

Exercise Sheet 2

This is the second of three sets of assessed exercises. It represents 40% of the continuously assessed component of the course (which in turn accounts for 25% of the overall credit for the course). The deadline for submission of solutions is 16:30, Friday 14th March. Please hand your solutions to ITO (Appleton Tower, Room 4.02).

The questions are not necessarily in increasing order of difficulty.

1. Prove that if $\text{NL} = \text{L}$, then $\text{NSPACE}(n) = \text{DSPACE}(n)$.
HINT: Use padding, but there will be an obstruction to doing this straightforwardly that you will need to get around.
2. The graph reachability language REACH consists of all $\langle G, s, t \rangle$, where G is a directed acyclic graph (in adjacency matrix representation) and s and t are vertices of the graph such that there is a path from s to t in G . Prove that REACH is complete for NL under log-space m-reductions.
3. Prove that there is a language L that can be decided by circuits of size n^3 but not by circuits of size n^2 .
HINT: Use a counting argument, together with the fact that any Boolean function for which a small number of inputs map to 1 has circuits of small size.
4. Show that $\text{NC}^1 \subseteq \text{L}$. Build on this to prove unconditionally that TQBF is not in NC.