CS3 Computability and Intractability (2012-2013)
Exercise Sheet 2

The deadline for this coursework is 4pm on Friday 9 November; submit your solutions to the ITO. Please note that multiple submissions are not allowed. Your work will be marked and returned, but the mark will not contribute to the overall mark for the course. Note that the marks for questions are not always related to their length or difficulty.

In your answers you should aim for clarity, conciseness and correctness; look at your answers with an objective eye, e.g., imagine that somebody else gave them to you to check. You should therefore make an early start to give yourself time to consider your answers before submitting the final version. This coursework should take about 10 hours of work to complete.

1. The Turing machine M has states $Q = \{q_0, q_1, q_2, q_3\}$, initial state q_0, input alphabet $\Sigma = \{\mathtt{0}, \mathtt{1}\}$, tape alphabet $\Gamma = \{\mathtt{0}, \mathtt{1}, \not{b}\}$, and transition function given by the tuples

$$\begin{aligned}\{\ & (q_0, \mathtt{0}, q_1, \not{b}, R), (q_0, \mathtt{1}, q_2, \not{b}, R), \\ & (q_1, \mathtt{0}, q_1, \mathtt{0}, R), (q_1, \mathtt{1}, q_2, \mathtt{0}, R), (q_1, \not{b}, q_3, \mathtt{0}, L), \\ & (q_2, \mathtt{0}, q_1, \mathtt{1}, R), (q_2, \mathtt{1}, q_2, \mathtt{1}, R), (q_2, \not{b}, q_3, \mathtt{1}, L)\ \}\end{aligned}$$

(The final state is of no interest and is therefore not declared.)

(a) Explain briefly what the machine does (i.e., if the machine halts, explain how the input string is transformed at the end and which square is being scanned). *[3 marks]*

(b) Use the encoding of Note 6 to code M as a word over $\{\mathtt{0}, \mathtt{1}, \mathtt{B}, *\}$. *[3 marks]*

(c) Download the Universal Turing Machine (UTM) from the dowloads section of the course web page at `http://www.inf.ed.ac.uk/teaching/courses/ci`. Use this to simulate the computation of M on the input `010`. Write down both the initial and final tape contents of the UTM (it will halt). (Set an appropriate speed so that you can observe the intermediate stages of the simulation; that is the main point of this part. In fact you might like to alter the speed for different parts of the simulation.)

Explain also how to deduce the state of M upon halting from the final tape contents of the UTM and write down the state. *[3 marks]*

(d) State the number of steps that M takes on the input `010` and the number of steps taken by the UTM for the simulation. *[2 marks]*

2. The language L_{all} consists of encodings of all binary Turing machines that accept every string, i.e.,

$$L_{\text{all}} = \{\langle M \rangle \mid L(M) = \{\mathtt{0}, \mathtt{1}\}^*\},$$

where $\langle M \rangle$ represents the encoding of a binary Turing machine.

(a) Using a reduction from L_{halt} (or otherwise), show that L_{all} is not recursive. [*6 marks*]

(b) Show that $\overline{L_{\text{all}}}$ is not recursively enumerable. [*5 marks*]

3. Let $\mathtt{ODD} \subseteq \{\mathtt{0}, \mathtt{1}\}^*$ be the set of all strings of odd length. The language L_{odd} is defined as follows:

$$L_{\text{odd}} = \{\langle M \rangle \mid L(M) \subseteq \mathtt{ODD}\}$$

(a) Prove that L_{odd} is not recursively enumerable.
HINT: start with a language that you know is not recursively enumerable. [*6 marks*]

(b) Prove that $\overline{L_{\text{odd}}}$ *is* recursively enumerable. [*5 marks*]

Rahul Santhanam, Thursday 25 October