Tutorial Sheet 4

1. Prove that the class of recursively enumerable languages is closed under union and intersection.

2. Let $A$ and $B$ be two disjoint languages. Say that language $C$ separates $A$ and $B$ if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that any two disjoint co-r.e. languages are separable by some decidable language.

3. Consider the language $L$ consisting of all polynomials with integer coefficients such that the polynomial evaluates to zero on some integer setting of its variables (the polynomial is represented by a list of all its co-efficients). Is $L$ r.e.? Justify your answer.

4. Let $A, B, C$ be languages such that $A$ reduces to $B$ and $B$ reduces to $C$. Show that $A$ reduces to $C$.

5. Let $S = \{(M) | L(M) \text{ is infinite} \}$. Prove that $S$ is not recursive. Is $S$ r.e.? Justify your answer.