Tutorial Sheet 3

1. Prove that for any recursive enumerable language $L$, there are infinitely many Turing machines which recognize $L$.

2. Prove that the language $L_{\text{somehalt}} = \{ \langle M \rangle | M$ is a binary machine, which halts on at least one input $\}$ is not recursive.

3. Consider $L = \{ \langle M \rangle | M$ makes at most 100 transitions on any input $\}$. Is $L$ recursive or not? Justify your answer.

4. Consider $L_{\text{equiv}} = \{ \langle M_1 \rangle S \langle M_2 \rangle | M_1$ and $M_2$ are binary machines which recognize the same language $\}$. Is $L$ recursive or not? Justify your answer.

5. In this question, we restrict ourselves to Turing machines with input alphabet $\{1\}$ and tape alphabet $\{1, b\}$. The busy beaver function $BB : \mathbb{N} \rightarrow \mathbb{N}$ is defined as follows: given a positive integer $n$, $BB(n)$ is the maximum, among all Turing machines with $n$ states which halt on the empty string, of the number of 1s on the tape when the machine halts on the empty string. Show that $BB$ is uncomputable.