Computer Graphics 9 - Environment mapping and mirroring

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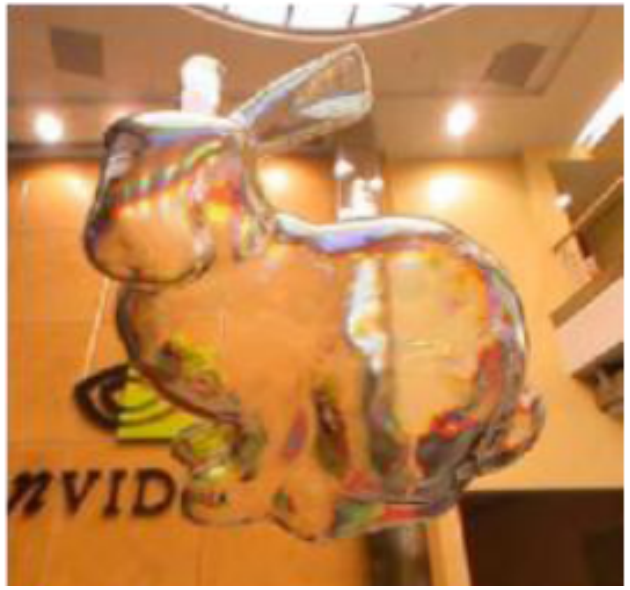
Slides courtesy of Taku Komura
www.inf.ed.ac.uk/teaching/courses/cg

Overview

- **Environment Mapping**
  - Introduction
  - Sphere mapping
  - Cube mapping
  - Refractive mapping
- Mirroring
  - Introduction
  - Reflection first
  - Stencil buffer
  - Reflection last

Environment Mapping: Background

- Many objects are glossy or transparent
- Glossy objects reflect the external world
- The world is refracted through transparent objects
- Important to make the scene appear realistic

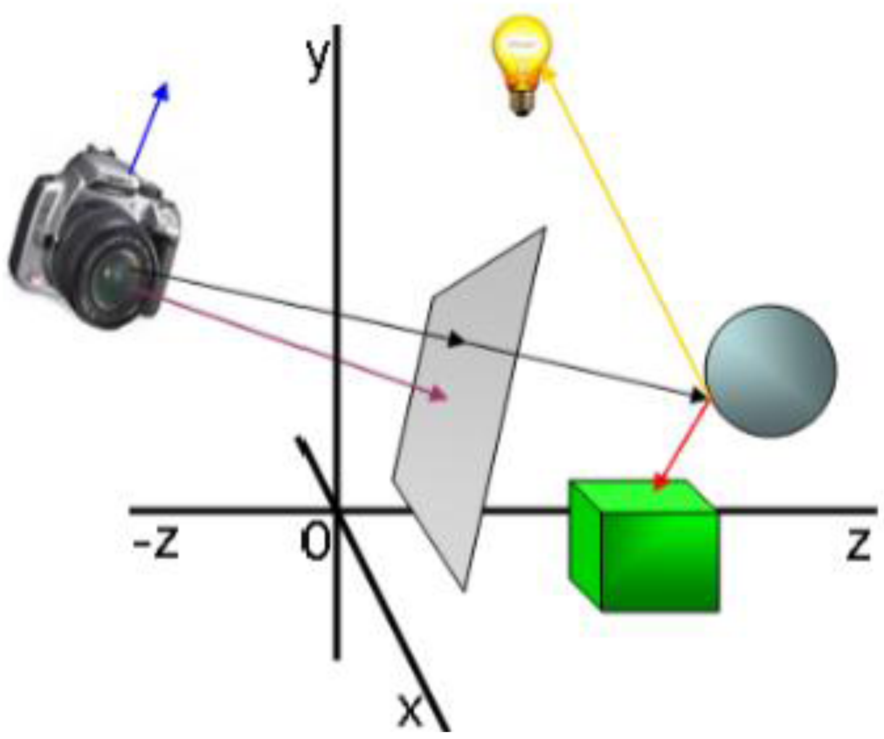



Example

Environment Mapping: Background

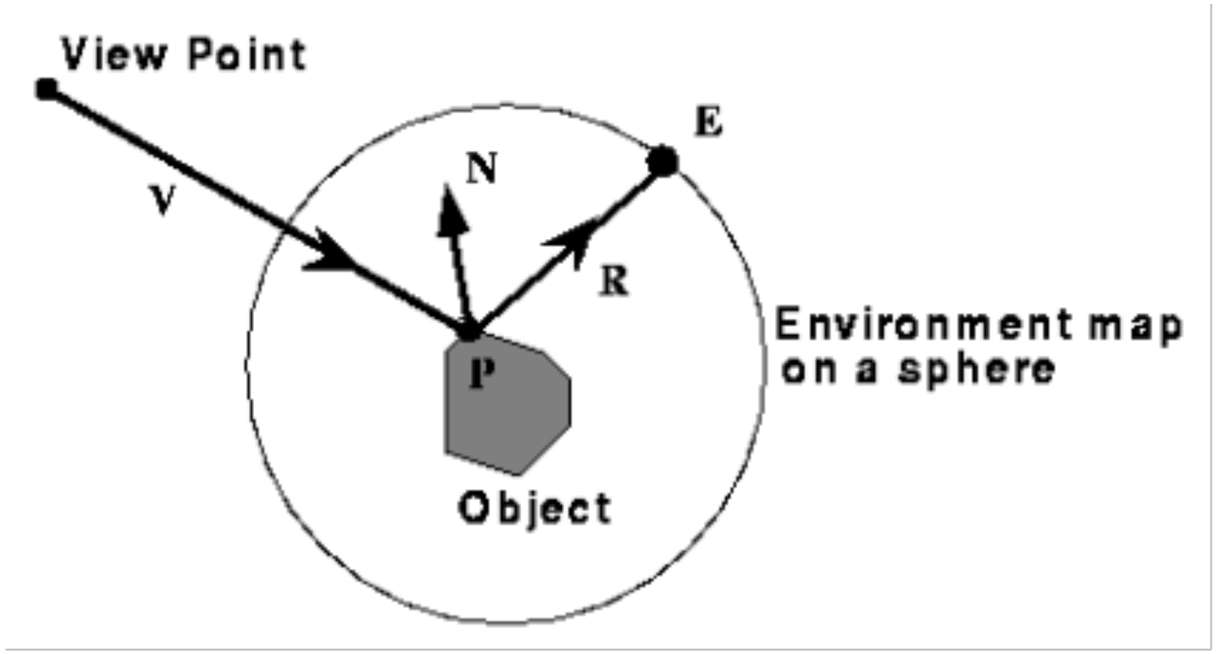
Precisely simulating such phenomena is computationally costly

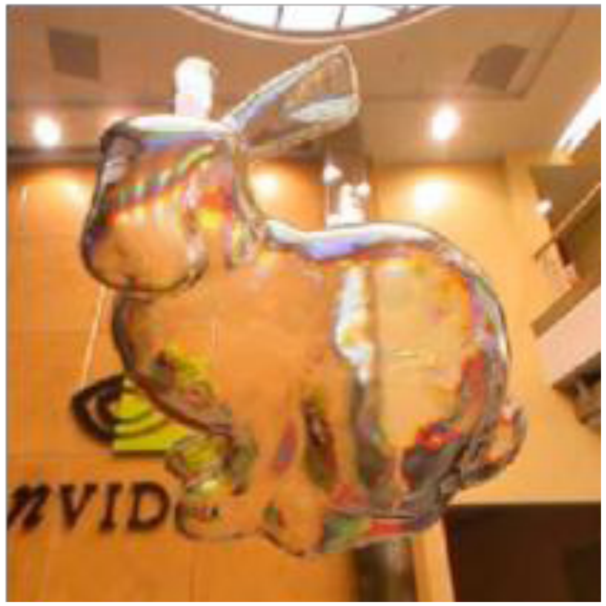
- Requires ray tracing, which can be expensive
- Tracking the rays, finding out where they collide, and doing another lighting computation



Environment Mapping

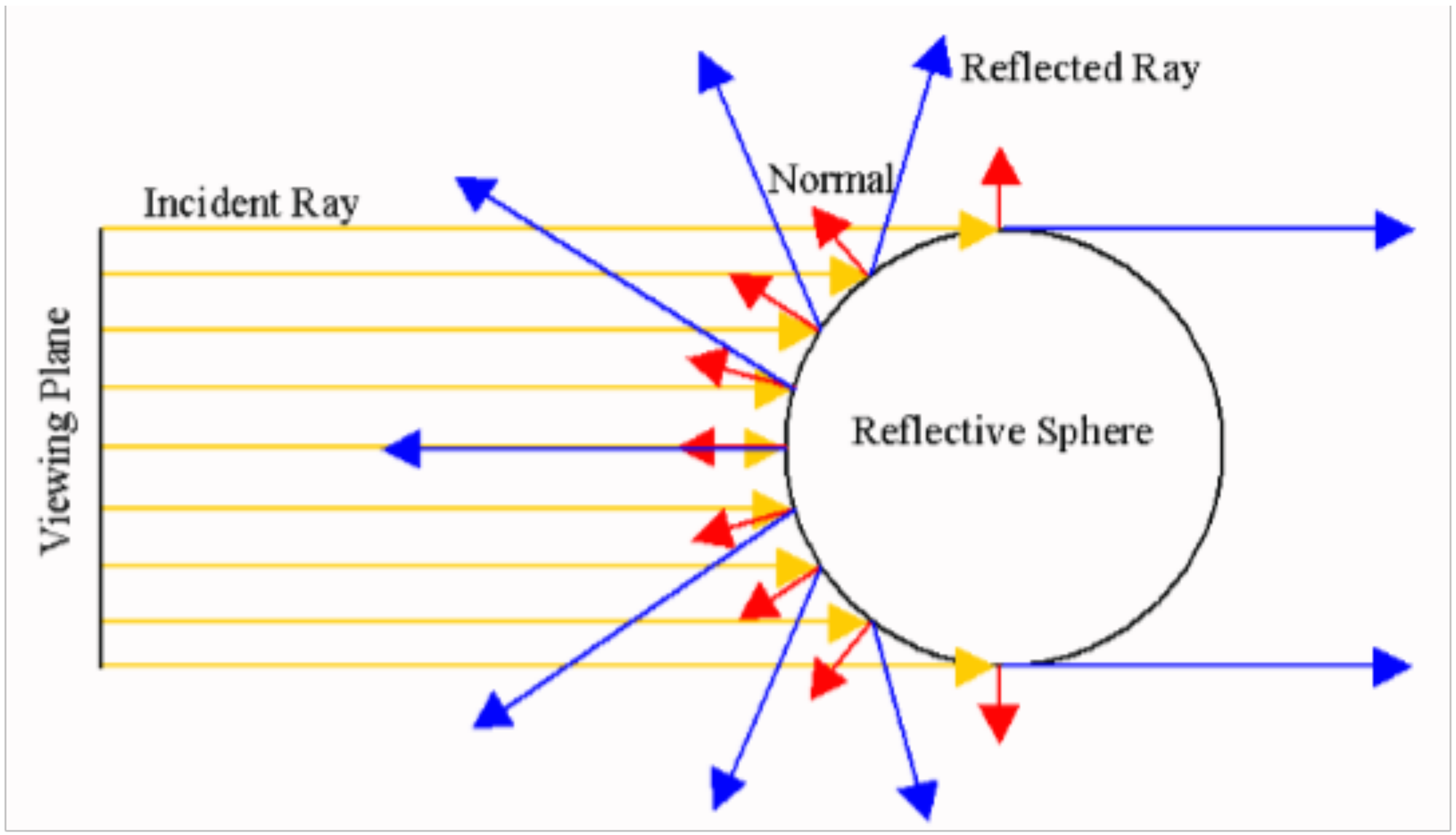
- Simple yet powerful method to generate reflections
- Simulate reflections by using the reflection vector to index a texture map at "infinity".





The original environment map was a sphere [by Jim Blinn '76]

Sphere maps

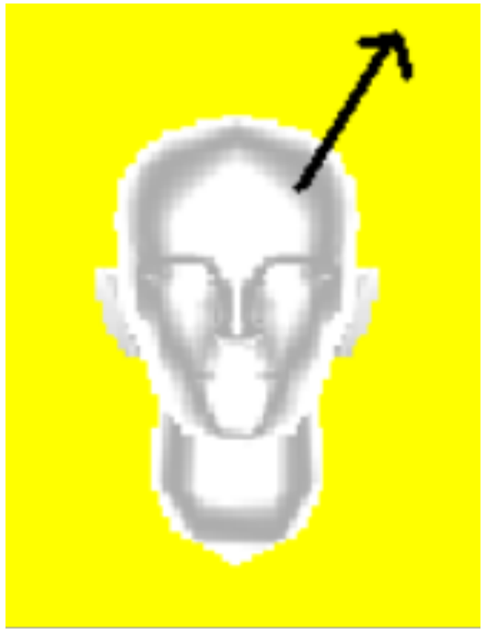


- A mapping between the reflection vector and a circular texture
- Contains the whole environment around a point in a single image
- Low resolution around edges

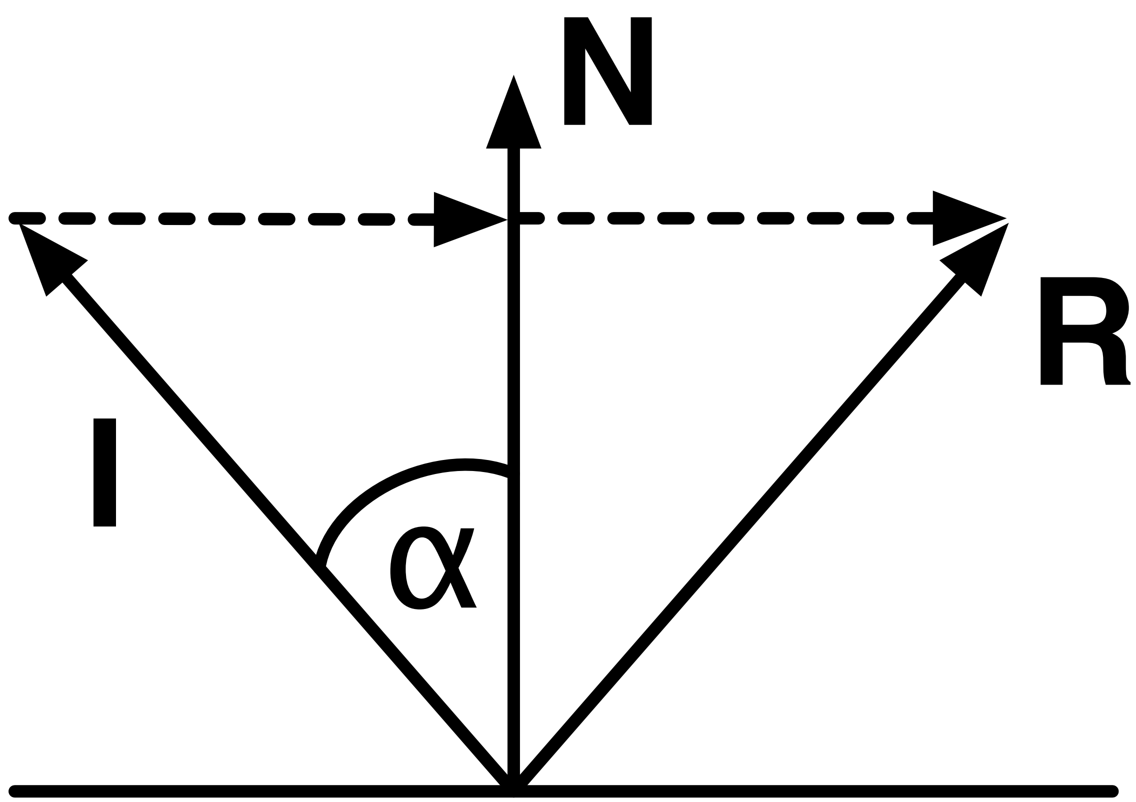
Sphere maps: overview

- Compute the reflection vector at the surface of the object
- Find the corresponding texture coordinates on the sphere map
- Use the texture to colour the surface of the object

Indexing sphere maps



- Calculate the reflection vector R based on direction to eye I



$$R = 2(N \cdot I)N - I$$

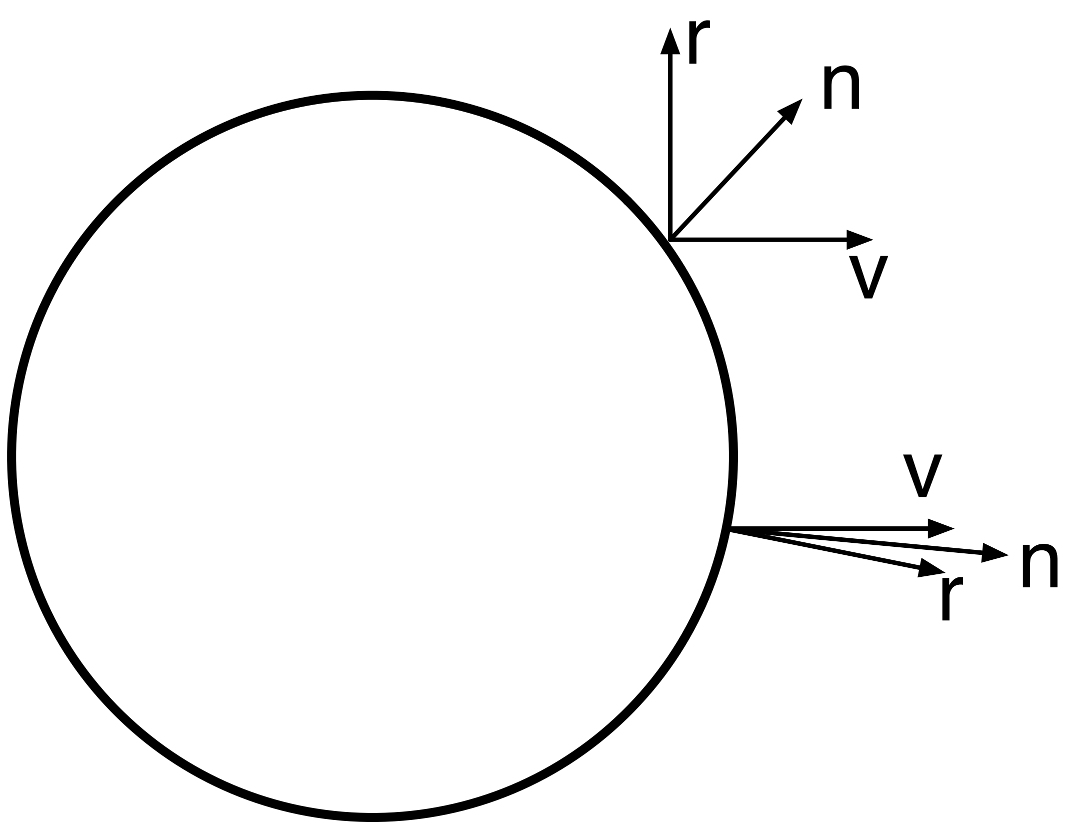
Indexing the sphere map

- Consider the mapping between reflection vectors on the sphere and the normal vector

- Assume that v is fixed at (0,0,1)

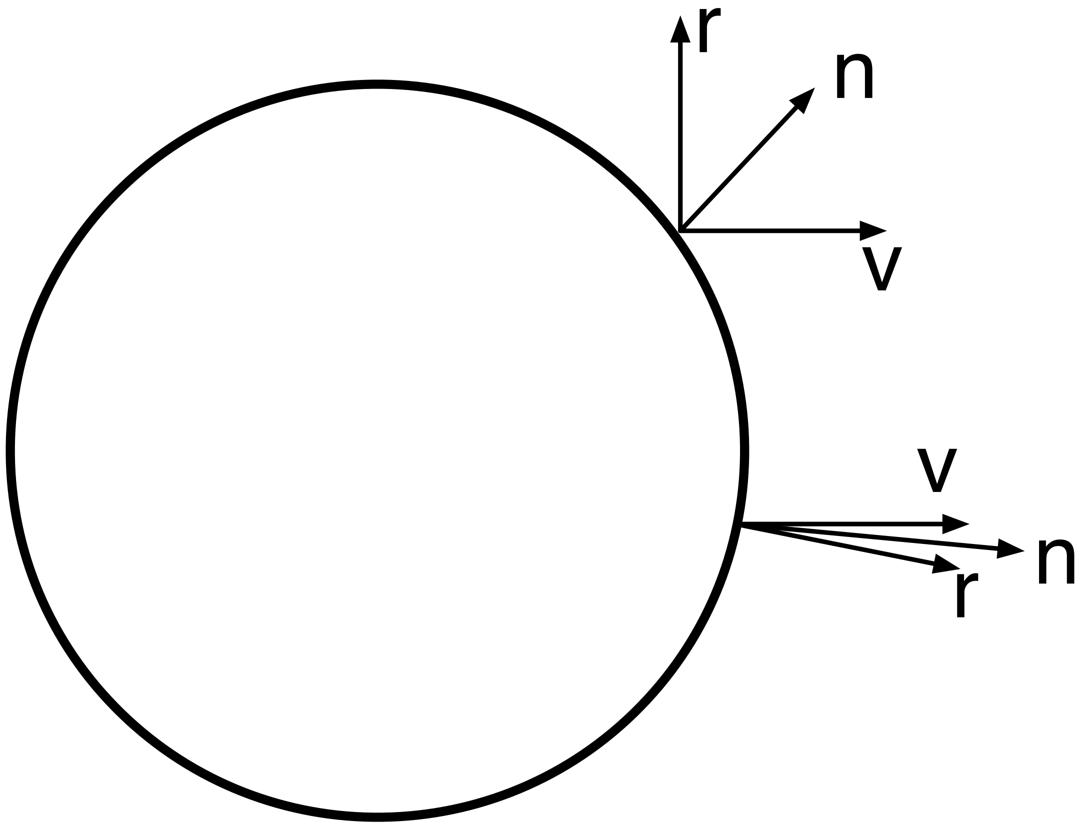
- An un-normalised normal vector n is then:

$$n = r + v$$
$$= (r_x, r_y, r_z + 1)$$



Indexing the sphere map

$$\overline{n} = (\frac{r_x}{m}, \frac{r_y}{m}, \frac{r_z + 1}{m})$$
$$m = \sqrt{r_x^2 + r_y^2 + (r_z + 1)^2}$$

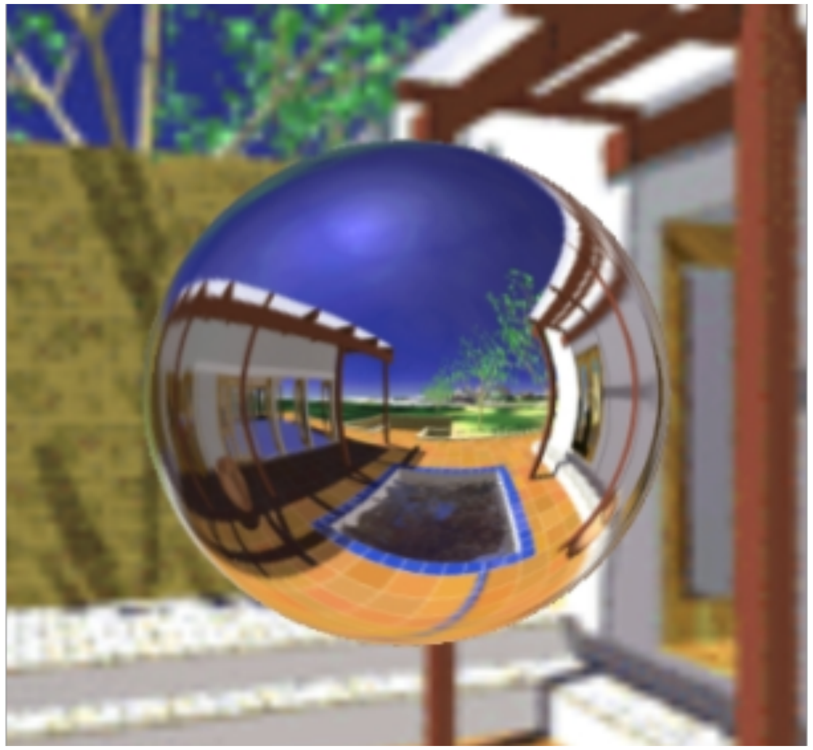


- Assume the sphere is of unit radius and centred at the origin

- We can index the sphere map using the x and y components of the normalised normal vector

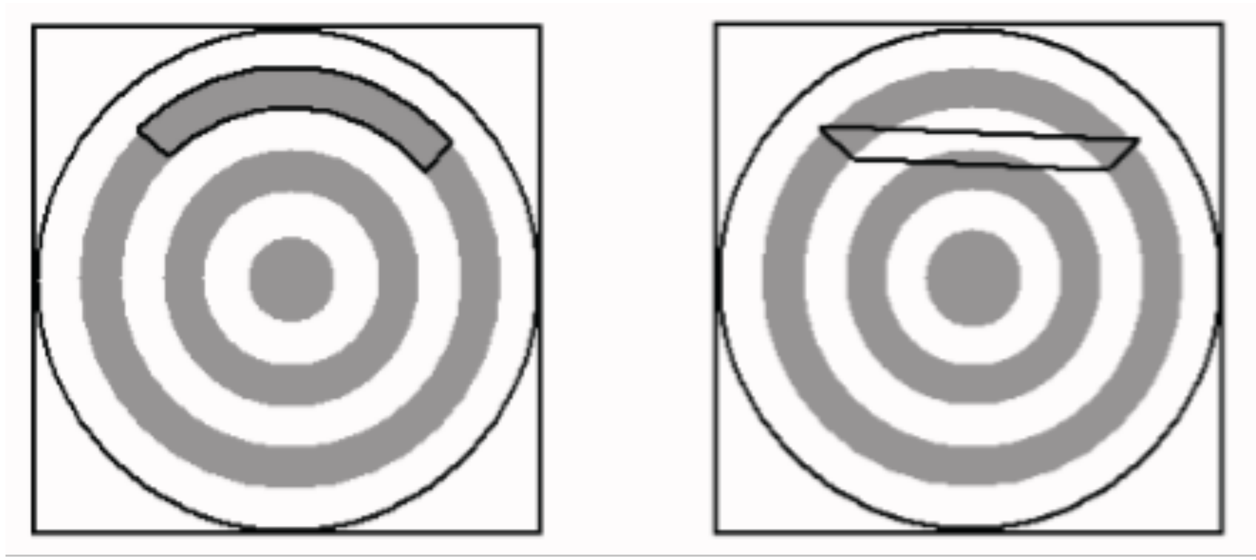
Generating sphere maps

- Take a photograph of a shiny sphere
- Mapping a cubic environment map onto a sphere
- For synthetic scenes, use ray tracing



Issues with sphere mapping

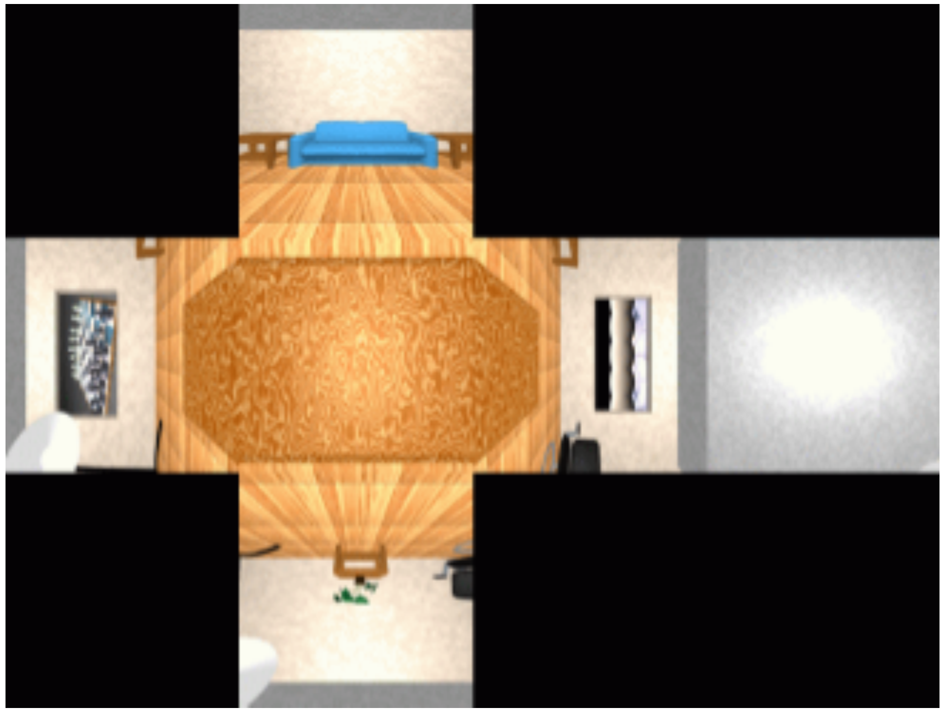
- Cannot change the viewpoint (requires recomputing the sphere map)
- Highly non-uniform sampling
- Highly non-linear mapping
- Linear interpolation of texture coordinates picks up the wrong texture pixels
- Do per-pixel sampling or use high resolution polygons

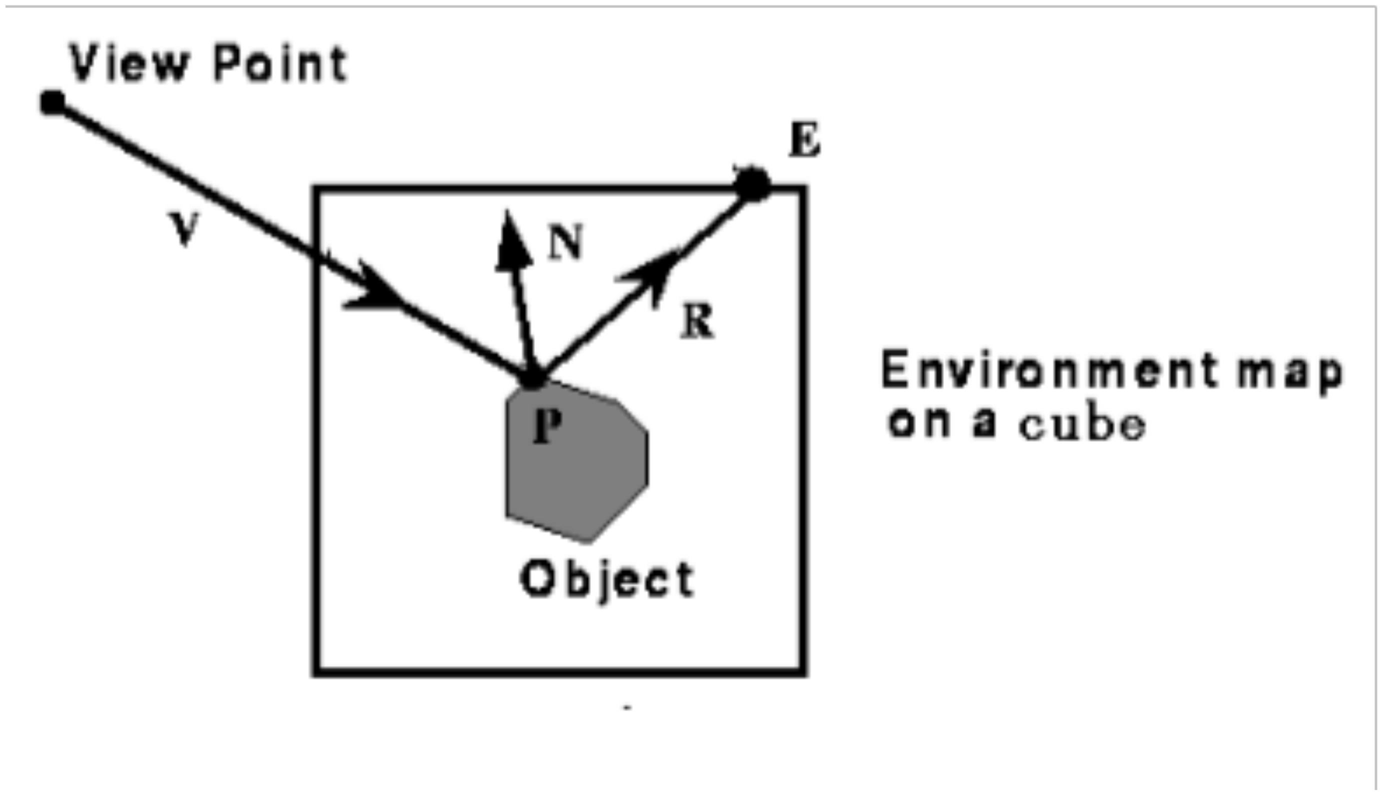


Correct Linear

Cube Mapping

- The map resides on the surfaces of a cube around the object
- Align the faces of the cube with the coordinate axes

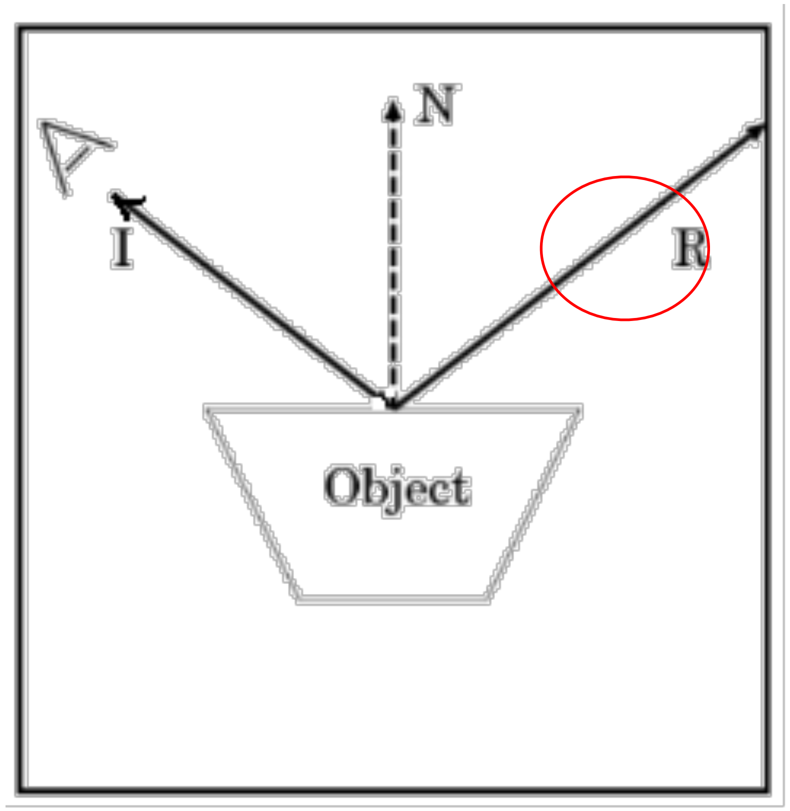




Procedure

During rasterisation, for every pixel,

1. Calculate the reflection vector R using the camera (incident) vector and the normal vector of the object N
2. Select the face of the environment map and the pixel on the face according to R
3. Colour the pixel with the colour of the environment map
   - Look up the environment map just using R



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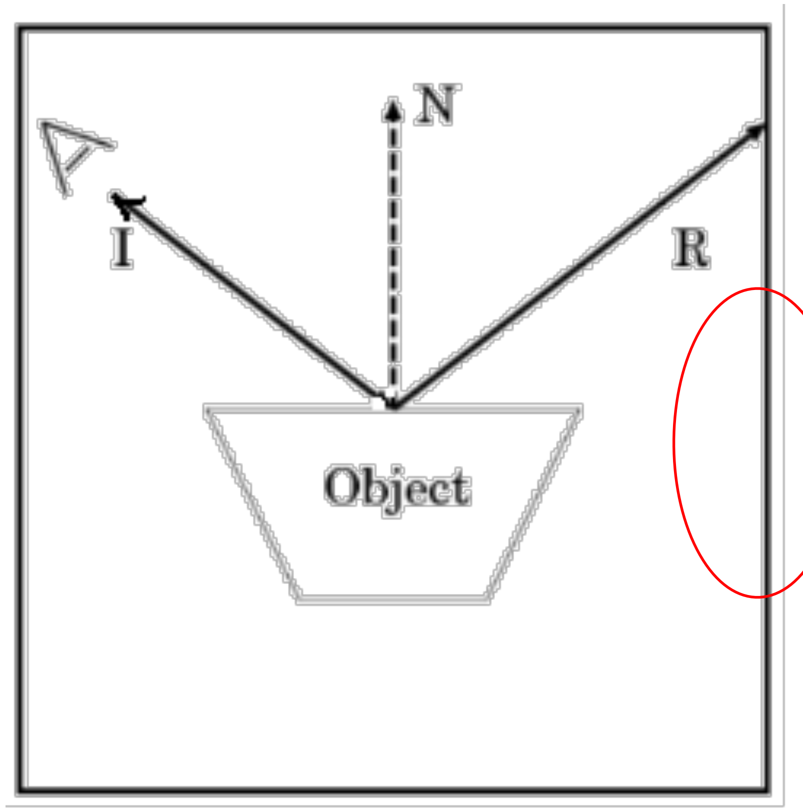

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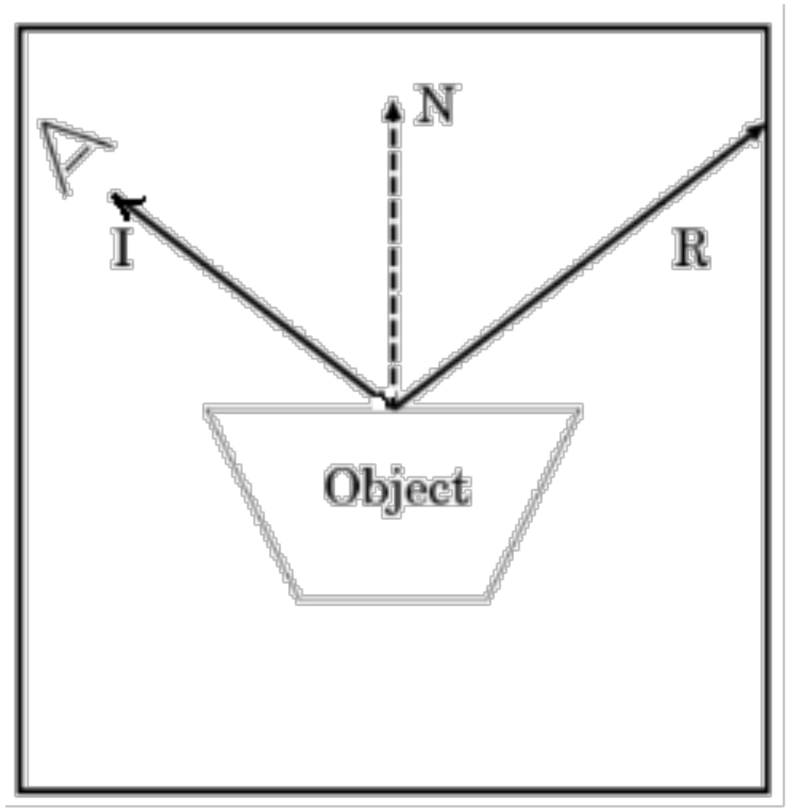
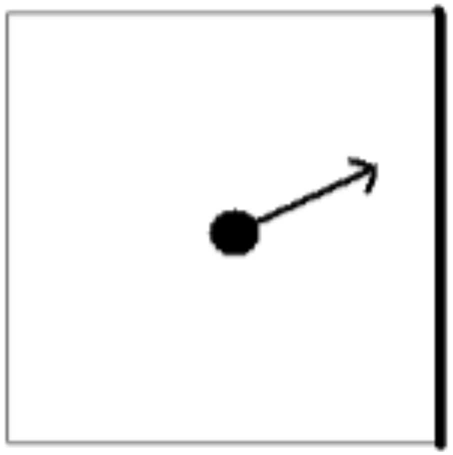


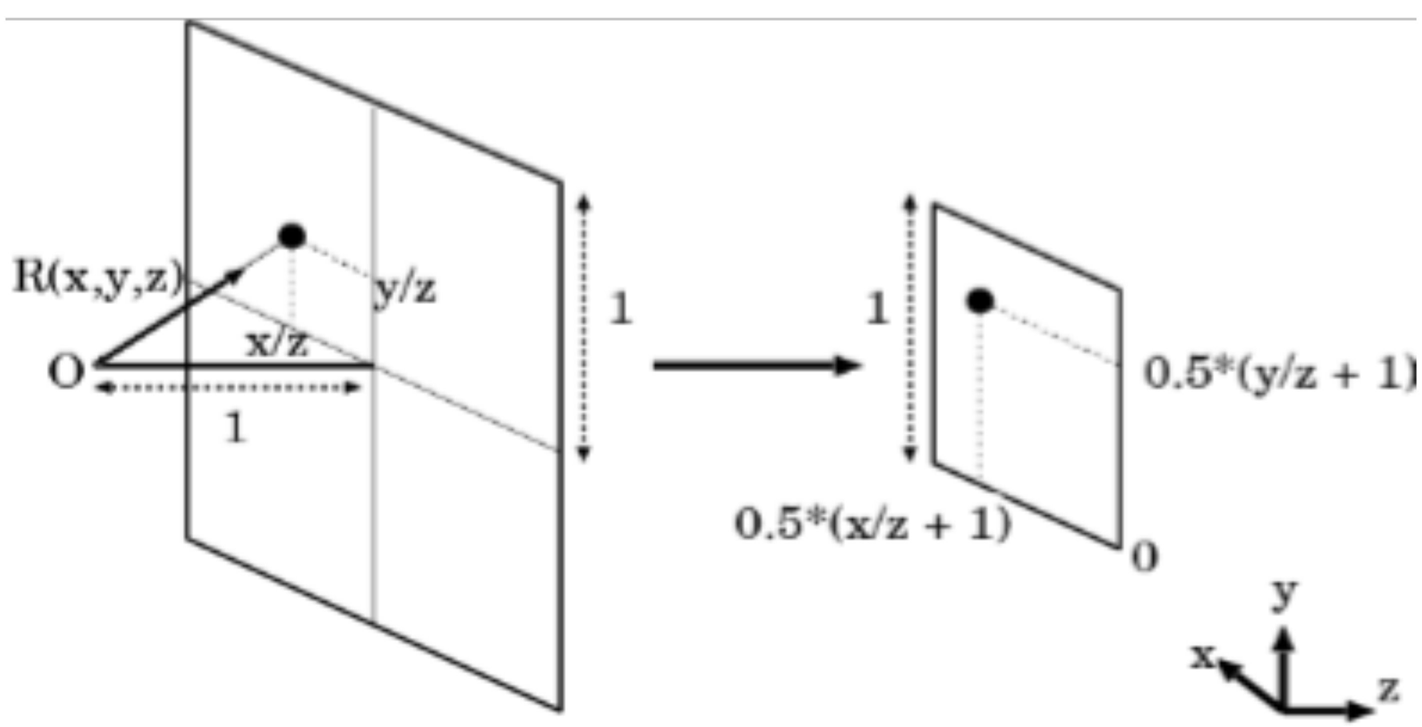
Indexing Cubic Maps

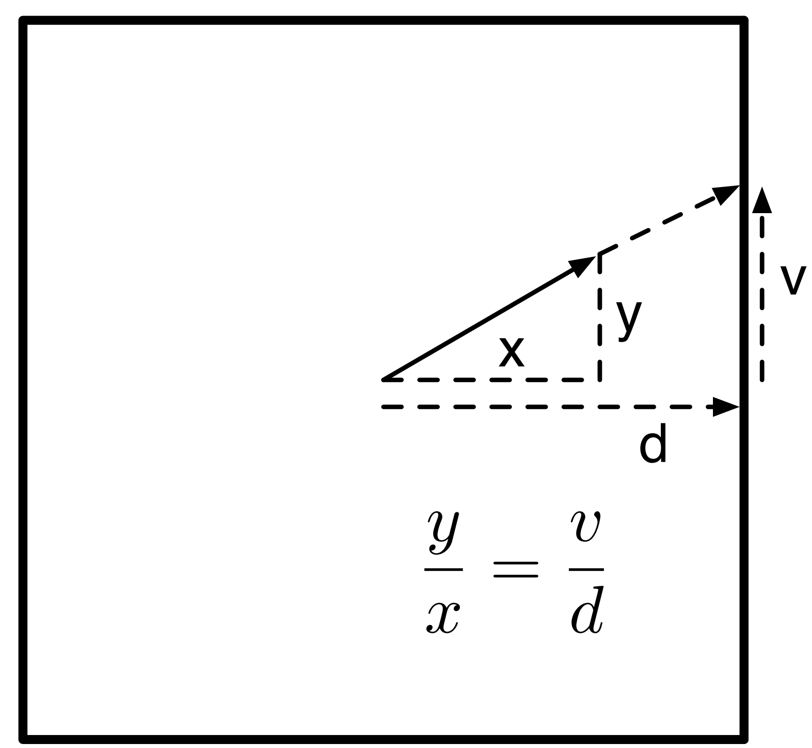
- Assume you have R and the cube's faces are aligned with the coordinate axes
- How do you decide which face to use?
- The reflection vector coordinate with the largest magnitude
- R=(0.3, 0.2, 0.8) -> facing in +z direction



Indexing Cubic Maps

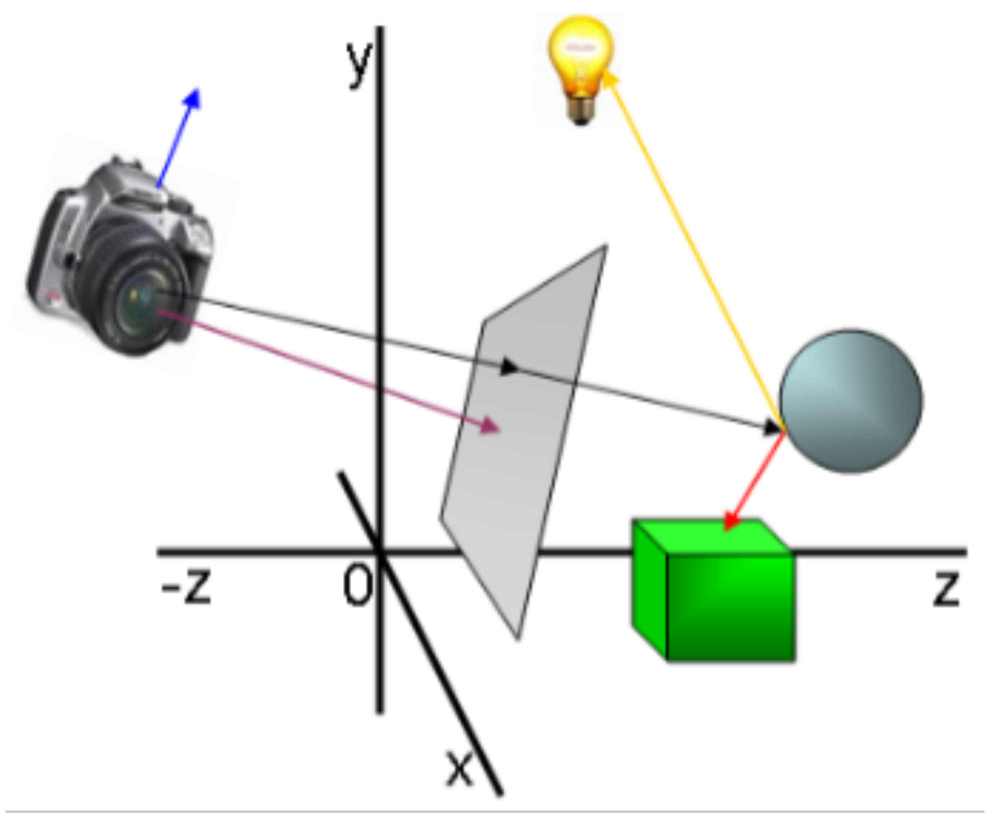
- How do you decide which texture coordinates to use?
- Divide by the coordinate with the largest magnitude
- Now have a value in the range [-1,1]
- Remapped to a value between 0 and 1.

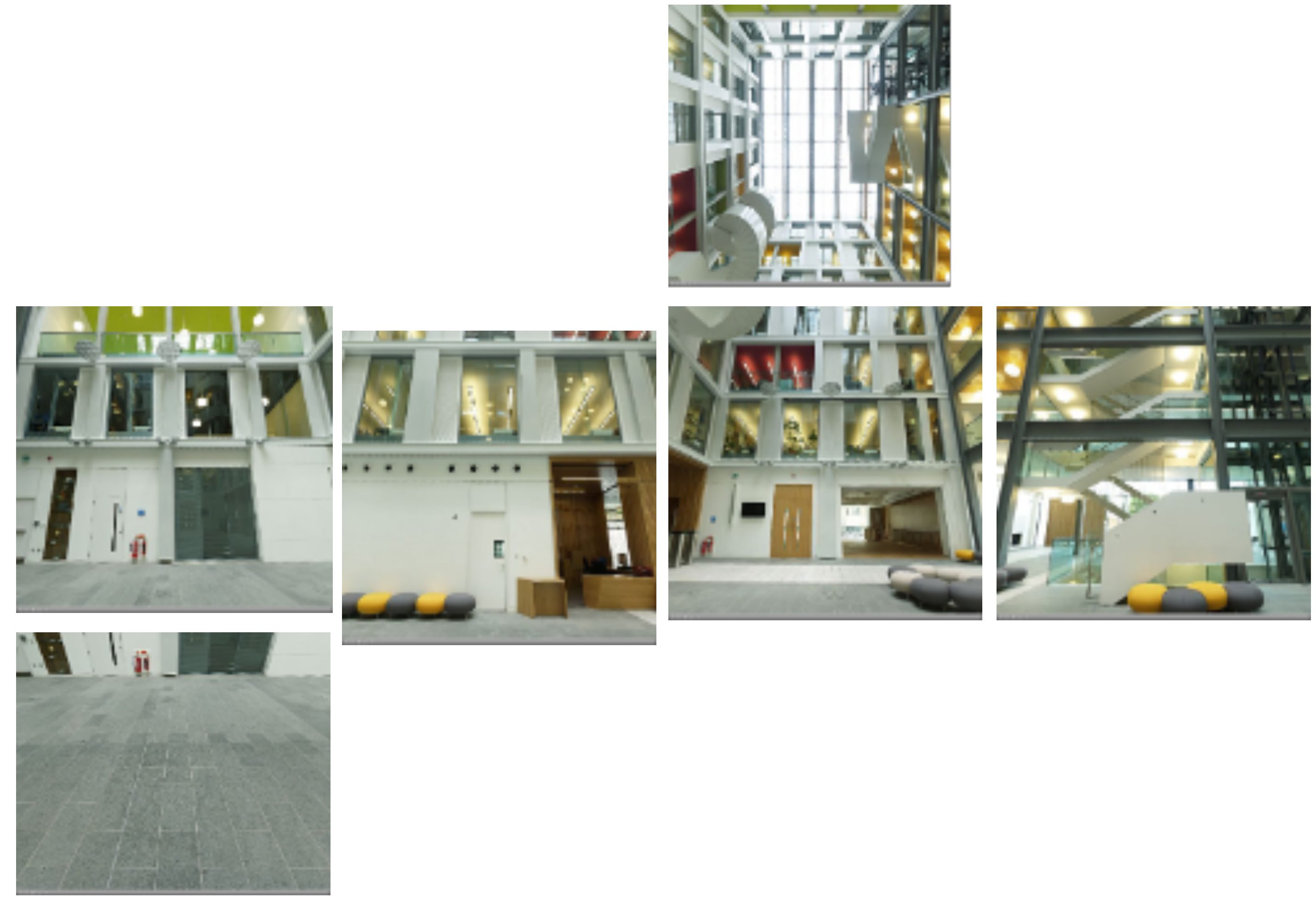




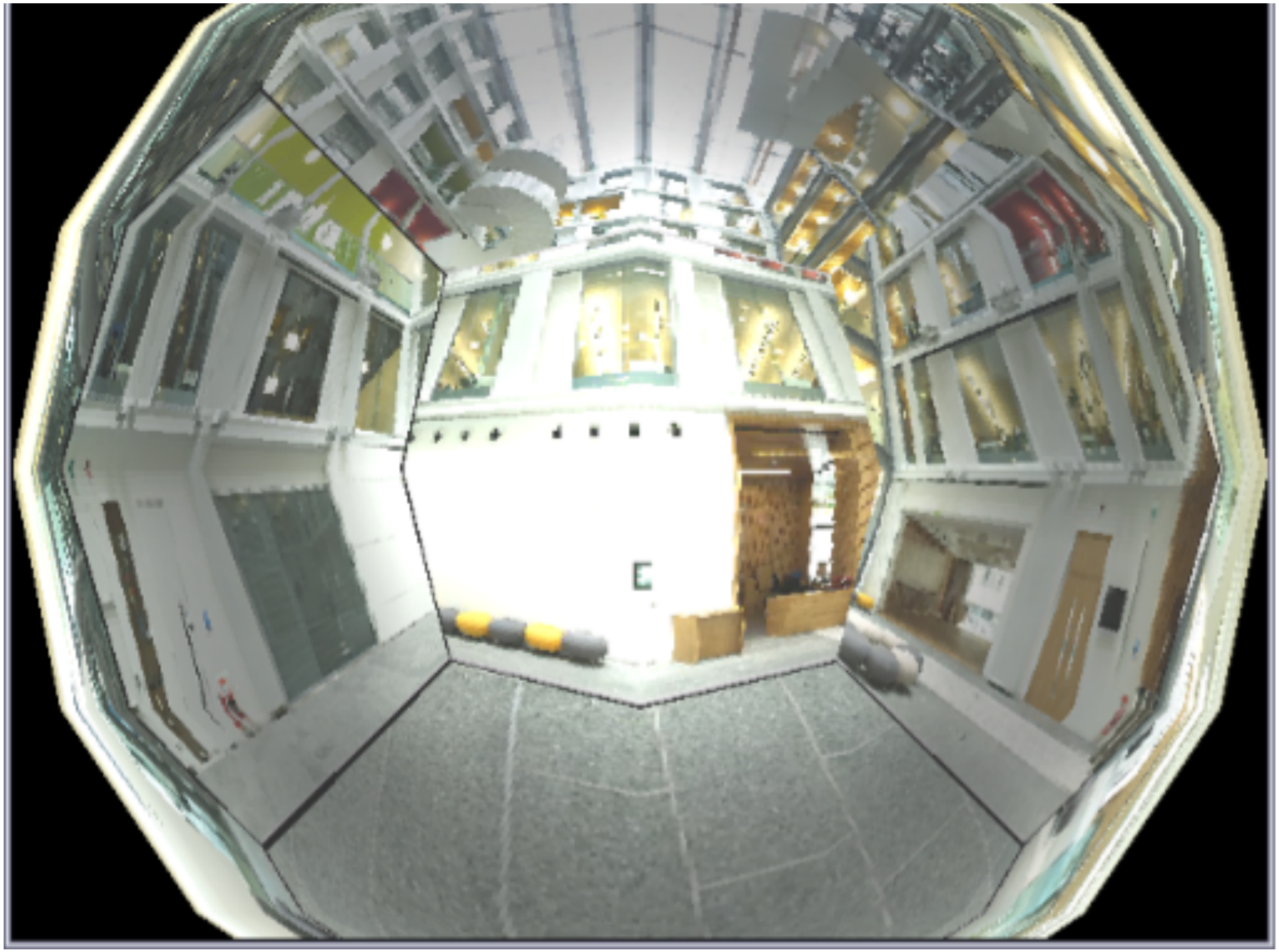
Cubic Mapping: How to make one?

- Draw with a computer
- Take 6 photos of a real environment with a camera in the object's position: much easier





Made from the Forum Images

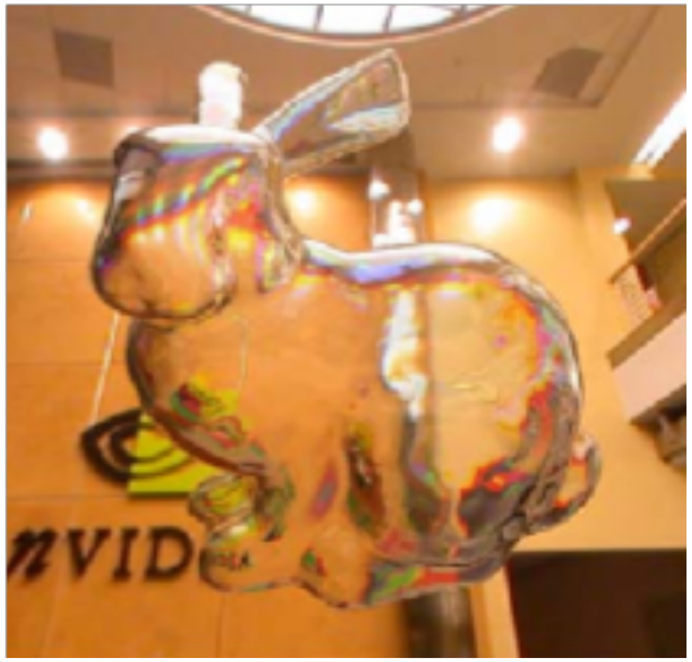


Pros and cons

- Advantages of cube mapping?
- Problems with sphere mapping?

Refractive environment mapping

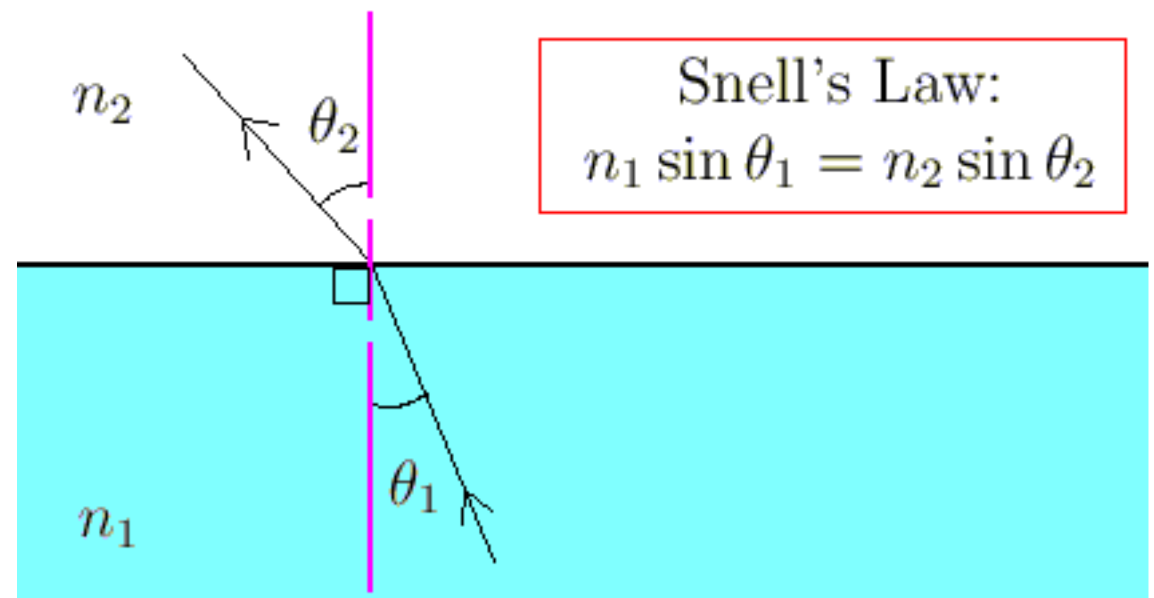
- When simulating effects mapping the refracted environment onto translucent materials such as ice or glass, we must use Refractive Environment Mapping



Snell's law

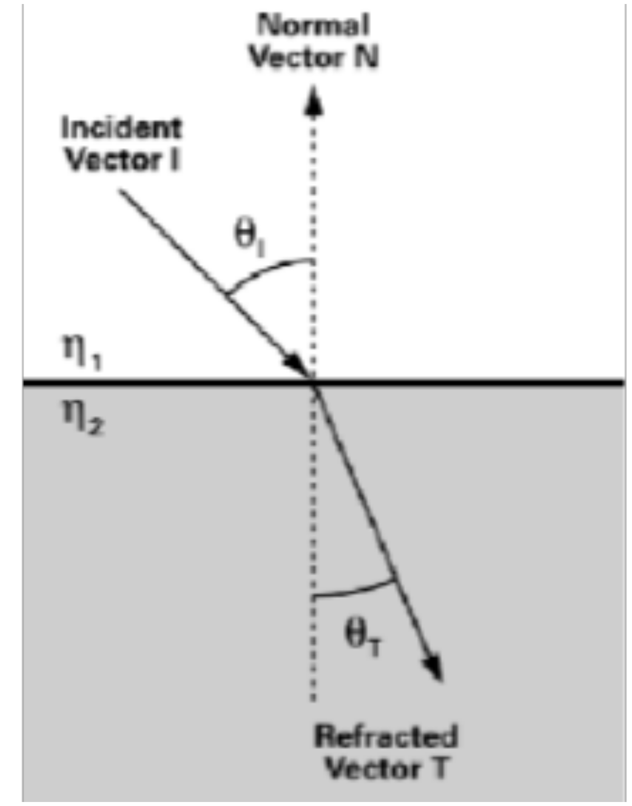
- Light travels at different speeds in different media

Material	Index of Refraction
Vacuum	1.0
Air	1.0003
Water	1.3333
Glass	1.5
Plastic	1.5
Diamond	
2.417	



Snell's Law

- When light passes through a boundary between two materials of different density (air and water, for example), the light's direction changes.
- The direction follows Snell's Law
- We can do environment mapping using the refracted vector T



$$\eta_1 \sin\theta_I = \eta_2 \sin\theta_T$$

Material	Index of Refraction
Vacuum	1.0
Air	1.0003
Water	1.3333
Glass	1.5
Plastic	1.5
Diamond	
2.417	

Snell's law

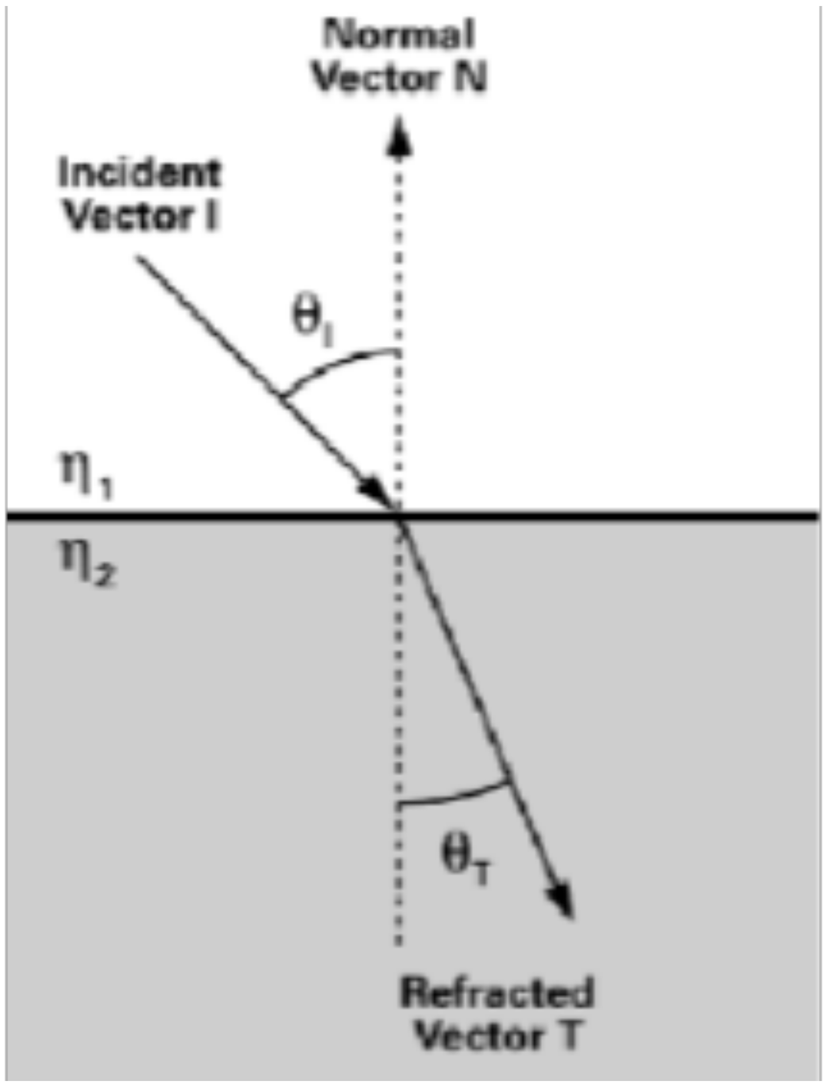
- Incoming vector I
- Refracted vector T

$$T = rI + (w - k)n$$

$$r = \frac{n_1}{n_2}$$

$$w = -(I \cdot n)r$$

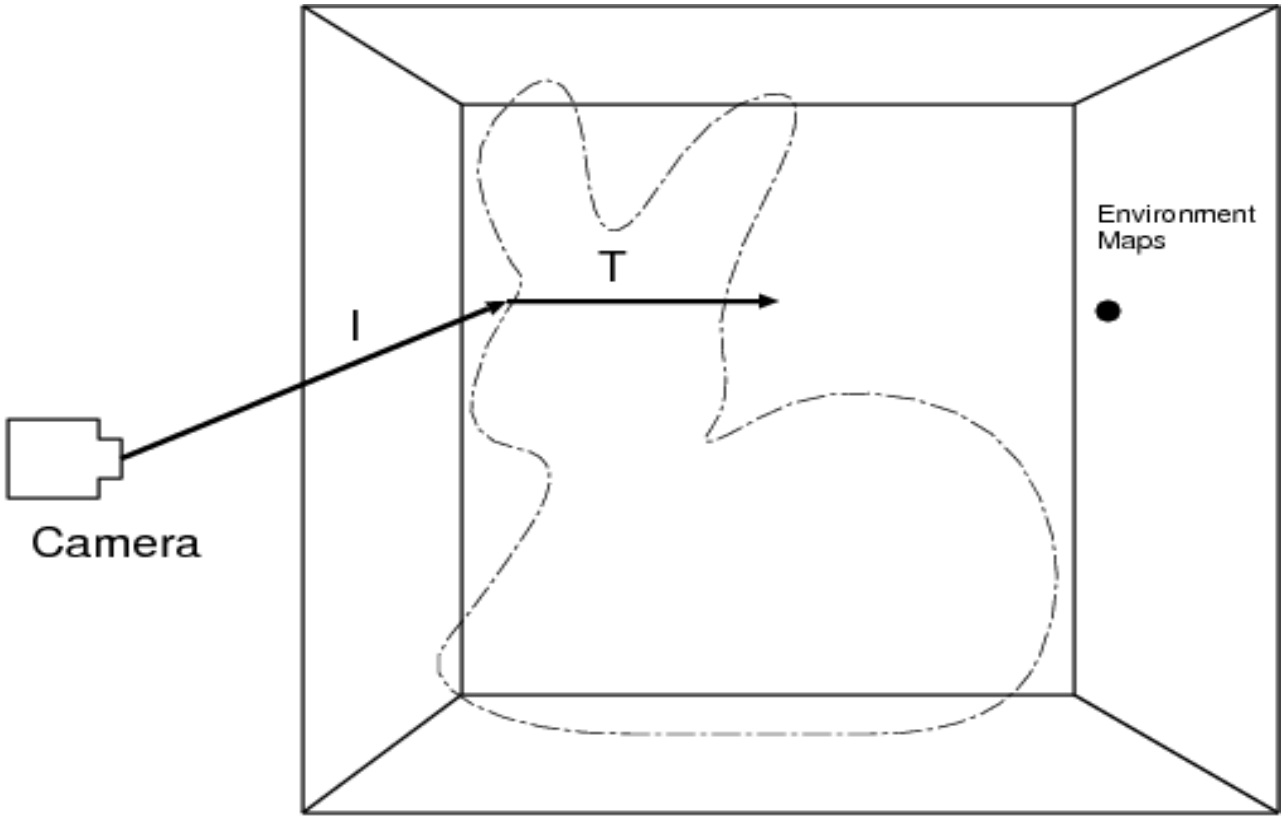
$$k = \sqrt{1 + (w - r)(w + r)}$$



Refractive environment mapping

- Use the refraction vector after the first hit as the index to the environment map

- Costly to compute the second refraction vector



Summary

- Environment mapping is a quick way to simulate the effects of reflecting the surrounding world on the surface of a glossy object
- Practical approaches are cube mapping and sphere mapping
- Can also be applied for simulating refraction

Overview

- Environment Mapping
  - Introduction
  - Sphere mapping
  - Cube mapping
  - Refractive mapping
- **Mirroring**
  - Introduction
  - Reflection first
  - Stencil buffer
  - Reflection last

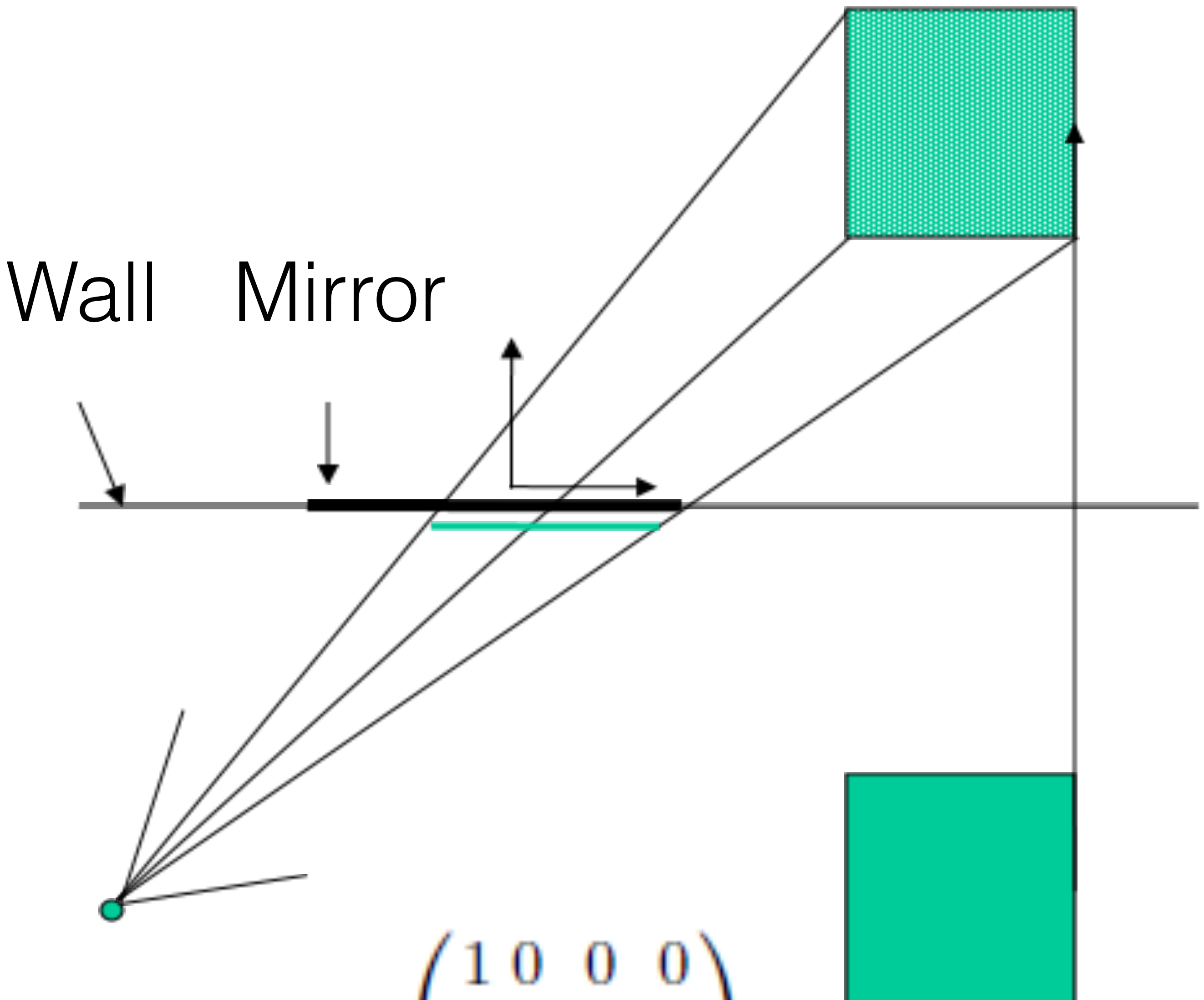
Flat Mirrors: Background

- Basic idea: Drawing a scene with mirrors
- Mirrors reflect the world
- A scene with a mirror can be drawn by rendering the world twice:
  - Draw original scene
  - Draw reflected scene

Flat Mirrors: Background

- Simply rendering the scene twice can result in problems
- Unless the mirrored world is hidden by the real world, the flipped world may appear outside of the mirror!
- We can avoid such problems using a “stencil buffer”

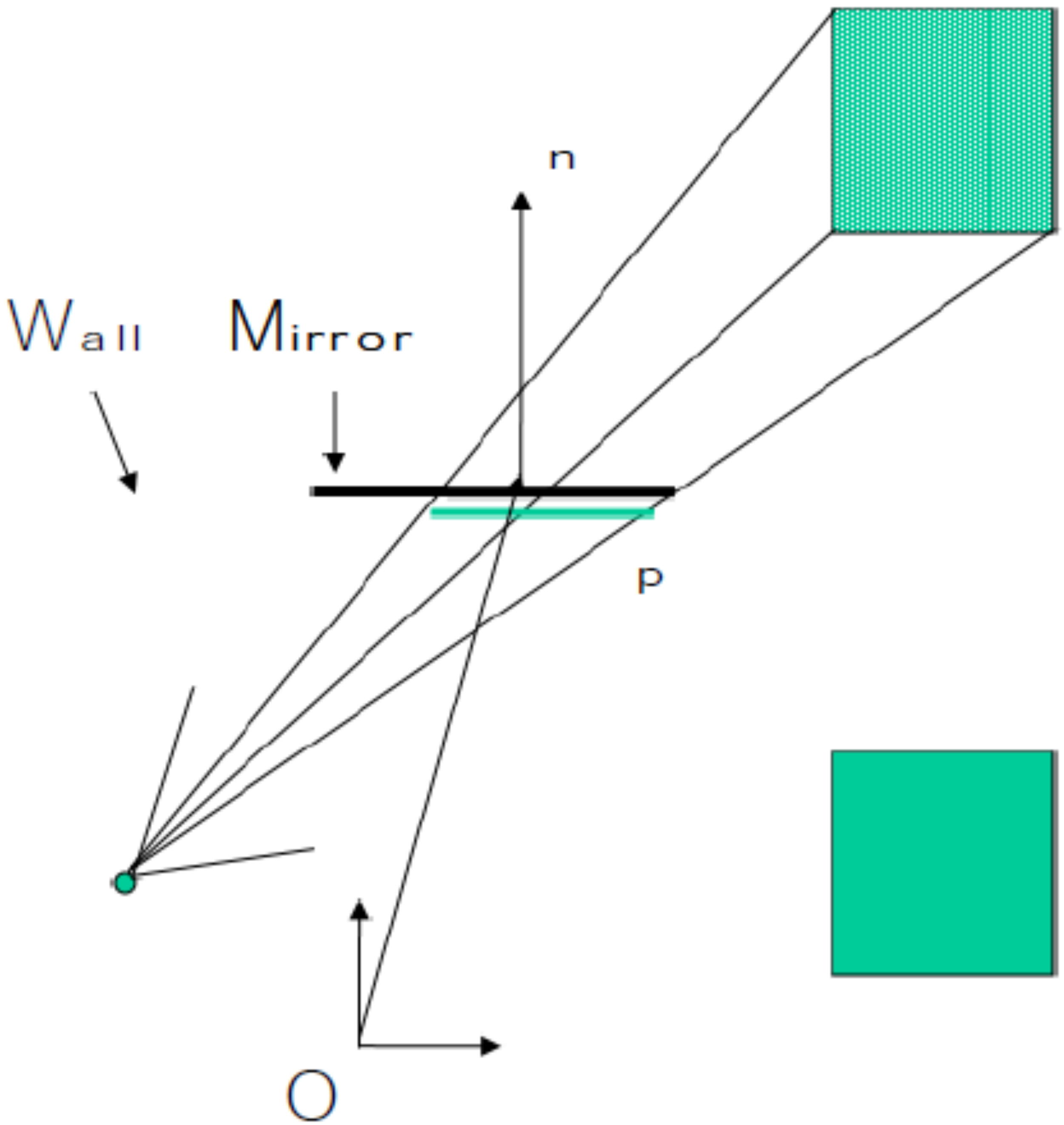
Reflecting objects



$$S = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- If the mirror passes through the origin, and is aligned with a coordinate axis, then just negate appropriate coordinate
- For example, if a reflection plane has a normal n=(0,1,0) and passes the origin, the reflected vertices can be obtained by scaling matrix S(1,-1,1)

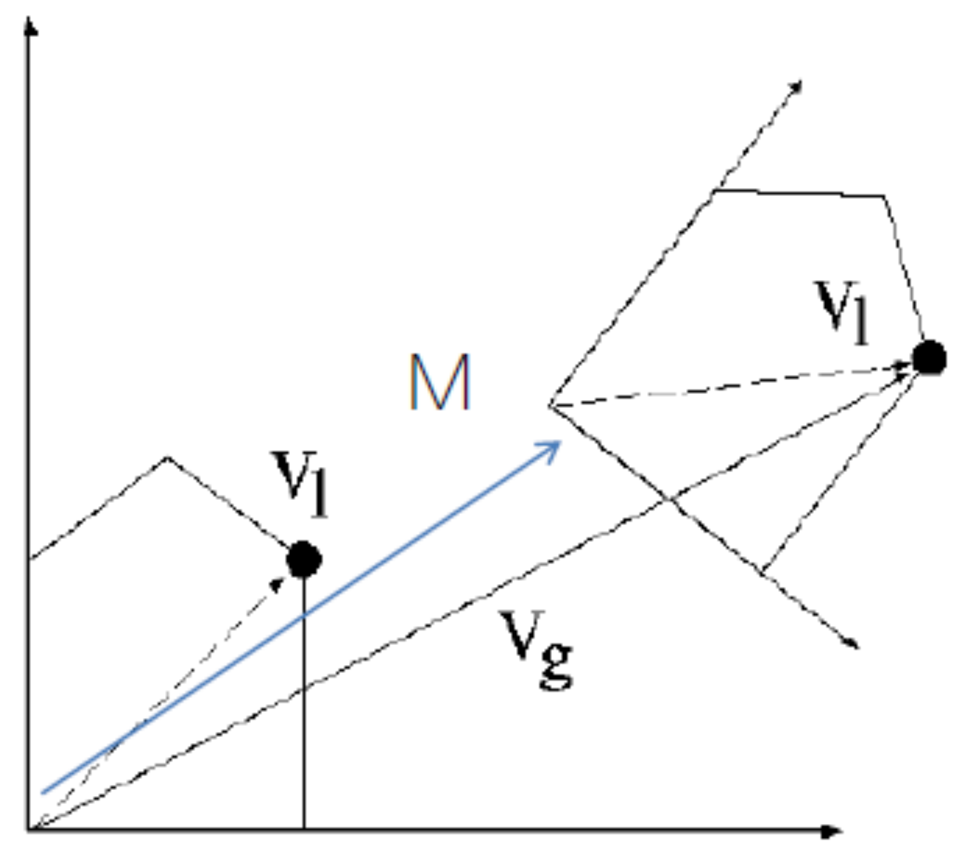
Reflecting objects



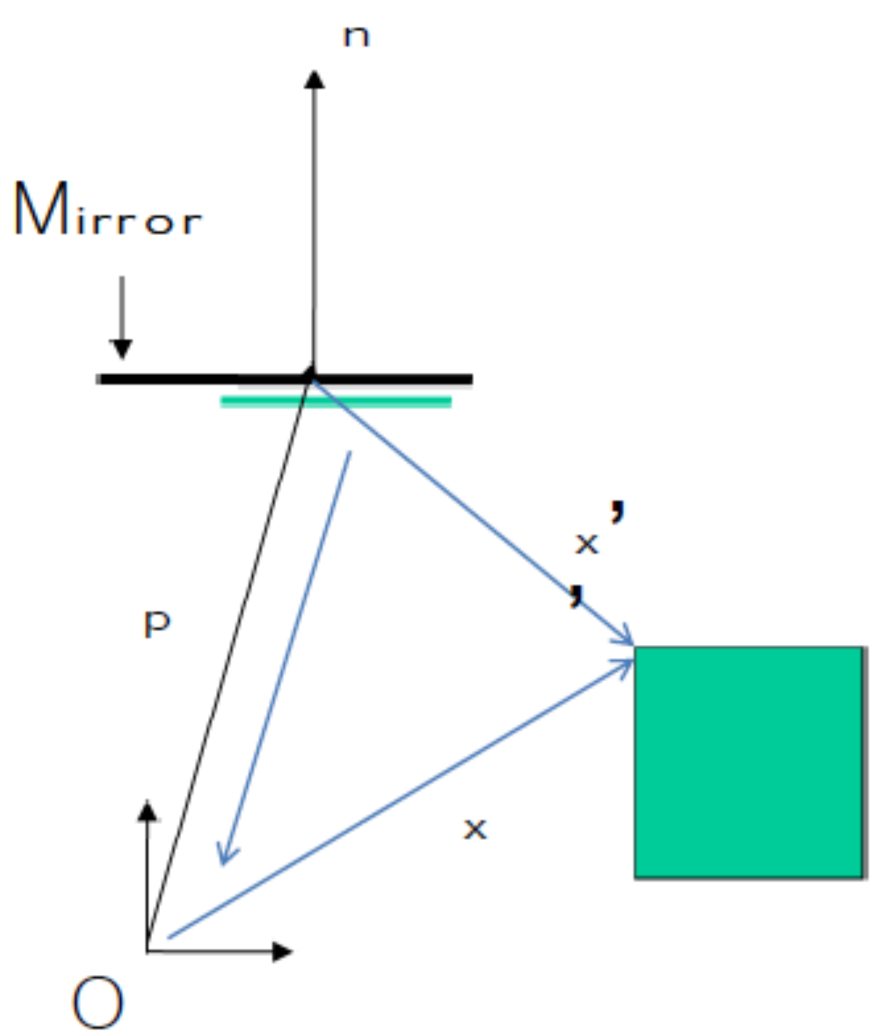
- What if the mirror is not on a plane that passes the origin?
- How do we compute the mirrored world?
- First, we need to compute the location of objects relative to the mirror

Recap:
Transformations between different coordinate systems

- We can interpret that the transformation matrix is converting the location of vertices between different coordinate systems
- $v_g = M v_l$
- $v_l = M^{-1} v^g$



Reflecting objects

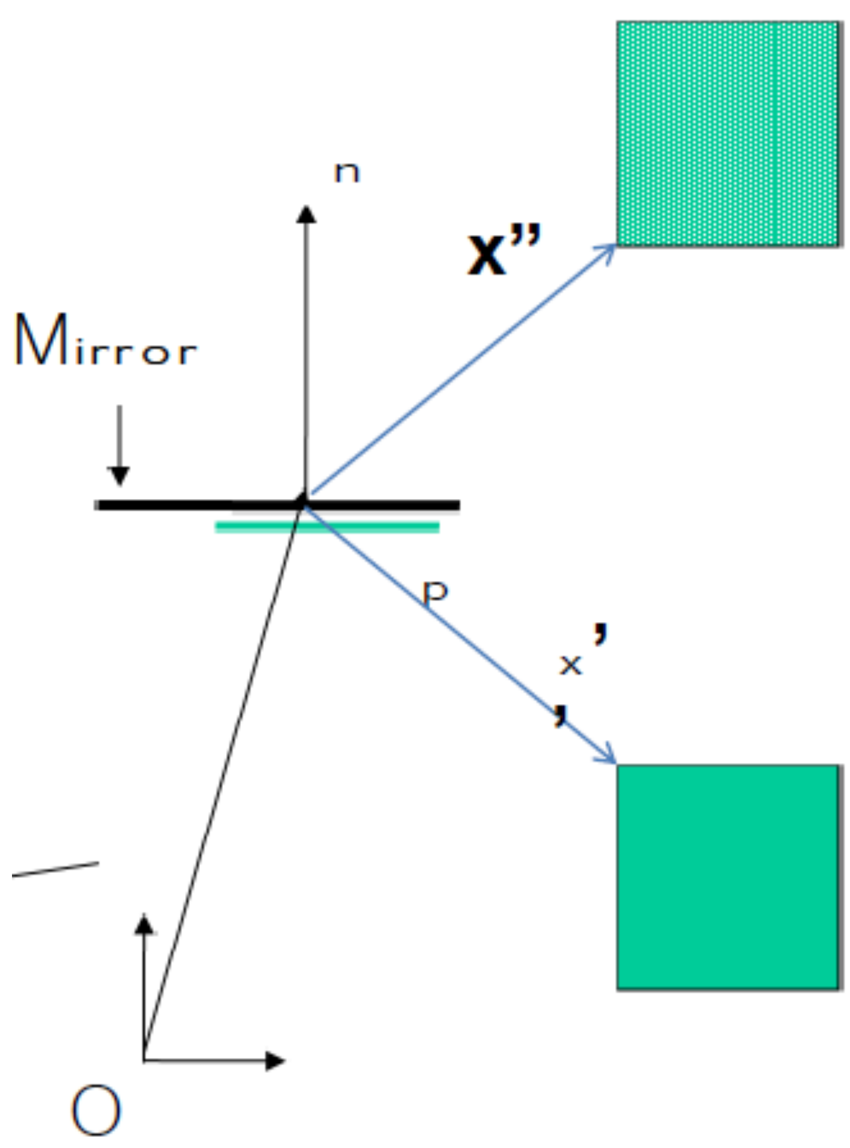


- To know the positions of objects with respect to the mirror coordinate

- We multiply by a transformation matrix from the world to the mirror coordinates

$$x' = R(n)^{-1}T(-p)\,x$$

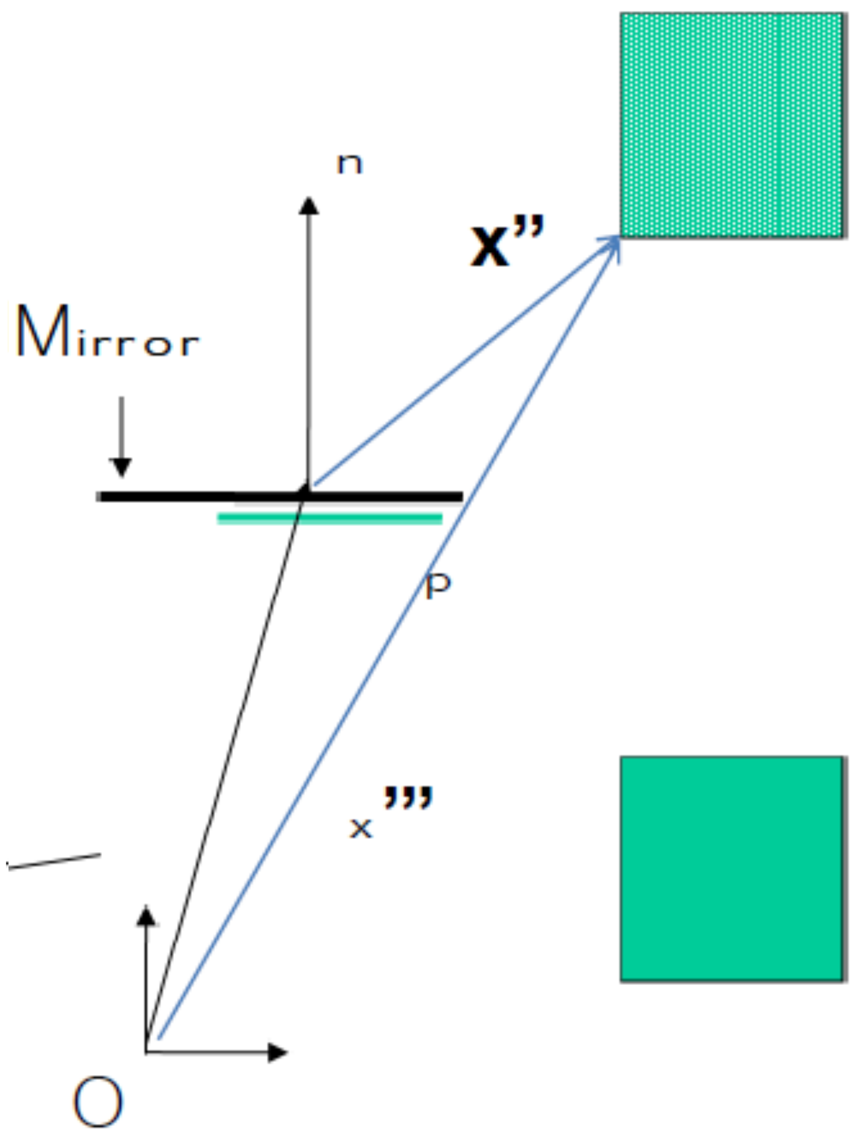
Reflecting objects



- For finding out the flipped location in the mirror coordinate, we multiply by the mirroring matrix

$$x'' = S(1,1,-1)\, x'$$

Reflecting objects

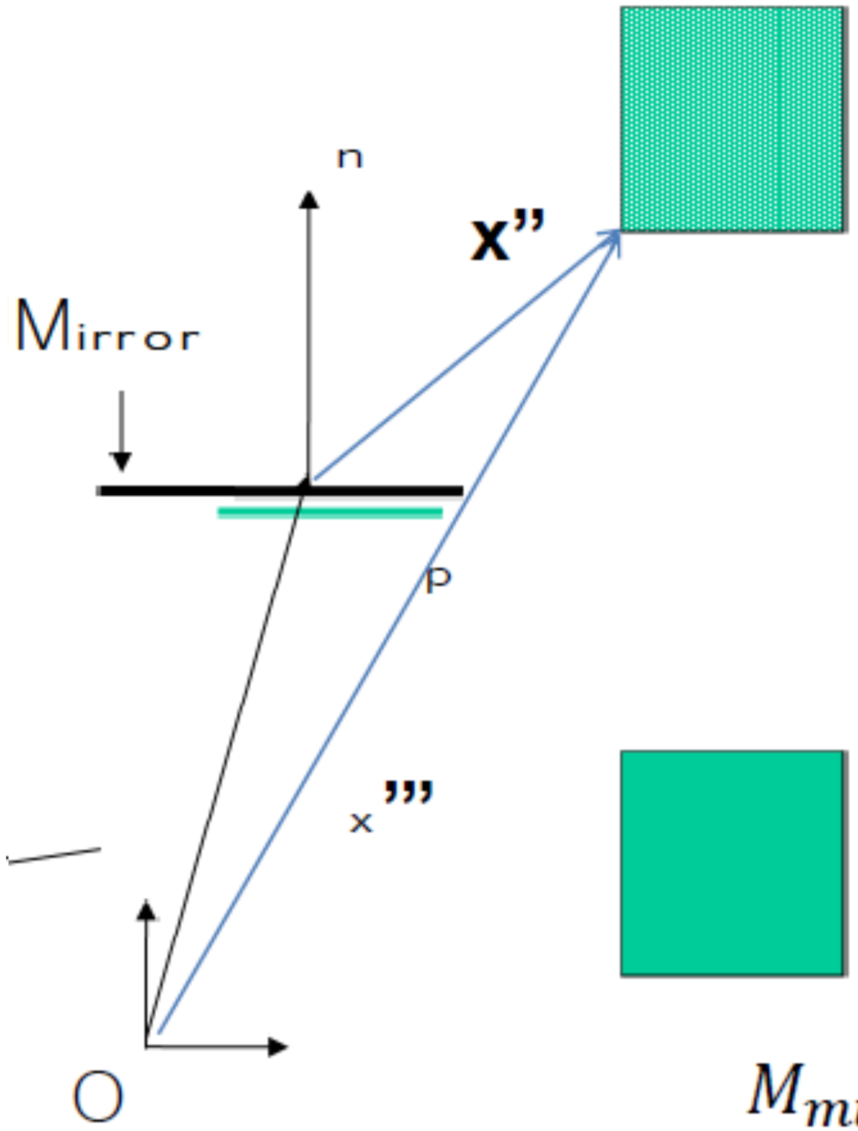


- Now we want to know where the flipped points are with respect to the world origin
- We can multiply x'' by the transformation matrix to move from the origin to the mirror to know where it is with respect to O

$$x''' = T(p)R(n)\, x''$$

Reflecting objects

- Combined:



$$x' = R(n)^{-1}T(-p)\,x$$

$$x'' = S(1,1,-1)\,x'$$

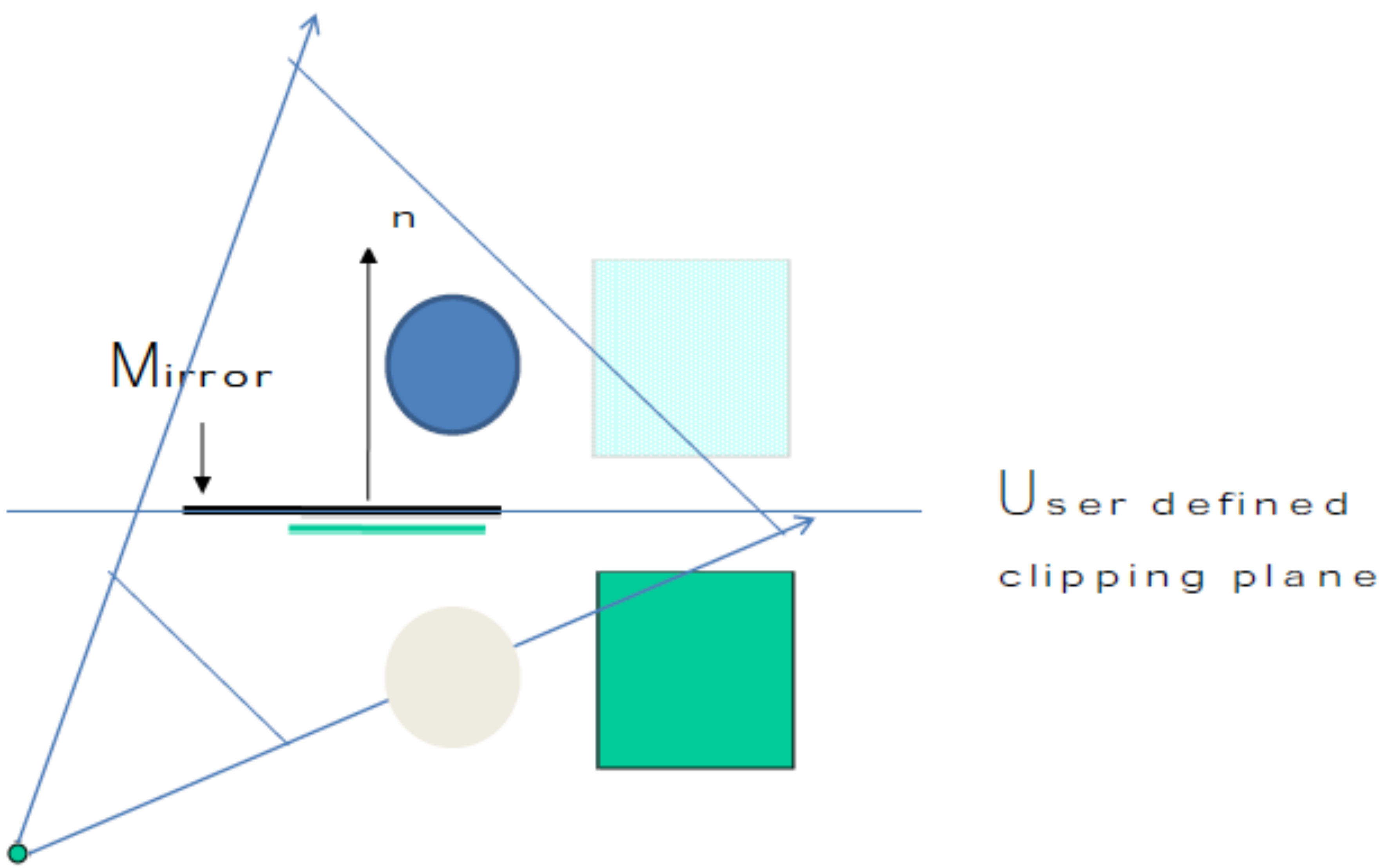
$$x''' = T(p)R(n)\,x''$$

$$x''' = T(p)R(n)S(1,1,-1)R(n)^{-1}T(-p)\,x$$

$$M_{mirrror} = T(p)R(n)S(1,1,-1)R(n)^{-1}T(-p)$$

Reflecting objects

- Need to avoid drawing objects behind the mirror in front of it
- Specify a clipping plane parallel to the mirror



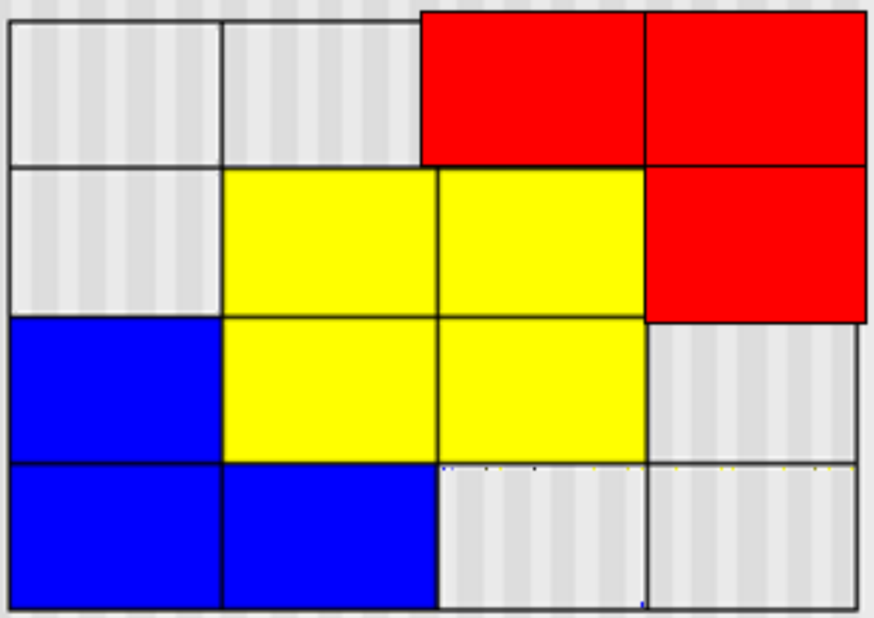
Drawing the mirrored world

- Draw the mirrored world first, then the real world
  - Only using the depth (Z) buffer
  - Does not work in some cases
- Draw the real-world first, and then the mirrored world
  - Requires using a stencil buffer

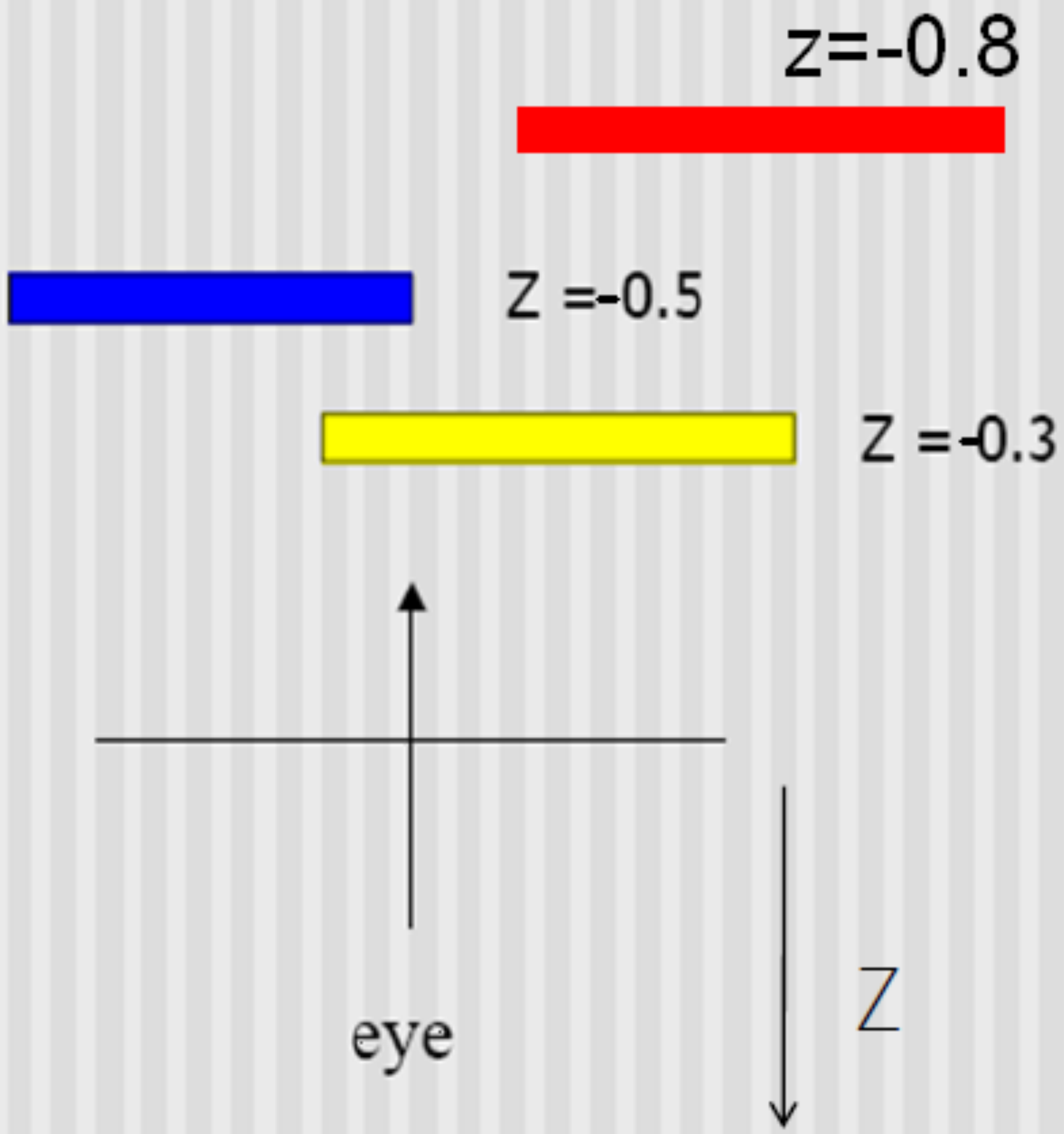
Z-buffer

- One method of hidden surface removal
- Basic Z-buffer idea: For every input polygon
  - For every pixel in the polygon interior, calculate its corresponding z value.
  - Compare the depth value with the closest value from a different polygon (largest z) so far
  - Paint the pixel (filling in the color buffer) with the color of the polygon if it is closer

Z buffer example



Correct Final image



Top View

Z buffer example

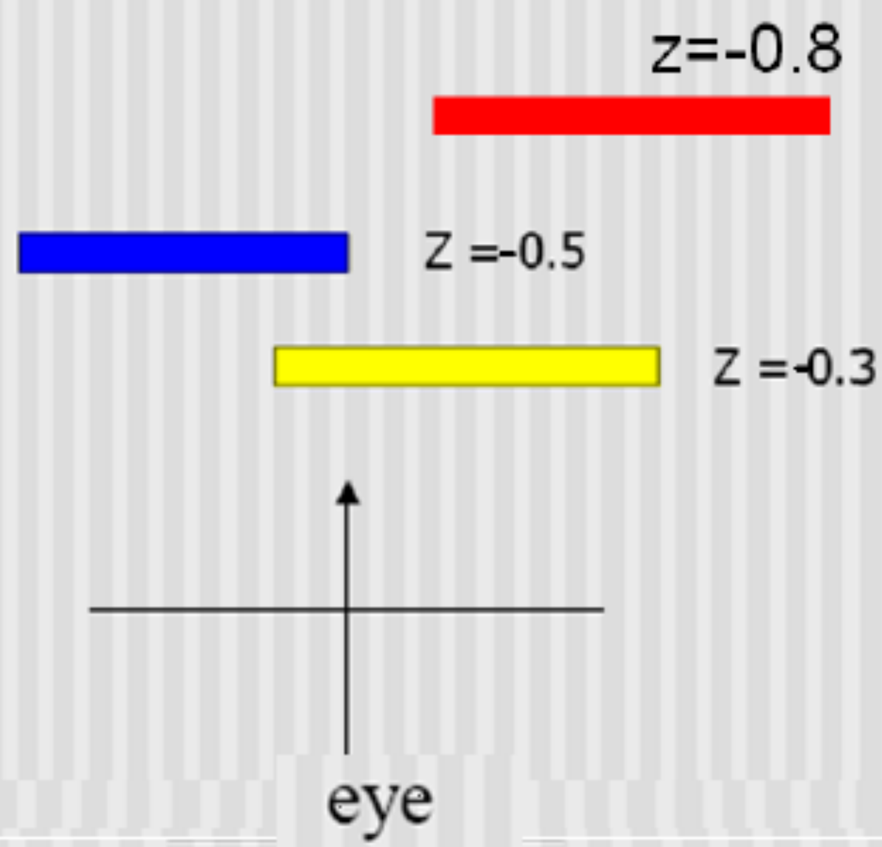
Step 1: Initialize the depth buffer

−1.0	−1.0	−1.0	−1.0
−1.0	−1.0	−1.0	−1.0
−1.0	−1.0	−1.0	−1.0
−1.0	−1.0	−1.0	−1.0

Z buffer example

Step 2: Draw the blue polygon (assuming the program draws blue polyon first – the order does not affect the final result any way).

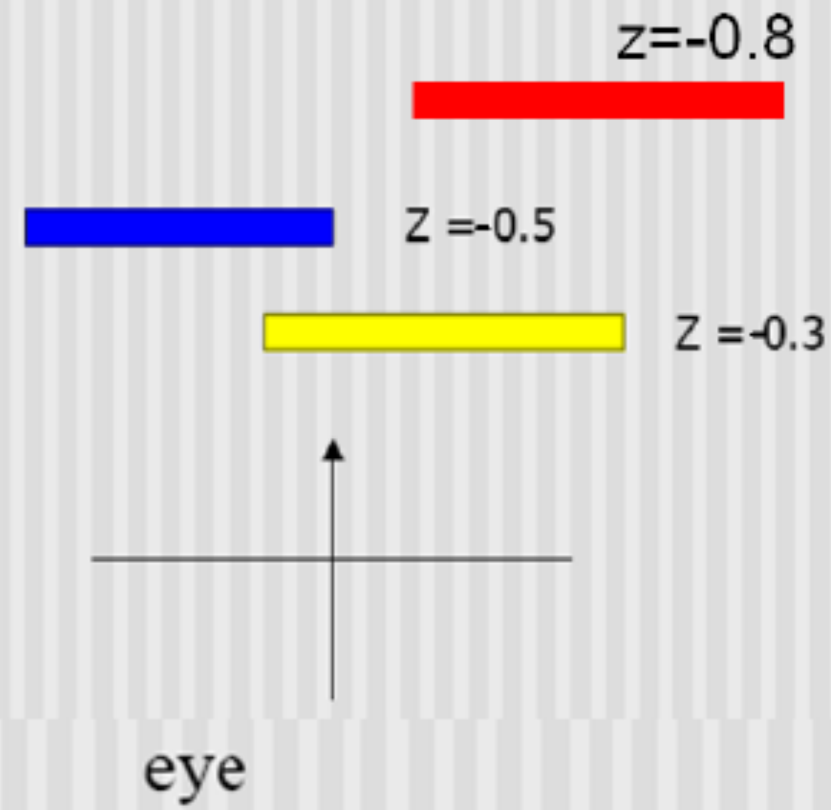
-1.0	-1.0	-1.0	-1.0
-1.0	-1.0	-1.0	-1.0
-0.5	-0.5	-1.0	-1.0
-0.5	-0.5	-1.0	-1.0



Z buffer example

Step 3: Draw the yellow polygon

-1.0	-1.0	-1.0	-1.0
-1.0	-0.3	-0.3	-1.0
-0.5	-0.3	-0.3	-1.0
-0.5	-0.5	-1.0	-1.0

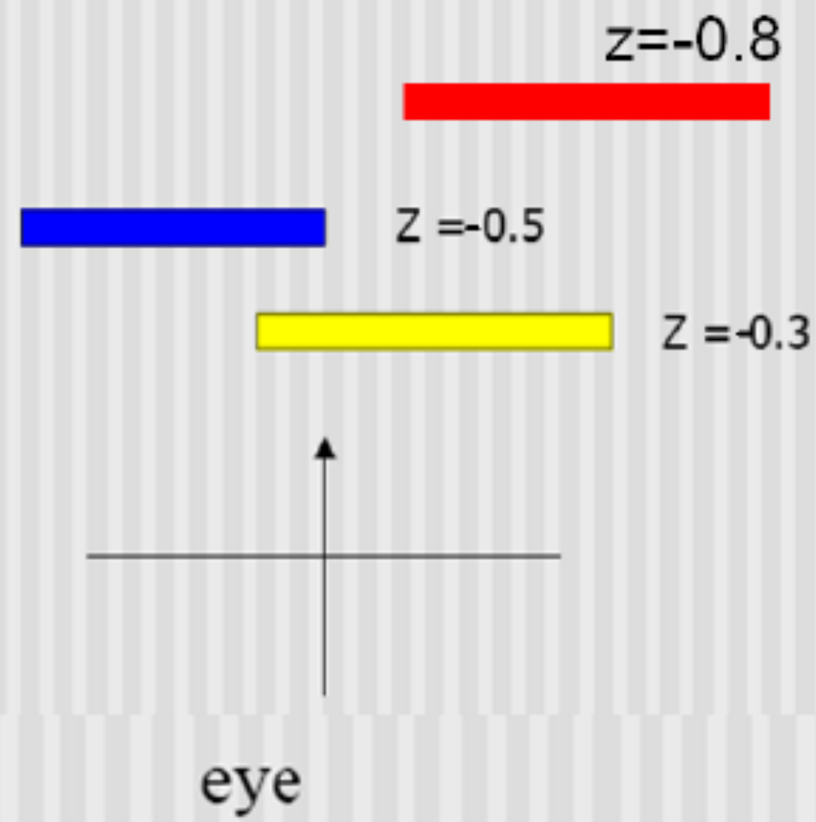


If the depth value is larger than that in the z-buffer, the pixel is coloured and value in the z-buffer is updated

Z buffer example

Step 4: Draw the red polygon

-1.0	-1.0	-0.8	-0.8
-1.0	-0.3	-0.3	-0.8
-0.5	-0.3	-0.3	-1.0
-0.5	-0.5	-1.0	-1.0



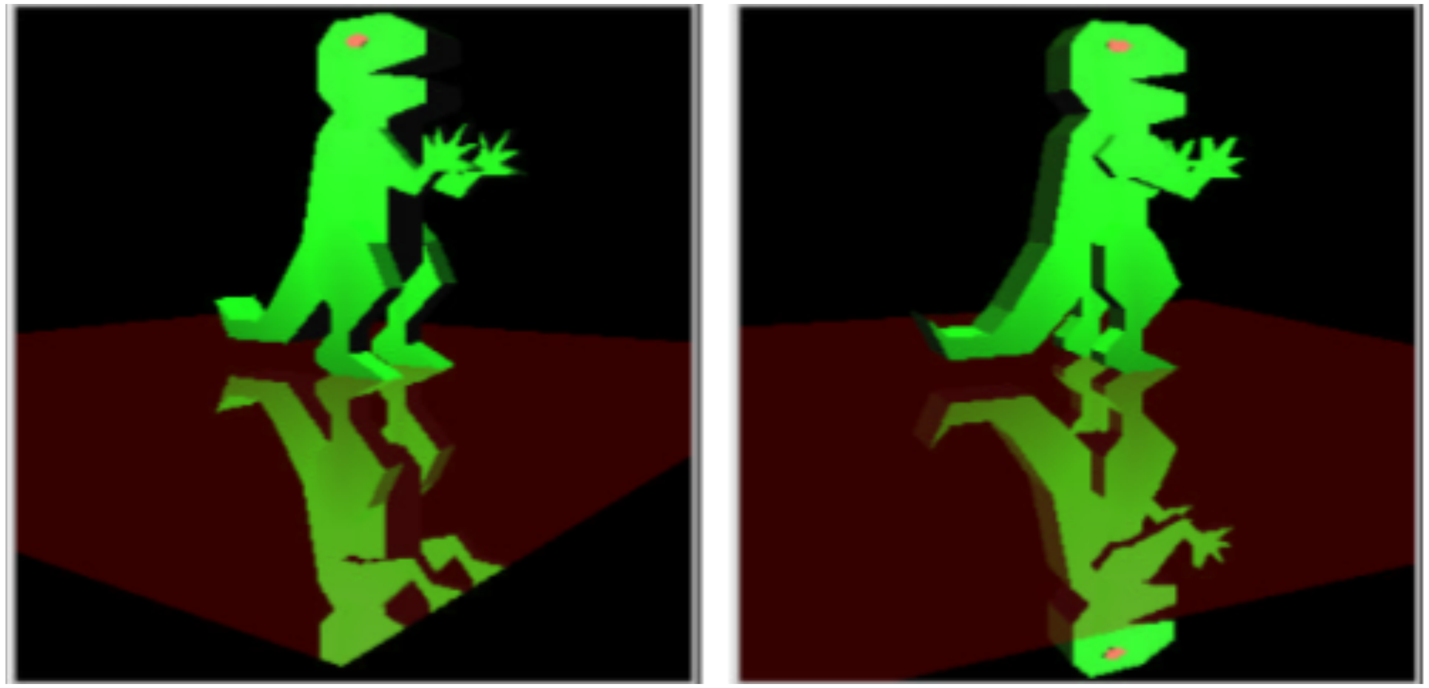
If the depth value is larger than that in the z-buffer, the pixel is coloured and value in the z-buffer is updated

Rendering Reflected Scene First

- First pass: Render the reflected scene without mirror, depth test on
- Second pass:
  - Disable the colour buffer, and render the mirror polygon (setting the Z-buffer values but not drawing pixel colours over reflected scene)
  - Now the Z buffer of the mirror region is set to the mirror's surface
- Third Pass:
  - Enable the colour buffer again
  - Render the original scene, without the mirror
  - Depth buffer stops us from writing over things in mirror

Rendering the reflected scene first

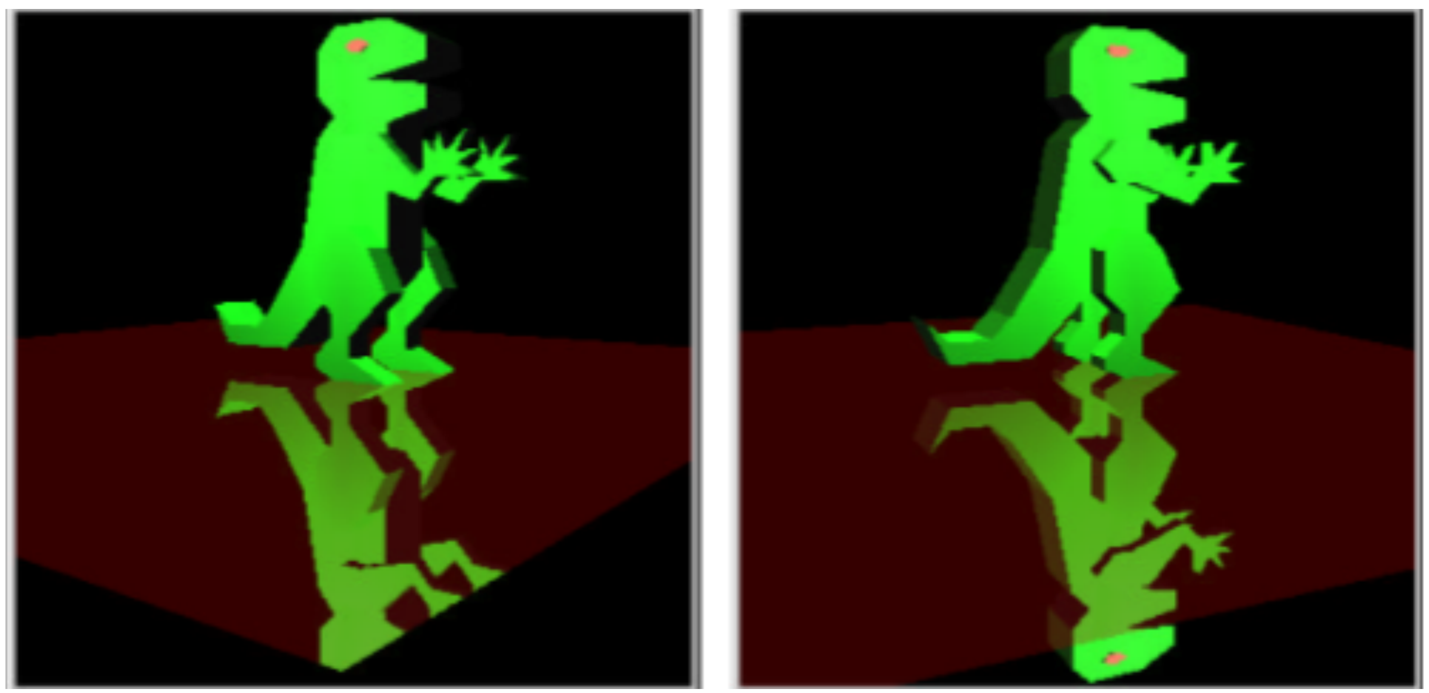
- The reflected area outside the mirror region is overwritten by the objects in the front



- Can't draw multiple mirrors or reflections of mirrors in mirrors (recursive reflections)

Using a stencil buffer

- The stencil buffer can help to prevent drawing outside of the mirror region



Using a stencil buffer

- The stencil buffer acts like a paint stencil - it lets some fragments through but not others

- It stores multi-bit values
- You specify two things:
  - The test that controls which fragments get through
  - The operations to perform on the buffer when the test passes or fails

Example

Procedure

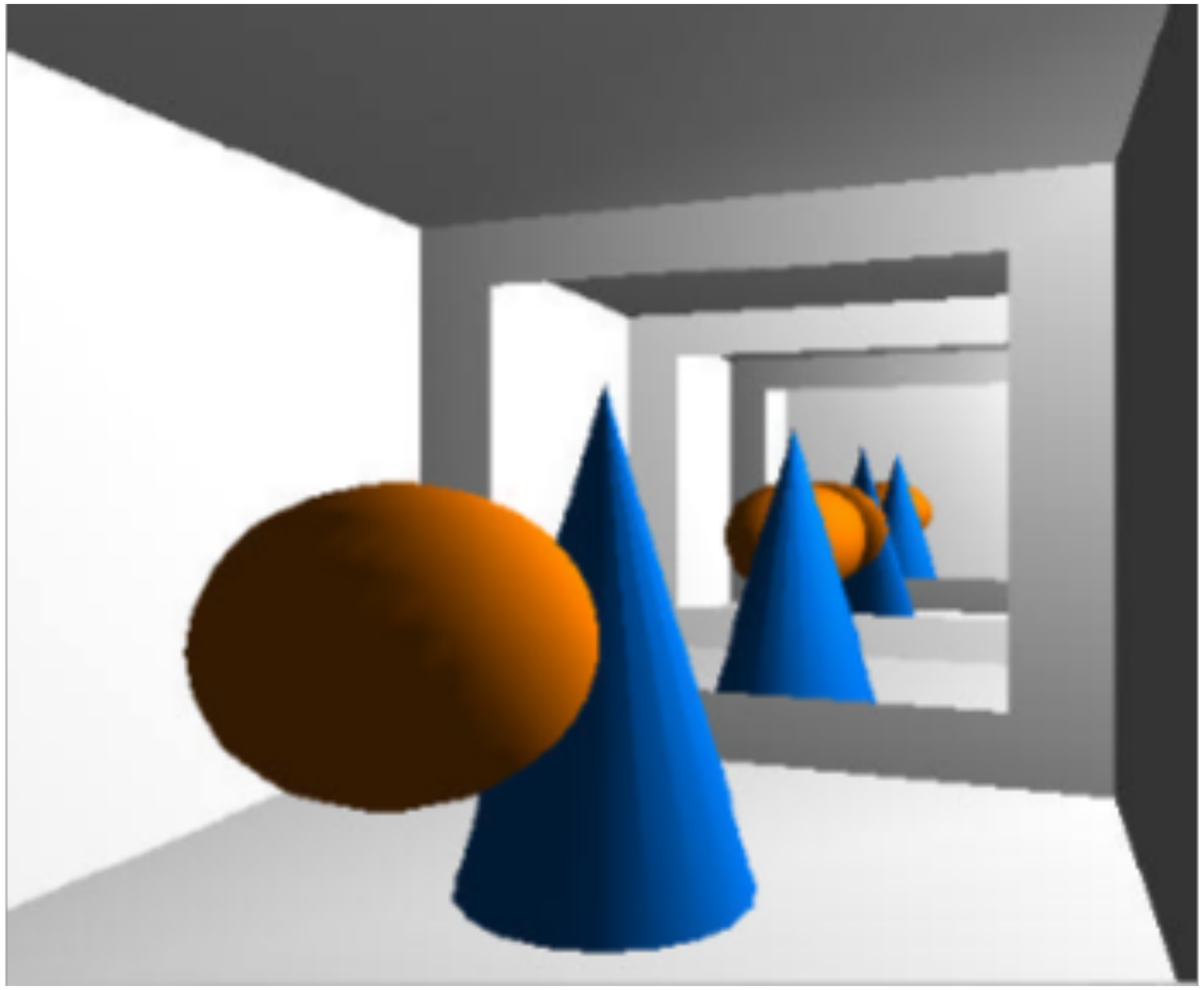
- First pass:
  - Render the scene without the mirror
- For each mirror:
  - Second pass:
    - Clear the stencil, disable the write to the colour buffer, render the mirror, setting the stencil to 1 if the depth test passes
  - Third pass:
    - Clear the depth buffer with the stencil active, passing things inside the mirror only
    - Reflect the world and draw using the stencil test. Only things seen in the mirror will be drawn
    - Combine it with the scene made during the first pass

Stencil buffer after the second pass

Render the mirrored scene into the stencil

Multiple mirrors

- Can manage multiple mirrors
- Render normal view, then do other passes for each mirror
- A recursive formulation exists for mirrors that see other mirrors
  - After rendering the reflected area inside the mirror surface, render the mirrors inside the mirror surface, and so on



References

- Akenine-Möller, Chapter 8.4 (Environment mapping)
- Akenine-Möller, Chapter 9.3.1 (Planar reflections)
- http://threejs.org/examples/#webgl_materials_cubemap
- http://www.pauldebevec.com/ReflectionMapping/