

§2. Turing's Thesis. As observed in NOTE 1, during the time when the fundamentals of computability were formulated a number of models were proposed with the aim of formalizing the notion of effective procedure, the Turing machine being one (we will give a formal definition⁸ in NOTE 3). This is such a simple model that we can have little doubt that all procedures that can be defined within it are effective in the algorithmic sense. However, the very simplicity of the model also leads us to question whether *all* the procedures that we would intuitively regard as effective can be described by Turing machines. Perhaps the model is *too* simple. Perhaps a modern programming language, such as Haskell or Java, can describe processes that cannot be carried out by a Turing machine.

Alan Turing argued that his model was a correct formulation of effective computability and defended the following proposition, which has come to be called *Turing's thesis*:

Any process which could naturally be called an effective procedure can be realized by a Turing machine.

Note that this is a *thesis* (i.e., an unproved proposition or hypothesis) and not a *theorem*, since 'effective procedure' is an intuitive notion.

One argument he advanced in favour of the thesis was an 'appeal to intuition'. In his original paper⁹ he fashions this appeal to intuition into a powerful case, which is reproduced verbatim below. Two things need to be borne in mind when reading this abstract more than 70 years after it was written. First, it will be seen that Turing is concerned with computations of a mathematical or numeric nature, whereas today we more often encounter computers in the role of data processing devices. However, there is nothing in the argument that does not carry over to the more general setting. Second, when Turing uses the word 'computer' he is clearly not using it in its modern sense; instead he means a *person* performing a computation as described in NOTE 1. The argument runs as follows:

"Computing is normally done by writing certain symbols on paper. We may suppose this paper is divided into squares like a child's arithmetic book. In elementary arithmetic the two-dimensional nature of the paper is sometimes used. But such a use is always avoidable, and I think it will be agreed that the two-dimensional character of the paper is no essential of computation. I assume that the computation

⁸There are variations but they are all equivalent in the sense that anything computable by one version is computable by any other.

⁹A. M. Turing, *On computable numbers, with an application to the Entscheidungsproblem*, Proc. London Math. Soc. **42** (1937).

is carried out on one-dimensional paper, i.e., on a tape divided into squares.¹⁰ I shall also suppose that the number of symbols which may be printed is finite. If we were to allow an infinity of symbols, then there would be symbols differing to an arbitrarily small extent. The effect of this restriction of the number of symbols is not very serious. It is always possible to use sequences of symbols in the place of single symbols. Thus an Arabic numeral such as 17 or 9999999999999999 is normally treated as a single symbol. Similarly in any European language words are treated as single symbols (Chinese, however, attempts to have an enumerable infinity of symbols.) The differences from our point of view between the single and compound symbols is that the compound symbols, if they are too lengthy, cannot be observed at a glance. This is in accordance with experience. We cannot tell at a glance whether 9999999999999999 and 9999999999999999 are the same.

“The behaviour of the computer at any moment is determined by the symbols which he is observing, and his ‘state of mind’ at that moment. We may suppose that there is a bound B to the number of symbols or squares which the computer can observe at one moment. If he wishes to observe more, he must use successive observations. We will also suppose that the number of states of mind which need to be taken into account is finite. The reasons for this are of the same character as those which restrict the number of symbols. If we admitted an infinity of states of mind, some of them will be ‘arbitrarily close’ and will be confused. Again, the restriction is not one which seriously affects the computation, since the use of more complicated states of mind can be avoided by writing more symbols on the tape.

“Let us imagine the operations to be performed by the computer to be split up into ‘simple operations’ which are so elementary that it is not easy to imagine them further divided. Every such operation consists of some change of the physical system consisting of the computer and his tape. We know the state of the system if we know the sequence of symbols on the tape, which of these are observed by the computer (possibly with a special order), and the state of mind of the computer. We may suppose that in a simple operation not more than one symbol is altered. Any other changes can be split up into changes of this kind. The situation in regard to the squares whose symbols may be altered in this way is the same as in regard to the observed squares. We may, therefore, without loss of generality, assume that the squares

¹⁰In fact, it can be rigorously demonstrated that a machine with a two-dimensional ‘tape’ is no more powerful than one with a one-dimensional tape.

whose symbols are changed are always the ‘observed’ squares.

“Besides these changes of symbols, the simple operations must include changes of distribution of the observed squares. The new observed squares must be immediately recognisable by the computer. I think it is reasonable to suppose that they can only be squares whose distance from the closest of the immediately previously observed squares does not exceed a certain fixed amount. Let us say that each of the new observed squares is within L squares of an immediately previously observed square.

“In connection with ‘immediate recognisability’, it may be thought that there are other kinds of squares which are immediately recognisable. In particular, squares marked by special symbols might be taken as immediately recognisable. Now if these squares are marked only by single symbols there can only be a finite number of them, and we should not upset our theory by adjoining these marked squares to the observed squares. If, on the other hand, they are marked by sequences of symbols, we cannot regard the process of recognition as a simple process. This is a fundamental point and should be illustrated. In most mathematical papers the equations and theorems are numbered. Normally the numbers do not go beyond (say) 1000. It is, therefore, possible to recognise a theorem at a glance by its number. But if the paper was very long, we might reach Theorem 157767733443477; then, further on in the paper, we might find ‘... hence (applying Theorem 157767733443477) we have ...’. In order to make sure which was the relevant theorem we should have to compare the two numbers figure by figure, possibly ticking the figures off in pencil to make sure of their not being counted twice. If in spite of this it is still thought that there are other ‘immediately recognisable’ squares, it does not upset my contention so long as those squares can be found by some process of which my type of machine is capable.

“The simple operations must therefore include:

- (a) Changes of the symbol on one of the observed squares.
- (b) Changes of one of the squares observed to another within L squares of the previously observed square.

“It may be that some of these changes necessarily involve a change of state of mind. The most general single operation must therefore be taken to be one of the following:

- (A) A possible change (a) of symbol together with a possible change of state of mind.

- (B) A possible change (b) of observed squares, together with a possible change of state of mind.

“The operation actually performed is determined, as has been suggested [above] by the state of mind of the computer and the observed symbols. In particular, they determine the state of mind of the computer after the operation.

“We may now construct a machine to do the work of this computer. To each state of mind of the computer corresponds an ‘ m -configuration’ of the machine. The machine scans B squares corresponding to the B squares observed by the computer. In any move the machine can change a symbol on a scanned square or can change any one of the scanned squares to another square distant not more than L squares from one of the other scanned squares. The move which is done, and the succeeding configuration, are determined by the scanned symbol and the m -configuration. The machines just described do not differ very essentially from computing machines defined [in NOTE 3] and corresponding to any machine of this type a [Turing] machine can be constructed to compute the same sequence, that is to say the sequence computed by the computer.”

Observe that in this careful argument Turing has justified each of the apparently limiting features of his model: that the tape is one-dimensional, that the set of states is finite, that the set of symbols is finite, or that only a fixed number of tape symbols are visible at any time. For some features he argues that it would be unreasonable to have things otherwise: an infinite set of states or an infinite tape alphabet would inevitably lead to different states or symbols being confused¹¹. For others he argues that no loss of generality is involved: any computation which can be carried out on a two-dimensional sheet of paper can equally well be carried out on a long thin tape.

Later in the course, we shall consider further evidence in support of Turing’s thesis.

¹¹I find this part of Turing’s argument problematic. Certainly at the level of modeling a person carrying out calculations it is convincing but if we enlarge our view of computing to physically realizable processes the point is not so clear. However to date nobody has produced a convincing counterexample. Marvin Minsky in his book *Computation, Finite and Infinite Machines*, Prentice-Hall (1972), makes the following observation: ‘... for sufficiently similar states there will be a chance of random transitions, e.g., because of thermal or quantum phenomena. There is a limit to the amount of information that can be recovered from any physical system of limited size. The same holds for whatever physical system is used to represent the the symbols within the ‘squares’ of fixed dimensions.’