Bells and whistles

Doubly infinite tape:

LEMMA A language L is accepted by a Turing machine with doubly infinite tape if and only if L is accepted by a Turing machine with singly infinite tape.

'If' part is easy.

'Only if' part needs more thought.

Suppose L accepted by

$$M = (Q, \Gamma, \Sigma, \not{b}, q_I, q_F, \delta)$$

with doubly infinite tape.

Must show: how to construct machine

$$\widehat{M} = (\widehat{Q}, \widehat{\Gamma}, \widehat{\Sigma}, \widehat{\not{b}}, \widehat{q}_I, \widehat{q}_F, \widehat{\delta}\,)$$

with singly infinite tape which accepts L

$\cdots$	s_{-5}	s_{-4}	s_{-3}	s_{-2}	s_{-1}	s_0	s_1	s_2	s_3	s_4	s_5	$\cdots$

The tape of M

\$	s_{-1}	s_{-2}	s_{-3}	s_{-4}	s_{-5}	$\cdots$
s_0	s_1	s_2	s_3	s_4	s_5	$\cdots$

The tape of $\widehat{M}$

$$\widehat{Q} = \{\widehat{q}_I, \widehat{q}_F\} \cup (Q \times \{0, 1\}),$$

$$\widehat{\Gamma} = \Gamma \times (\Gamma \cup \{\$\}),$$

$$\widehat{\Sigma} = \Sigma \times \{\not{b}\} \subset \widehat{\Gamma}.$$

1 :	\$	s_{-1}	s_{-2}	s_{-3}	s_{-4}	s_{-5}	$\cdots$
0 :	s_0	s_1	s_2	s_3	s_4 s_5	$\cdots$	

(1) Transitions from the initial state:

$$\widehat{\delta}(\widehat{q}_I, \langle s, \not{b} \rangle) = \begin{cases} (\langle q', \mathsf{0} \rangle, \langle s', \$ \rangle, R), \\ \qquad \text{if } \delta(q_I, s) = (q', s', R); \\ (\langle q', \mathsf{1} \rangle, \langle s', \$ \rangle, R), \\ \qquad \text{if } \delta(q_I, s) = (q', s', L). \end{cases}$$

[The special symbol $ is written to mark the end of the tape while, simultaneously, the first move of M is simulated.]

(2) Transitions to the final state:

$$\widehat{\delta}(\langle q_F, \cdot \rangle, \langle s_0, s_1 \rangle) = (\widehat{q}_F, \langle s_0, s_1 \rangle, R).$$

[If M enters its accepting state, then $\widehat{M}$ enters its accepting state on the following move.]

(3) Transitions when M is scanning square 0:

$$\widehat{\delta}(\langle q, \cdot\,\rangle, \langle s, \$\rangle) = \begin{cases} (\langle q', 0\rangle, \langle s', \$\rangle, R), \\ \qquad\qquad \text{if } \delta(q, s) = (q', s', R); \\ (\langle q', 1\rangle, \langle s', \$\rangle, R), \\ \qquad\qquad \text{if } \delta(q, s) = (q', s', L). \end{cases}$$

[If the head of M moves right the simulation is continued on the lower track, otherwise on the upper track.]

(4) Transitions when M is scanning a square with positive index:

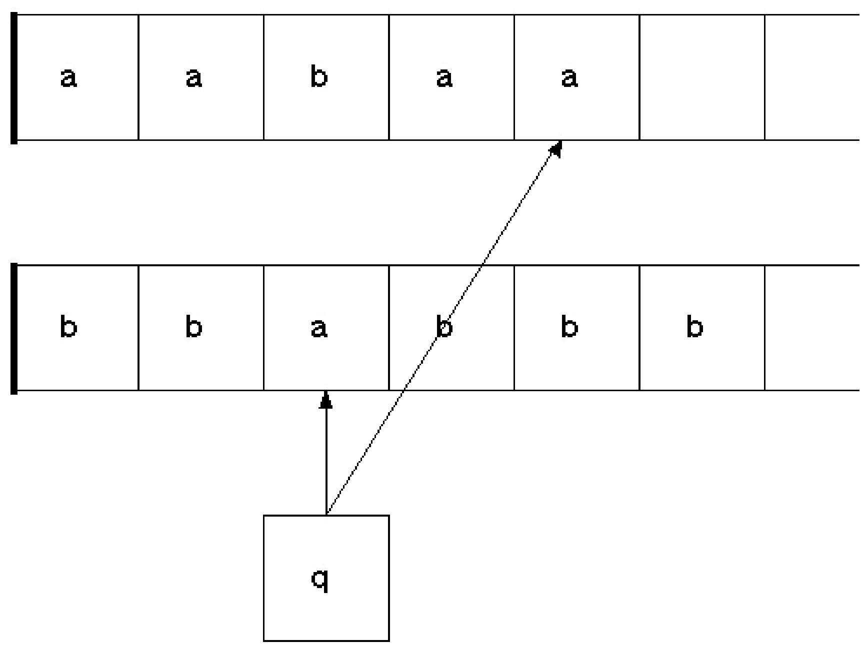
$$\widehat{\delta}(\langle q, 0\rangle, \langle s_0, s_1\rangle) = \begin{cases} (\langle q', 0\rangle, \langle s'_0, s_1\rangle, L), \\ \qquad\qquad \text{if } \delta(q, s_0) = (q', s'_0, L); \\ (\langle q', 0\rangle, \langle s'_0, s_1\rangle, R), \\ \qquad\qquad \text{if } \delta(q, s_0) = (q', s'_0, R). \end{cases}$$

(5) Transitions when M is scanning a square with negative index:

$$\widehat{\delta}(\langle q, 1\rangle, \langle s_0, s_1\rangle) = \begin{cases} (\langle q', 1\rangle, \langle s_0, s_1'\rangle, R), \\ \qquad\qquad \text{if } \delta(q, s_1) = (q', s_1', L); \\ (\langle q', 1\rangle, \langle s_0, s_1'\rangle, L), \\ \qquad\qquad \text{if } \delta(q, s_1) = (q', s_1', R). \end{cases}$$

[Note that $\widehat{M}$ must move its head in the opposite direction to that of M.]

Several tapes



k (singly infinite) tapes, k heads.

1. the finite control moves to a new state;
2. each tape head prints a new symbol on the tape square it currently scans;
3. each tape head moves (independently) one square left or right.

Start with input on first tape at the left.

Simulation of a 2-tape machine

s_0	s_1	s_2	s_3	s_4	s_5	$\cdots$
	$\wedge$					

t_0	t_1	t_2	t_3	t_4	t_5	$\cdots$
				$\wedge$		

The tapes of M

s_0	s_1	s_2	s_3	s_4	s_5	$\cdots$
$\not b$	$\wedge$	$\not b$	$\not b$	$\not b$	$\not b$	$\cdots$
t_0	t_1	t_2	t_3	t_4	t_5	$\cdots$
$\not b$	$\not b$	$\not b$	$\not b$	$\wedge$	$\not b$	$\cdots$

The tape of $\widehat{M}$

Other variants

- Two-dimensional array or *page* of squares, in place of the one-dimensional array of squares.

- k heads each moving independently of each other (on the same tape).

- Many others.