Communication and Concurrency
Lectures 10 & 11

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Showing bisimilarity

To establish $E \sim F$

1. Present a candidate relation R with $(E, F) \in R$
2. Prove that indeed it obeys the hereditary conditions

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Example: $(A|B)\backslash c \sim C_1$

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\begin{aligned}
A &\stackrel{\mathrm{def}}{=} a.\overline{\mathrm{c}}.A \\
B &\stackrel{\mathrm{def}}{=} c.\overline{\mathrm{b}}.B \\
C_0 &\stackrel{\mathrm{def}}{=} \overline{\mathrm{b}}.C_1 + a.C_2 \\
C_1 &\stackrel{\mathrm{def}}{=} a.C_3 \\
C_2 &\stackrel{\mathrm{def}}{=} \overline{\mathrm{b}}.C_3 \\
C_3 &\stackrel{\mathrm{def}}{=} \tau.C_0
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\end{array}
$$

R below is a bisimulation

$$
\begin{array}{l}
\{((A|B)\backslash c, C_1), ((\overline{\mathtt{c}}.A|B)\backslash c, C_3) \\
((A|\overline{\mathtt{b}}.B)\backslash c, C_0), ((\overline{\mathtt{c}}.A|\overline{\mathtt{b}}.B)\backslash c, C_2)\}
\end{array}
$$

Showing Bisimilarity II

Same sort of argument establishes that $\sim$ is a congruence.

1. if $E \sim F$ then $G|E \sim G|F$
2. Proof: Assume that $E \sim F$, so there is a bisimulation B with $(E, F) \in B$.
3. Let C be the relation

$$\{(H|E', H|F') : (E', F') \in B\}$$

4. Show that C is a bisimulation ...

Some Results

$$
\begin{array}{lcl}
Id & = & \{(E,E)\} \\
B^{-1} & = & \{(E,F) \,:\, (F,E) \in B\} \\
B_1 B_2 & = & \{(E,G) \,:\, \text{there is } F.\ (E,F) \in B_1 \\
 & & \text{and } (F,G) \in B_2\}
\end{array}
$$

Proposition Assume B_i $(i = 1, 2, \ldots)$ is a bisimulation. Then the following are bisimulations:

1. Id
2. B_i^{-1}
3. $B_1 B_2$
4. $\bigcup\{B_i \,:\, i \geq 1\}$

Corollary $\sim$ is the largest bisimulation

A bigger example: $\mathtt{Cnt} \sim \mathtt{Ct}'_0$

$$
\begin{array}{lcl}
\mathtt{Cnt} & \stackrel{\mathrm{def}}{=} & \mathtt{up}.(\mathtt{Cnt} \mid \mathtt{down}.0) \\
\mathtt{Ct}'_0 & \stackrel{\mathrm{def}}{=} & \mathtt{up}.\mathtt{Ct}'_1 \\
\mathtt{Ct}'_{i+1} & \stackrel{\mathrm{def}}{=} & \mathtt{up}.\mathtt{Ct}'_{i+2} + \mathtt{down}.\mathtt{Ct}'_i \quad i \geq 0.
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\end{aligned}
$$

$$
\begin{aligned}
P_0 &= \{\texttt{Cnt} \mid 0^j \;:\; j \geq 0\} \\
P_{i+1} &= \{E \mid 0^j \mid \texttt{down}.0 \mid 0^k \;:\; E \in P_i \text{ and } j \geq 0 \text{ and } k \geq 0\}
\end{aligned}
$$

where $F \mid 0^0 = F$ and $F \mid 0^{i+1} = F \mid 0^i \mid 0$ and brackets are dropped between parallel components.

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$$B \ = \ \{(E, \mathtt{Ct}'_i) \ : \ i \geq 0 \text{ and } E \in P_i\} \text{ is a bisimulation}$$

More Properties I

Proposition

1. $E + F \sim F + E$
2. $E + (F + G) \sim (E + F) + G$
3. $E + 0 \sim E$
4. $E + E \sim E$

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1. $E|F \sim F|E$
2. $E|(F|G) \sim (E|F)|G$
3. $E|0 \sim E$

More Properties II

Proposition

1. $(E + F)\backslash K \sim E\backslash K + F\backslash K$
2. $(a.E)\backslash K \sim 0$ if $a \in K \cup \overline{K}$
3. $(a.E)\backslash K \sim a.(E\backslash K)$ if $a \notin K \cup \overline{K}$

Expansion law

- Assume $x_i \sim \sum\{a_{ij}.x_{ij} \; : \; 1 \leq j \leq n_i\}$ for $i : 1 \leq i \leq m$

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- Example

$$\begin{aligned} x_1 &\sim a.x_{11} + b.x_{12} + a.x_{13} \\ x_2 &\sim \overline{a}.x_{21} + c.x_{22}, \end{aligned}$$

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$$\begin{aligned} x_1 &\sim a.x_{11} + b.x_{12} + a.x_{13} \\ x_2 &\sim \overline{a}.x_{21} + c.x_{22}, \end{aligned}$$
- $$\begin{aligned} x_1|x_2 \quad \sim \quad & a.(x_{11}|x_2) + b.(x_{12}|x_2) + a.(x_{13}|x_2) + \\ & \overline{a}.(x_1|x_{21}) + \\ & c.(x_1|x_{22}) + \tau.(x_{11}|x_{21}) + \tau.(x_{13}|x_{21}). \end{aligned}$$

Weak (observable) bisimulations

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- if $F \stackrel{a}{\Longrightarrow} F'$ then $E \stackrel{a}{\Longrightarrow} E'$ for some E' such that $(E', F') \in B$
- Two processes E and F are weak bisimulation equivalent (or weakly bisimilar) if there is a weak bisimulation relation B such that $(E, F) \in B$. We write $E \approx F$ if E and F are weakly bisimilar

Exercise

Which of the following are weakly bisimilar?

		Y/N
$a.\tau.b.0$	$a.b.0$	
$a.(b.0 + \tau.c.0)$	$a.(b.0 + c.0)$	
$a.(b.0 + \tau.c.0)$	$a.(b.0 + \tau.c.0) + a.c.0$	
$a.0 + b.0 + \tau.b.0$	$a.0 + \tau.b.0$	
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$a.(b.0 + \tau.c.0)$	$a.(b.0 + \tau.c.0) + a.c.0$	Y
$a.0 + b.0 + \tau.b.0$	$a.0 + \tau.b.0$	Y
$a.0 + b.0 + \tau.b.0$	$a.0 + b.0$	N
$a.(b.0 + \tau.b.0)$	$a.b.0$	Y

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4. $A_0 \approx B_1$

$$\{(A_0, B_1), (A_1, B_1), (A_2, B_2)\}$$

is a weak bisimulation

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- Proposition
  1. B is a weak bisim if, and only if B is an ob bisim
  2. $\approx \, = \, \approx'$

Protocol that may lose messages

$$\begin{aligned}
\texttt{Sender} &\stackrel{\text{def}}{=} \texttt{in}(x).\overline{\texttt{sm}}(x).\texttt{Send1}(x)\\
\texttt{Send1}(x) &\stackrel{\text{def}}{=} \texttt{ms}.\overline{\texttt{sm}}(x).\texttt{Send1}(x) + \texttt{ok}.\texttt{Sender}\\
\texttt{Medium} &\stackrel{\text{def}}{=} \texttt{sm}(y).\texttt{Med1}(y)\\
\texttt{Med1}(y) &\stackrel{\text{def}}{=} \overline{\texttt{mr}}(y).\texttt{Medium} + \tau.\overline{\texttt{ms}}.\texttt{Medium}\\
\texttt{Receiver} &\stackrel{\text{def}}{=} \texttt{mr}(x).\overline{\texttt{out}}(x).\overline{\texttt{ok}}.\texttt{Receiver}
\end{aligned}$$

$$\texttt{Protocol} \equiv (\texttt{Sender} \mid \texttt{Medium} \mid \texttt{Receiver})\backslash\{\texttt{sm}, \texttt{ms}, \texttt{mr}, \texttt{ok}\}$$

$$\texttt{Cop} \stackrel{\text{def}}{=} \texttt{in}(x).\overline{\texttt{out}}(x).\texttt{Cop}$$

$\texttt{Protocol} \approx \texttt{Cop}$

Let B be the following relation

$$
\begin{aligned}
&\{(\texttt{Protocol}, \texttt{Cop})\} \cup \\
&\{((\texttt{Send1}(m) \mid \texttt{Medium} \mid \overline{\texttt{ok}}.\texttt{Receiver})\backslash J, \\
&\qquad \texttt{Cop}) \;:\; m \in D\} \cup \\
&\{((\overline{\texttt{sm}}(m).\texttt{Send1}(m) \mid \texttt{Medium} \mid \texttt{Receiver})\backslash J, \\
&\qquad \overline{\texttt{out}}(m).\texttt{Cop}) \;:\; m \in D\} \cup \\
&\{((\texttt{Send1}(m) \mid \texttt{Med1}(m) \mid \texttt{Receiver})\backslash J, \\
&\qquad \overline{\texttt{out}}(m).\texttt{Cop}) \;:\; m \in D\} \cup \\
&\{((\texttt{Send1}(m) \mid \texttt{Medium} \mid \overline{\texttt{out}}(m).\overline{\texttt{ok}}.\texttt{Receiver})\backslash J, \\
&\qquad \overline{\texttt{out}}(m).\texttt{Cop}) \;:\; m \in D\} \cup \\
&\{((\texttt{Send1}(m) \mid \overline{\texttt{ms}}.\texttt{Medium} \mid \texttt{Receiver})\backslash J, \\
&\qquad \overline{\texttt{out}}(m).\texttt{Cop}) \;:\; m \in D\}
\end{aligned}
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B is a weak bisimulation

Properties of weak bisimulation

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\begin{array}{lcl}
Id & = & \{(E,E)\} \\
B^{-1} & = & \{(E,F) : (F,E) \in B\} \\
B_1 B_2 & = & \{(E,G) : \text{there is } F.\ (E,F) \in B_1 \\
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Proposition Assume B_i $(i = 1, 2, \ldots)$ is a weak bisimulation. Then the following are weak bisimulations:

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Proposition If $E \sim F$ then $E \approx F$

Tau laws

1. $a.\tau.E \approx a.E$
2. $E + \tau.E \approx \tau.E$
3. $a.(E + \tau.F) + a.F \approx a.(E + \tau.F)$

But

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