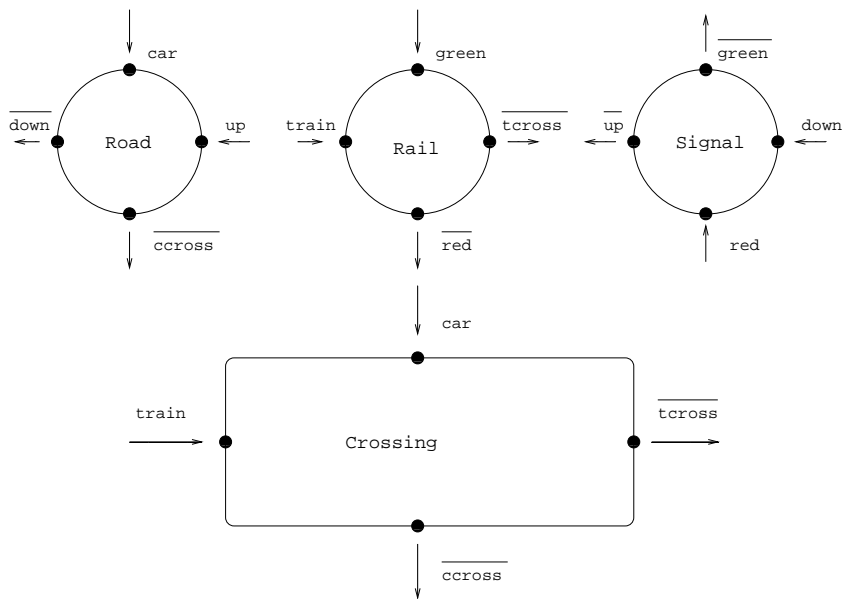
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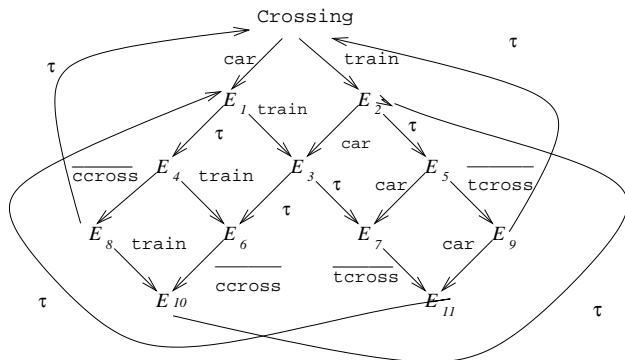
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Road	$\stackrel{\text{def}}{=}$	$\text{car.up}.\overline{\text{ccross}}.\overline{\text{down}}.\text{Road}$
Rail	$\stackrel{\text{def}}{=}$	$\text{train.green}.\overline{\text{tcross}}.\overline{\text{red}}.\text{Rail}$
Signal	$\stackrel{\text{def}}{=}$	$\overline{\text{green}}.\text{red}.\text{Signal} + \overline{\text{up}}.\text{down}.\text{Signal}$
Crossing	\equiv	$(\text{Road} \mid \text{Rail} \mid \text{Signal}) \setminus K$
K	$=$	$\{\text{green}, \text{red}, \text{up}, \text{down}\}$

Flow graphs



Transition graph



CCS model of Peterson's solution to mutual exclusion

B1f	$\stackrel{\text{def}}{=}$	$\overline{b1rf}.B1f + b1wf.B1f + b1wt.B1t$
B1t	$\stackrel{\text{def}}{=}$	$\overline{b1rt}.B1t + b1wt.B1t + b1wf.B1f$
B2f	$\stackrel{\text{def}}{=}$	$\overline{b2rf}.B2f + b2wf.B2f + b2wt.B2t$
B2t	$\stackrel{\text{def}}{=}$	$\overline{b2rt}.B2t + b2wt.B2t + b2wf.B2f$
K1	$\stackrel{\text{def}}{=}$	$\overline{kr1}.K1 + kw1.K1 + kw2.K2$
K2	$\stackrel{\text{def}}{=}$	$\overline{kr2}.K2 + kw2.K2 + kw1.K1$
P1	$\stackrel{\text{def}}{=}$	$\overline{b1wt}.req1.\overline{kw2}.P11$
P11	$\stackrel{\text{def}}{=}$	$b2rt.P11 + b2rf.P12 + kr2.P11 + kr1.P12$
P12	$\stackrel{\text{def}}{=}$	$enter1.exit1.\overline{b1wf}.P1$
P2	$\stackrel{\text{def}}{=}$	$\overline{b2wt}.req2.\overline{kw1}.P21$
P21	$\stackrel{\text{def}}{=}$	$b1rf.P22 + b1rt.P21 + kr1.P21 + kr2.P22$
P22	$\stackrel{\text{def}}{=}$	$enter2.exit2.\overline{b2wf}.P2$
Peterson	\equiv	$(P1 \mid P2 \mid K1 \mid B1f \mid B2f) \setminus L$

L all actions except req_i , $enter_i$ and $exit_i$

Protocol that may lose messages

$$\begin{aligned} \text{Sender} & \stackrel{\text{def}}{=} \text{in}(x).\overline{\text{sm}}(x).\text{Send1}(x) \\ \text{Send1}(x) & \stackrel{\text{def}}{=} \text{ms}.\overline{\text{sm}}(x).\text{Send1}(x) + \text{ok}.\text{Sender} \\ \text{Medium} & \stackrel{\text{def}}{=} \text{sm}(y).\text{Med1}(y) \\ \text{Med1}(y) & \stackrel{\text{def}}{=} \overline{\text{mr}}(y).\text{Medium} + \tau.\overline{\text{ms}}.\text{Medium} \\ \text{Receiver} & \stackrel{\text{def}}{=} \text{mr}(x).\overline{\text{out}}(x).\overline{\text{ok}}.\text{Receiver} \\ \\ \text{Protocol} & \equiv (\text{Sender} \mid \text{Medium} \mid \text{Receiver}) \setminus \{\text{sm}, \text{ms}, \text{mr}, \text{ok}\} \end{aligned}$$