Communication and Concurrency
Lecture 2

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Concurrent composition: $E \mid F$

$$\mathrm{R}(\mid \mathrm{com}) \quad \frac{E \mid F \xrightarrow{\tau} E' \mid F'}{E \xrightarrow{a} E' \quad F \xrightarrow{\overline{a}} F'}$$

$$\mathrm{R}(\mid) \quad \frac{E \mid F \xrightarrow{a} E' \mid F}{E \xrightarrow{a} E'} \qquad \frac{E \mid F \xrightarrow{a} E \mid F'}{F \xrightarrow{a} F'}$$

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Example: user of a copier

$$\begin{array}{lcl}
\texttt{Cop} & \stackrel{\mathrm{def}}{=} & \texttt{in}(x).\overline{\texttt{out}}(x).\texttt{Cop} \\
\texttt{User} & \stackrel{\mathrm{def}}{=} & \texttt{write}(x).\texttt{User}_x \\
\texttt{User}_\texttt{v} & \stackrel{\mathrm{def}}{=} & \overline{\texttt{in}}(v).\texttt{User}
\end{array}$$

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$$\frac{\texttt{Cop} \mid \texttt{User}_\texttt{v} \xrightarrow{\tau} \overline{\texttt{out}}(v).\texttt{Cop} \mid \texttt{User}}{\dfrac{\texttt{Cop} \xrightarrow{\texttt{in}(v)} \overline{\texttt{out}}(v).\texttt{Cop}}{\texttt{in}(x).\overline{\texttt{out}}(x).\texttt{Cop} \xrightarrow{\texttt{in}(v)} \overline{\texttt{out}}(v).\texttt{Cop}} \qquad \dfrac{\texttt{User}_\texttt{v} \xrightarrow{\overline{\texttt{in}}(v)} \texttt{User}}{\overline{\texttt{in}}(v).\texttt{User} \xrightarrow{\overline{\texttt{in}}(v)} \texttt{User}}}$$

More users

$$\begin{aligned}
\mathtt{Cop} &\stackrel{\text{def}}{=} \mathtt{in}(x).\overline{\mathtt{out}}(x).\mathtt{Cop}\\
\mathtt{User} &\stackrel{\text{def}}{=} \mathtt{write}(x).\mathtt{User}_x\\
\mathtt{User}_v &\stackrel{\text{def}}{=} \overline{\mathtt{in}}(v).\mathtt{User}
\end{aligned}$$

$$\dfrac{\dfrac{\dfrac{}{\mathtt{in}(x).\overline{\mathtt{out}}(x).\mathtt{Cop} \xrightarrow{\mathtt{in}(v1)} \overline{\mathtt{out}}(v1).\mathtt{Cop}}}{\mathtt{Cop} \xrightarrow{\mathtt{in}(v1)} \overline{\mathtt{out}}(v1).\mathtt{Cop}} \qquad \dfrac{\dfrac{\overline{\mathtt{in}}(v1).\mathtt{User} \xrightarrow{\overline{\mathtt{in}}(v1)} \mathtt{User}}{\mathtt{User}_{v1} \xrightarrow{\overline{\mathtt{in}}(v1)} \mathtt{User}}}{\mathtt{User}_{v1} \,|\, \mathtt{User}_{v2} \xrightarrow{\overline{\mathtt{in}}(v1)} \mathtt{User} \,|\, \mathtt{User}_{v2}}}{\mathtt{Cop} \,|\, (\mathtt{User}_{v1} \,|\, \mathtt{User}_{v2}) \xrightarrow{\tau} \overline{\mathtt{out}}(v1).\mathtt{Cop} \,|\, (\mathtt{User} \,|\, \mathtt{User}_{v2})}$$

Exercise

1. What are the possible initial transitions of

$$\mathtt{Cop} \,|\, (\mathtt{User}_{v1} \,|\, \mathtt{User}_{v2})$$

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2. Draw the transition graph of Cnt

$$\mathtt{Cnt} \stackrel{\text{def}}{=} \mathtt{up}.(\mathtt{Cnt} \,|\, \mathtt{down}.0)$$

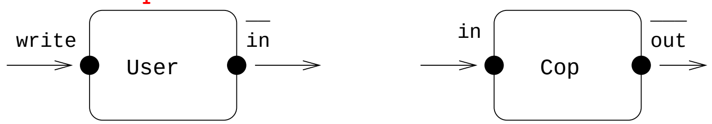
And compare it with $\mathtt{Ct}_0$

$$\begin{aligned}
\mathtt{Ct}_0 &\stackrel{\text{def}}{=} \mathtt{up}.\mathtt{Ct}_1 + \mathtt{round}.\mathtt{Ct}_0\\
\mathtt{Ct}_{i+1} &\stackrel{\text{def}}{=} \mathtt{up}.\mathtt{Ct}_{i+2} + \mathtt{down}.\mathtt{Ct}_i
\end{aligned}$$

Flow graphs

Summarizes potential movement of information flowing into and out of ports.

- User and Cop



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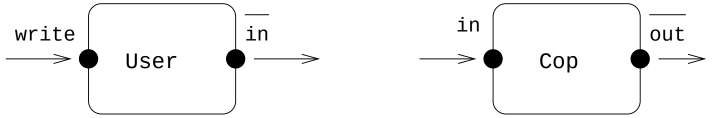
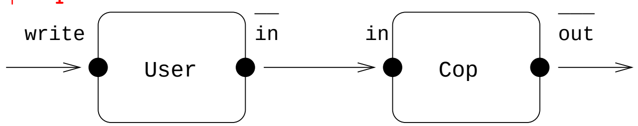
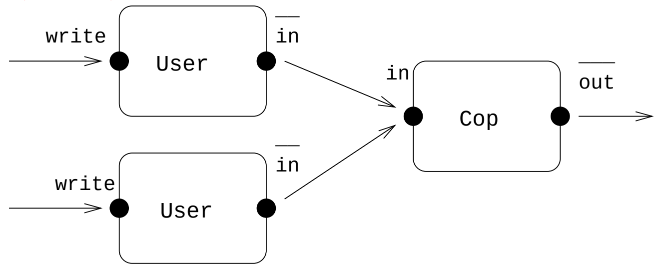


- User | Cop



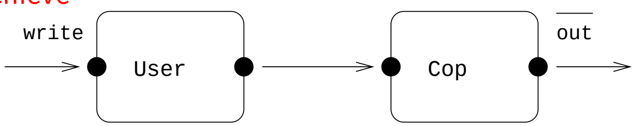
- User | User | Cop

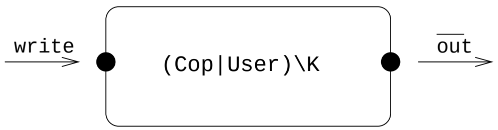


- And so on with more users

A private copier?

- Like to achieve





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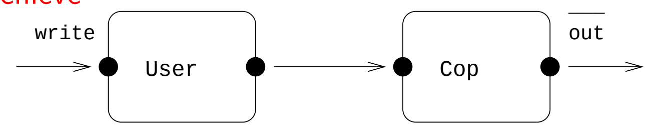

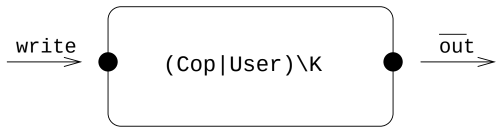

- Operation $\backslash K$: Restriction

  $K = \{\texttt{in}(v) \,:\, v \in D\}$ abbreviate to `in`

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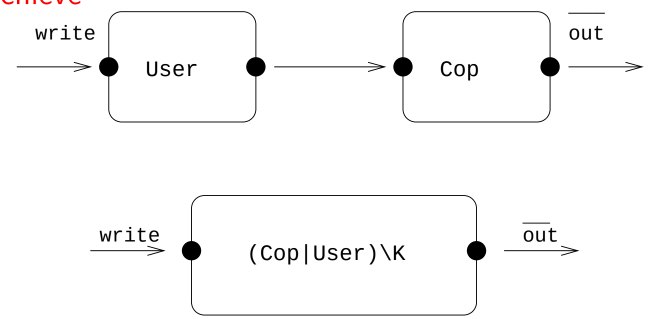

- Operation $\backslash K$: Restriction
  $K = \{\texttt{in}(v) \,:\, v \in D\}$ abbreviate to $\texttt{in}$
- $(\texttt{User} \mid \texttt{Cop})\backslash\texttt{in}$

Transition rule for $\backslash J$

Assume $\tau \notin J$ and $\overline{J}$ is $\{\overline{a} \,:\, a \in J\}$

$$\frac{E\backslash J \xrightarrow{a} F\backslash J}{E \xrightarrow{a} F} \; a \notin J \cup \overline{J}$$

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Example

$$\dfrac{(\texttt{Cop} \mid \texttt{User}_{\texttt{v}})\backslash\texttt{in} \xrightarrow{\tau} (\overline{\texttt{out}}(v).\texttt{Cop} \mid \texttt{User})\backslash\texttt{in}}{\dfrac{\texttt{Cop} \mid \texttt{User}_{\texttt{v}} \xrightarrow{\tau} \overline{\texttt{out}}(v).\texttt{Cop} \mid \texttt{User}}{\dfrac{\texttt{Cop} \xrightarrow{\texttt{in}(v)} \overline{\texttt{out}}(v).\texttt{Cop}}{\texttt{in}(x).\overline{\texttt{out}}(x).\texttt{Cop} \xrightarrow{\texttt{in}(v)} \overline{\texttt{out}}(v).\texttt{Cop}} \quad \dfrac{\texttt{User}_{\texttt{v}} \xrightarrow{\overline{\texttt{in}}(v)} \texttt{User}}{\overline{\texttt{in}}(v).\texttt{User} \xrightarrow{\overline{\texttt{in}}(v)} \texttt{User}}}}$$

Abbreviations

Process descriptions can become large, especially when they consist of multiple components.
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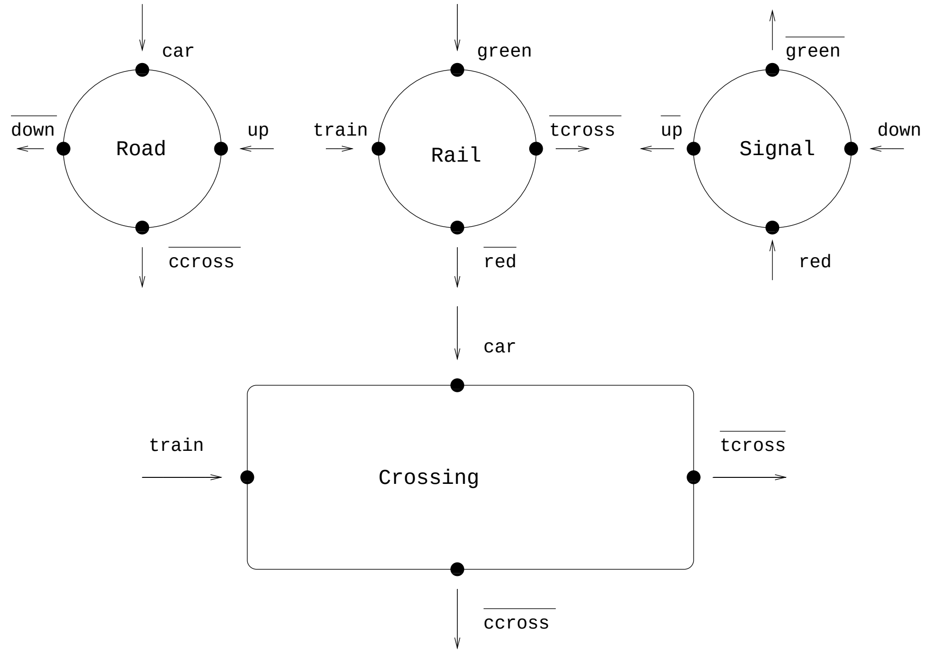
$$\mathtt{Road} \stackrel{\text{def}}{=} \mathtt{car}.\mathtt{up}.\overline{\mathtt{ccross}}.\overline{\mathtt{down}}.\mathtt{Road}$$
$$\mathtt{Rail} \stackrel{\text{def}}{=} \mathtt{train}.\mathtt{green}.\overline{\mathtt{tcross}}.\overline{\mathtt{red}}.\mathtt{Rail}$$
$$\mathtt{Signal} \stackrel{\text{def}}{=} \overline{\mathtt{green}}.\mathtt{red}.\mathtt{Signal} + \overline{\mathtt{up}}.\mathtt{down}.\mathtt{Signal}$$

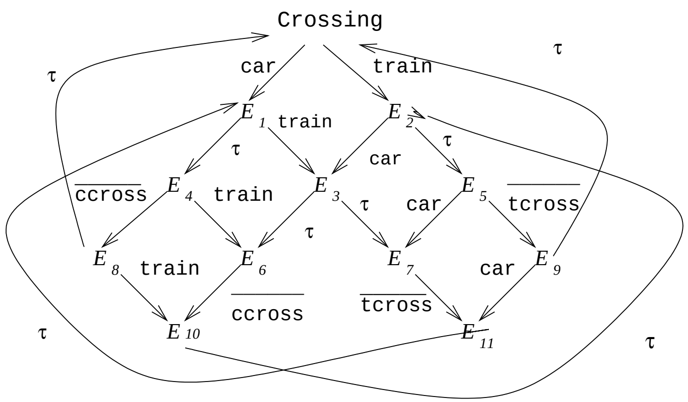
$$\mathtt{Crossing} \equiv (\mathtt{Road} \mid \mathtt{Rail} \mid \mathtt{Signal})\backslash K$$

$$K = \{\mathtt{green}, \mathtt{red}, \mathtt{up}, \mathtt{down}\}$$

Flow graphs



Transition graph



CCS model of Peterson's solution to mutual exclusion

$$\mathtt{B1f} \stackrel{\text{def}}{=} \overline{\mathtt{b1rf}}.\mathtt{B1f} + \mathtt{b1wf}.\mathtt{B1f} + \mathtt{b1wt}.\mathtt{B1t}$$
$$\mathtt{B1t} \stackrel{\text{def}}{=} \overline{\mathtt{b1rt}}.\mathtt{B1t} + \mathtt{b1wt}.\mathtt{B1t} + \mathtt{b1wf}.\mathtt{B1f}$$

$$\mathtt{B2f} \stackrel{\text{def}}{=} \overline{\mathtt{b2rf}}.\mathtt{B2f} + \mathtt{b2wf}.\mathtt{B2f} + \mathtt{b2wt}.\mathtt{B2t}$$
$$\mathtt{B2t} \stackrel{\text{def}}{=} \overline{\mathtt{b2rt}}.\mathtt{B2t} + \mathtt{b2wt}.\mathtt{B2t} + \mathtt{b2wf}.\mathtt{B2f}$$

$$\mathtt{K1} \stackrel{\text{def}}{=} \overline{\mathtt{kr1}}.\mathtt{K1} + \mathtt{kw1}.\mathtt{K1} + \mathtt{kw2}.\mathtt{K2}$$
$$\mathtt{K2} \stackrel{\text{def}}{=} \overline{\mathtt{kr2}}.\mathtt{K2} + \mathtt{kw2}.\mathtt{K2} + \mathtt{kw1}.\mathtt{K1}$$

$$\mathtt{P1} \stackrel{\text{def}}{=} \overline{\mathtt{b1wt}}.\mathtt{req1}.\overline{\mathtt{kw2}}.\mathtt{P11}$$
$$\mathtt{P11} \stackrel{\text{def}}{=} \mathtt{b2rt}.\mathtt{P11} + \mathtt{b2rf}.\mathtt{P12} + \mathtt{kr2}.\mathtt{P11} + \mathtt{kr1}.\mathtt{P12}$$
$$\mathtt{P12} \stackrel{\text{def}}{=} \mathtt{enter1}.\mathtt{exit1}.\overline{\mathtt{b1wf}}.\mathtt{P1}$$

$$\mathtt{P2} \stackrel{\text{def}}{=} \overline{\mathtt{b2wt}}.\mathtt{req2}.\overline{\mathtt{kw1}}.\mathtt{P21}$$
$$\mathtt{P21} \stackrel{\text{def}}{=} \mathtt{b1rf}.\mathtt{P22} + \mathtt{b1rt}.\mathtt{P21} + \mathtt{kr1}.\mathtt{P21} + \mathtt{kr2}.\mathtt{P22}$$
$$\mathtt{P22} \stackrel{\text{def}}{=} \mathtt{enter2}.\mathtt{exit2}.\overline{\mathtt{b2wf}}.\mathtt{P2}$$

$$\mathtt{Peterson} \equiv (\mathtt{P1} \mid \mathtt{P2} \mid \mathtt{K1} \mid \mathtt{B1f} \mid \mathtt{B2f})\backslash L$$

L all actions except reqi, enteri and exiti

Protocol that may lose messages

$$
\begin{array}{lcl}
\texttt{Sender} & \stackrel{\text{def}}{=} & \texttt{in}(x).\overline{\texttt{sm}}(x).\texttt{Send1}(x) \\
\texttt{Send1}(x) & \stackrel{\text{def}}{=} & \texttt{ms}.\overline{\texttt{sm}}(x).\texttt{Send1}(x) + \texttt{ok}.\texttt{Sender} \\
\texttt{Medium} & \stackrel{\text{def}}{=} & \texttt{sm}(y).\texttt{Med1}(y) \\
\texttt{Med1}(y) & \stackrel{\text{def}}{=} & \overline{\texttt{mr}}(y).\texttt{Medium} + \tau.\overline{\texttt{ms}}.\texttt{Medium} \\
\texttt{Receiver} & \stackrel{\text{def}}{=} & \texttt{mr}(x).\overline{\texttt{out}}(x).\overline{\texttt{ok}}.\texttt{Receiver} \\
 & & \\
\texttt{Protocol} & \equiv & (\texttt{Sender} \mid \texttt{Medium} \mid \texttt{Receiver}) \backslash \{\texttt{sm}, \texttt{ms}, \texttt{mr}, \texttt{ok}\}
\end{array}
$$