Modeling the Visual System

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Where do we begin?

Modeling approaches

- Compartmental neuron model
- Integrate-and-fire / firing-rate model of the network

One approach: model single cells extremely well
Our approach: many, many simple single-cell models

Dense connectivity

Remember that the actual network is far denser than in the previous slides, with many opportunities for contact between neurons and neurites.
Adult retina and LGN cell models

- Standard model of adult RGC or LGN cell activity: Difference of Gaussians weight matrix
- Firing rate: dot product of weight and input matrices
- Can be tuned for quantitative match to firing rate
- Can add temporal component (transient+sustained)

Effect of DoG

ON:
- c0.5 s1.5
- c1 s3
- c3 s9
- c10 s30
- c30 s90

OFF:
- c1.5 s0.5
- c3 s1
- c9 s3
- c30 s10
- c90 s30

Each DoG, if convolved with the image, performs edge detection at a certain size scale (spatial frequency band)

Adult V1 cell model: Gabor

Standard model of adult V1 simple cell spatial preferences: Gabor (Gaussian times sine grating) (Daugman 1980)

Adult V1 cell model: CGE

- Gabor model fits spatial preferences
- Simple response function: dot product
- To match observations: need to add numerous nonlinearities
- Example: CGE model (Geisler & Albrecht 1997)
**Adult V1 cell model: Energy**

- Spatiotemporal energy:
  - Standard model of complex direction cell (Adelson & Bergen 1985)
- Combines inputs from a quadrature pair (two simple cell motion models out of phase)
- Achieves phase invariance, direction selectivity

**Retina/LGN development models**

Relatively rare, but more in recent years:

- Retinal wave generation
  - e.g. Feller et al. 1997; Godfrey & Swindale 2007; Hennig et al. 2009
- RGC development based on retinal waves
  - e.g. Eglen & Willshaw 2002
- Retinogeniculate pathway based on retinal waves
  - e.g. Eglen 1999; Haith 1998

Because of the wealth of data from the retina, such models can now become quite detailed.

**Our focus: Cortical map models**

Basic architecture: input surface mapped to cortical surface + some form of lateral interaction

**Kohonen SOM: Feedforward**

Popular computationally tractable map model (Kohonen 1982)

Feedforward activity of unit \((i, j)\):

\[
η_{ij} = \| \vec{V} - \vec{W}_{ij} \| \tag{1}
\]

(distance between input vector \(\vec{V}\) and weight vector \(\vec{W}\))

Not particularly biologically plausible, but easy to compute, widely implemented, and has some nice properties.

Note: Activation function is not typically a dot product; the CMVC book is confusing about that.
Kohonen SOM: Lateral

Abstract model of lateral interactions:

- Pick winner \((r, s)\)
- Assign it activity \(\eta_{\text{max}}\)
- Assume that activity of unit \((i, j)\) can be described by a neighborhood function, such as a Gaussian:

\[
h_{rs,ij} = \eta_{\text{max}} \exp\left(-\frac{(r-i)^2 + (s-j)^2}{\sigma^2_h}\right),
\]

(2)

Models lateral interactions that depend only on distance from winning unit.

Kohonen SOM: Learning

Inspired by basic Hebbian rule (Hebb 1949):

\[
w' = w + \alpha \eta \chi
\]

(3)

where the weight increases in proportion to the product of the input and output activities.

In SOM, the weight vector is shifted toward the input vector based on the Euclidean difference:

\[
w'_{k,ij} = w_{k,ij} + \alpha (\chi_{k} - w_{k,ij}) h_{rs,ij}.
\]

(4)

Hebb-like, but depending on distance from winning unit

SOM example: Input

- SOM will be trained with unoriented Gaussian activity patterns
- Random \((x, y)\) positions anywhere on retina
- 576-dimensional input, but the \(x\) and \(y\) locations are the only source of variance

SOM: Weight vector self-org

Combination of input patterns; eventually settles to an exemplar
SOM: Retinotopy self-org

Iteration 0: Initial
Initially bunched (all average to zero)
Unfolds as neurons differentiate

Iteration 1000: Unfolding

SOM: Retinotopy self-org

Iteration 5000: Expanding
Expands to cover usable portion of input space.

Iteration 40,000: Final

Magnification of dense input areas

Gaussian distribution
Density of units receiving input from a particular region
depends on input pattern statistics

Two long Gaussians

Principal components of data distributions

(a) Linear distribution
(b) Nonlinear distribution

PCA: linear approximation, good for linear data
Nonlinear distributions: principal curves, folding

Principal curve
Folded curve

Generalization of idea of PCA to pick best-fit curve(s)

Multiple possible curves

Three-dimensional model of ocular dominance

Representing the third dimension by folding
Visualization of ocular dominance

Feature maps: Discrete approximations to principal surfaces?

Role of density of input sheet

- Gaussian inputs are nearly band-limited (since Fourier transform is also Gaussian)
- Density of input sampling unimportant, if it's greater than 2X highest frequency in input (Nyquist theorem)

Role of density of SOM sheet

SOM sheet acts as a discrete approximation to a two-dimensional surface.

How many units are needed for the SOM depends on how nonlinear the input distribution is — a smoothly varying input distribution requires fewer units to represent the shape.

Only loosely related to the input density — input density limits how quickly the input varies across space, but only for wideband stimuli.
Other relevant models

**ICA** Independent Component Analysis yields realistic RFs (Olshausen & Field 1996); also can be applied to maps (Hyvärinen & Hoyer 2001).

**InfoMax** Information maximization can lead to RFs (Linsker 1986b,c) and basic maps (Kozloski et al. 2007; Linsker 1986a)

**Elastic net** Achieving good coverage and continuity leads to realistic feature maps (Carreira-Perpiñán et al. 2005; Goodhill & Cimponeriu 2000)

This course focuses on a mechanistic circuit models, not normative models (ICA, Infomax, PCA, principal surfaces) or feature space models (elastic net), both of which are hard to relate directly to the underlying biological systems.

Summary

- Basic intro to visual modeling
- Adult models are well established, but vision-specific
- SOM: maps multiple dimensions down to two
- Feature maps: Principal surfaces?

References


