1. Show that $\text{NP}$ is closed under union and intersection. In other words,
   (a) If $L_1 \in \text{NP}$, $L_2 \in \text{NP}$, then $L_1 \cup L_2 \in \text{NP}$. [2]
   (b) If $L_1 \in \text{NP}$, $L_2 \in \text{NP}$, then $L_1 \cap L_2 \in \text{NP}$. [2]

2. Show that if $\text{SAT} \leq_p \text{Taut}$, then $\text{PH} = \text{NP}$. [4]

3. Recall that $\text{TQBF}$ is the problem of determining the validity of a totally quantified Boolean formula. Show that $\text{TQBF} \not\in \text{L}$. [4]

4. Let $\oplus(\cdot)$ be the parity function. Namely, $\oplus(x) = \sum_{i=1}^{n} x_i \mod 2$ for a vector $x = \{x_1, \ldots, x_n\}$ and for all $i \in [n]$, $x_i \in \{0, 1\}$.
   Show that to express $\oplus(\cdot)$ as a CNF or DNF formula, at least $2^{n-1}$ clauses are required. [4]
   **Hint:** first show that every clause must have size $n$.
   What about expressing $\oplus(\cdot)$ as an arbitrary (not necessarily CNF or DNF) formula? For simplicity, you may only give the construction for $n$ being a power of 2. Try to minimize the size of your construction. [3]
   **Hint:** use a recursive construction.

5. (a) Show that if a function $f : \{0, 1\}^n \rightarrow \{0, 1\}$ has a support of size $k$, then it can be decided by a circuit of size $O(nk)$. [2]
   *(The support of $f$ is the set of inputs that map to 1, namely $\text{Supp}(f) = \{x \mid f(x) = 1\}$.)*
   (b) Show that, for sufficiently large $n$, there is a language that can be decided by circuits of size $n^3$ but not $n^2$. [4]
   **Hint:** use a counting argument.