Tutorial Sheet 3

1. Prove that for any recursive enumerable language L, there are infinitely many Turing machines which recognize L.

2. Prove that the language $L_{somehalt} = \{\langle M \rangle | M$ is a binary machine, which halts on at least one input$\}$ is not recursive.

3. Consider $L = \{\langle M \rangle | M$ makes at most 100 transitions on any input$\}$. Is L recursive or not? Justify your answer.

4. Consider $L_{equiv} = \{\langle M_1 \rangle \$ \langle M_2 \rangle | M_1$ and M_2 are binary machines which recognize the same language$\}$. Is L recursive or not? Justify your answer.

5. In this question, we restrict ourselves to Turing machines with input alphabet $\{1\}$ and tape alphabet $\{1, b\}$. The busy beaver function $BB : \mathbb{N} \rightarrow \mathbb{N}$ is defined as follows: given a positive integer n, $BB(n)$ is the maximum, among all Turing machines with n states which halt on the empty string, of the number of 1s on the tape when the machine halts on the empty string. Show that BB is uncomputable.

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