Tutorial Sheet 1

1. Prove that the set of languages over $\{0, 1\}$ has a larger cardinality than the set of Turing machines. Use this to conclude that there is an uncomputable language.

2. English languages phrases such as "one thousand and one" and "the fifth prime number" describe specific numbers, while phrases such as "the quick brown fox that jumped over the lazy dog" self-evidently do not. Consider the following phrase: "the first number not named by an English language phrase less than twenty words long". Why does this lead to a paradox, and what is the resolution of the paradox?

3. For each of the following Turing machines, say what is the language accepted by the machine. We assume $\Sigma = \{0, 1\}, \Gamma = \Sigma \cup \{b\}$, where b is the blank symbol. Also q_I is understood to be the initial state and q_F the accepting state:

   (a) $Q = \{q_I, q_F, q_1, q_2, q_3\}$, $\delta = \{\}$.

   (b) $Q = \{q_I, q_F\}$, $\delta = \{(q_I, 0, q_I, 0, R), (q_I, 1, q_I, 1, R), (q_I, b, q_F, b, R)\}$.

   (c) $Q = \{q_I, q_F, q_1, q_2\}$, $\delta = \{(q_I, 0, q_1, 0, R), (q_1, 0, q_I, 0, L), (q_1, 1, q_2, 0, R), (q_2, 0, q_2, 0, R), (q_2, 1, q_2, 1, R), (q_2, b, q_F, b, L)\}$

4. Design a Turing machine transducer with alphabet $\{1\}$, which on input 1^n outputs 1^{n+1} if n is odd and 1^n if n is even, where $n \geqslant 0$ is any non-negative integer.

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