Tutorial Sheet 4

1. Prove that the class of recursively enumerable languages is closed under union and intersection.

2. Let A and B be two disjoint languages. Say that language C *separates* A and B if $A \subseteq C$ and $\mathsf{B} \subseteq \bar{C}$. Show that any two disjoint co-r.e. languages are separable by some decidable language.

3. Consider the language L consisting of all polynomials with integer co-efficients such that the polynomial evaluates to zero on some integer setting of its variables (the polynomial is represented by a list of all its co-efficients). Is L r.e.? Justify your answer.

4. Let A, B, C be languages such that A reduces to B and B reduces to C. Show that A reduces to C.

5. Let $S = \{\langle M \rangle | L(M) \text{ is infinite}\}$. Prove that S is not recursive. Is S r.e.? Justify your answer.