# Exercise Sheet 1

This is the first of three sets of assessed exercises. It represents 40% of the continuously assessed component of the course (which in turn accounts for 25% of the overall credit for the course). The deadline for submission of solutions is 16:30, Friday 14th February. Please hand your solutions to ITO (Appleton Tower, Room 4.02).

The questions are not necessarily in increasing order of difficulty.

1. An *infinite-state Turing machine* is a Turing machine defined in the usual manner, except that the state set $Q$ is infinite. The input and tape alphabets, though, remain finite. Show that for *any* language $L \subseteq \{0,1\}^*$, there is an infinite-state Turing machine deciding $L$ in linear time.

2. Let $L = \{xy \mid |x| = |y| \text{ and } \Sigma_{i=0}^{|x|} x_i y_i = 1 \bmod 2\}$. Prove:

   (a) $L \in \mathsf{DTIME}(n)$

   (b) $L \in \mathsf{DSPACE}(\log(n))$

   Note: You do not need to specify the Turing machines accepting $L$ in full detail, but you need to give a clear high-level description and argue that the resource bounds are as claimed.

3. Show that $\mathsf{P} \neq \mathsf{NSPACE}(n)$.
   HINT: Consider the closures of these classes under polynomial-time reductions, .

4. Let $L = \{< M, x, t >\mid M \text{ is a deterministic TM accepting } x \text{ within } t \text{ steps}\}$. Note that here $t$ is represented in binary. Prove that $L \notin \mathsf{P}$.

Ilias Diakonikolas and Rahul Santhanam, February 2014