Computer Graphics 14 - Global illumination 1

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Slides courtesy of Taku Komura
www.inf.ed.ac.uk/teaching/courses/cg

Overview

- **Global illumination and light transport**
- **Monte-Carlo integration**
- **Monte-Carlo Ray Tracing**
  - -**Path Tracing**
  - -**Bidirectional Path Tracing**
- Photon Mapping

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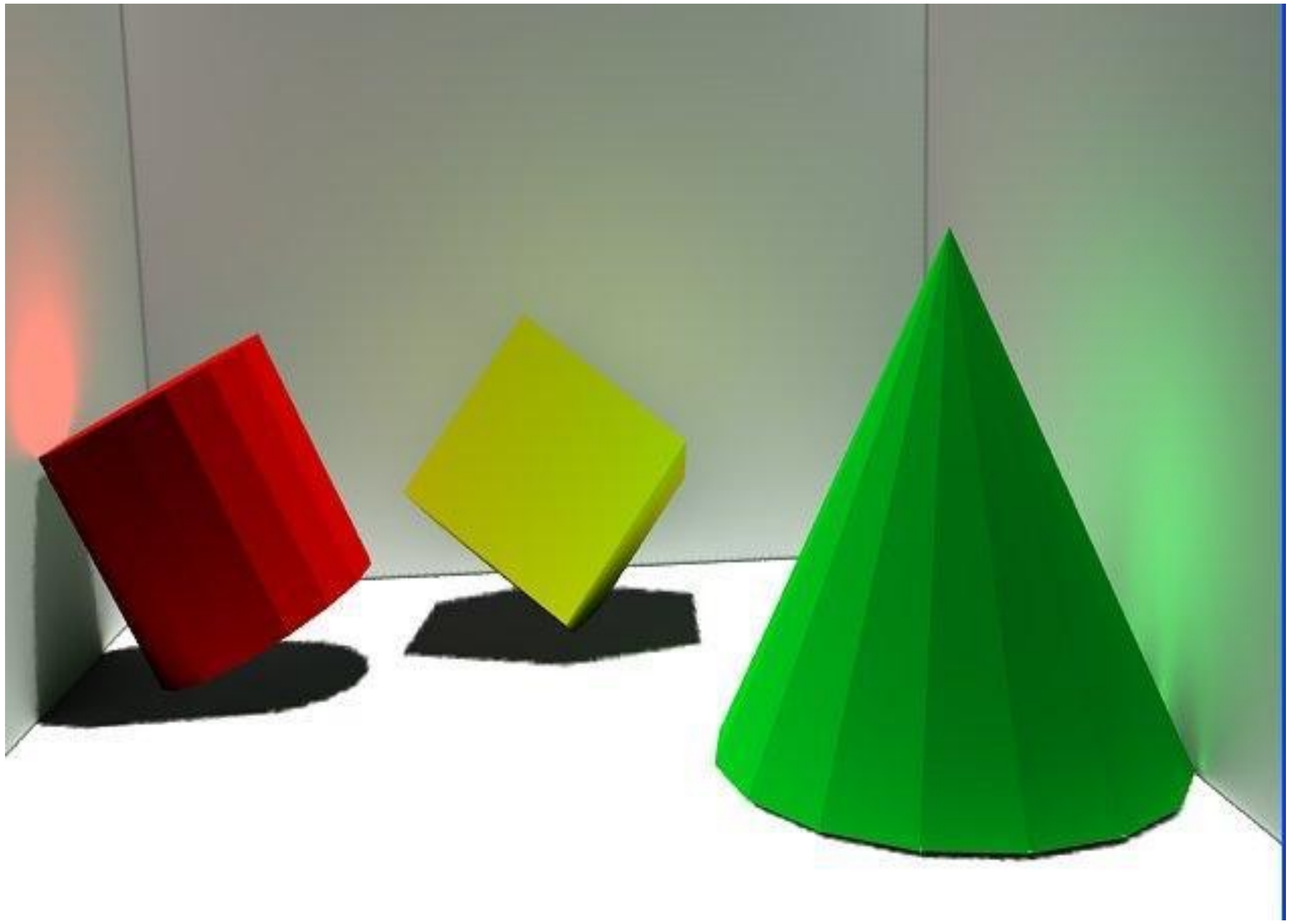
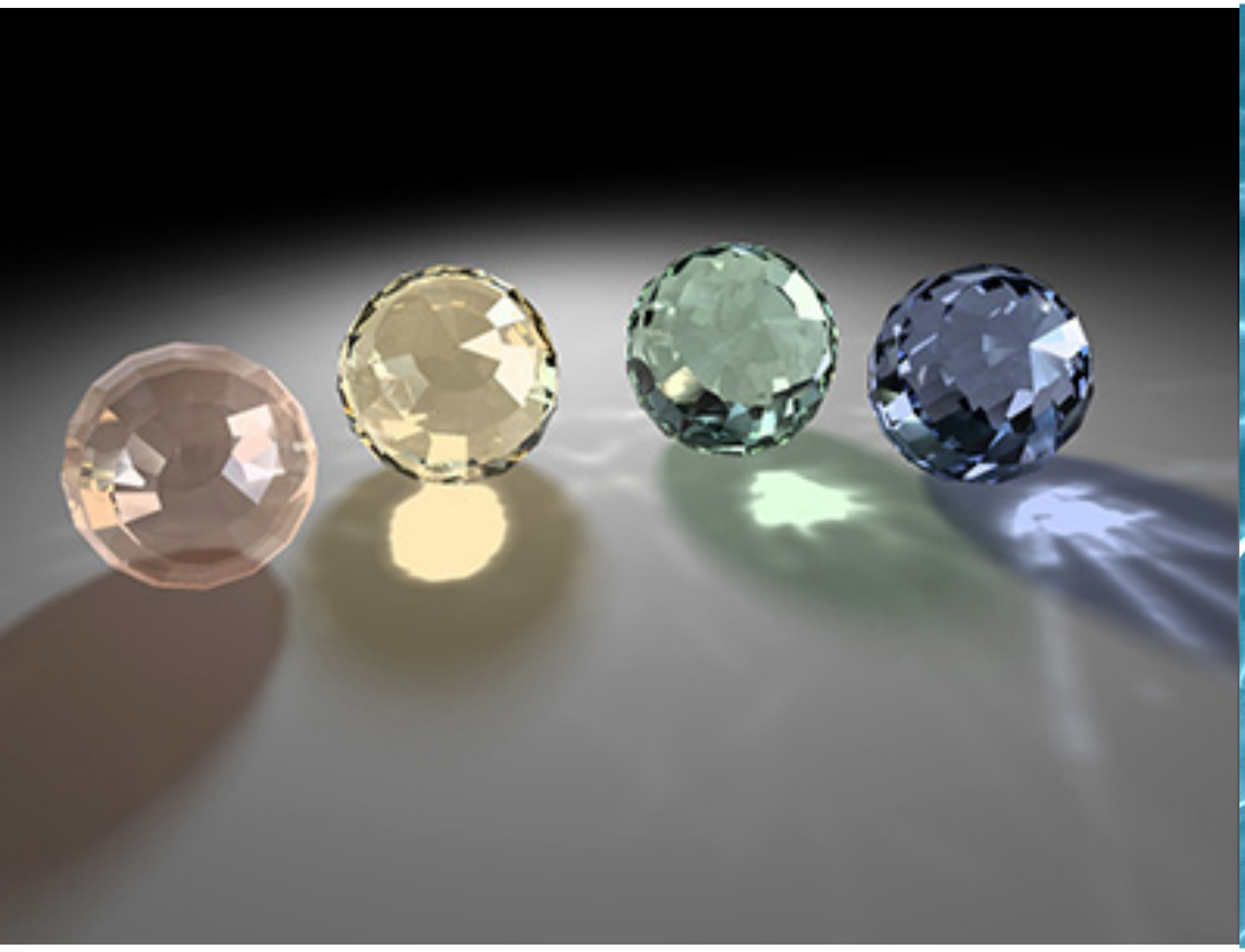
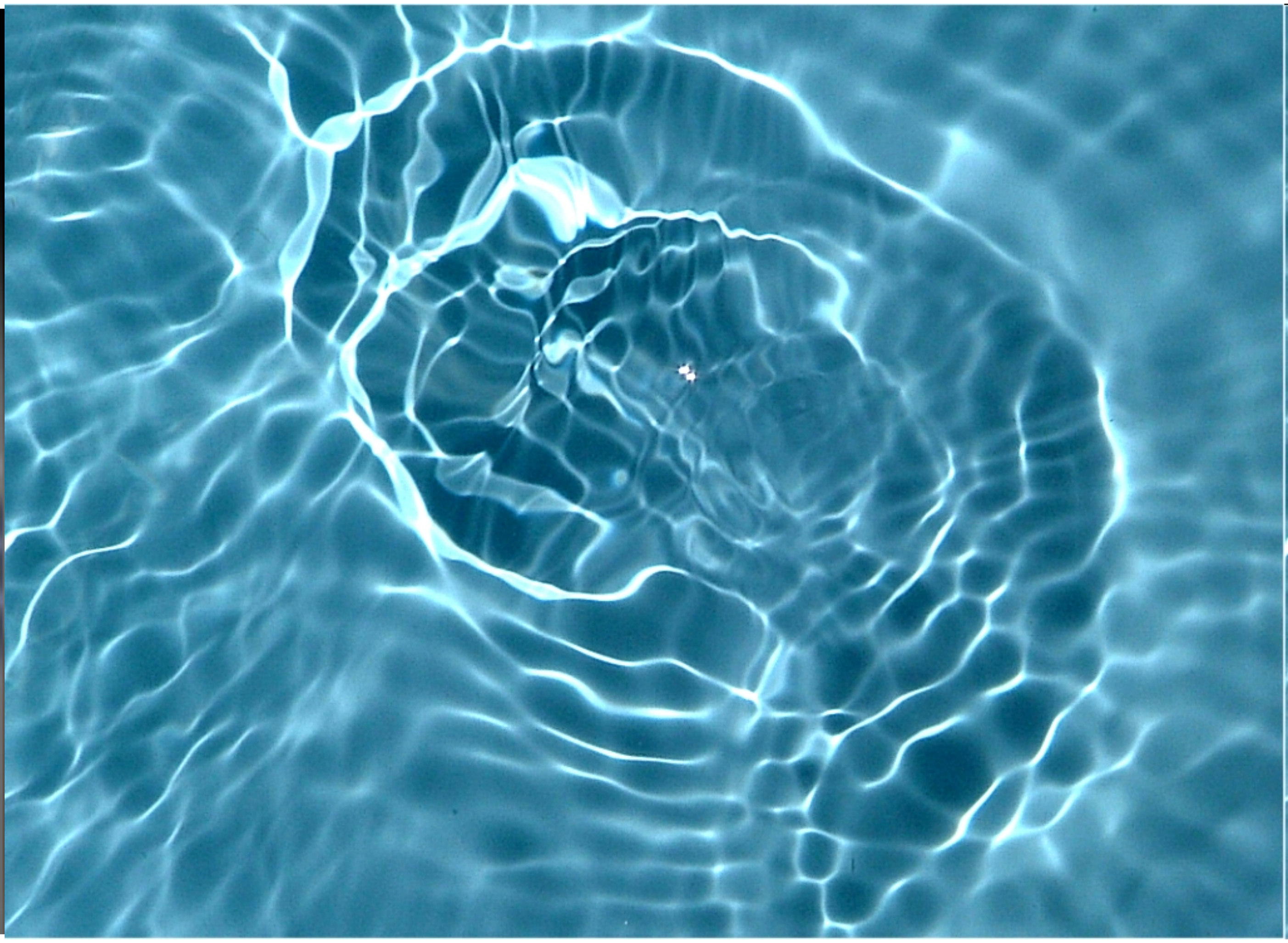
Colour bleeding



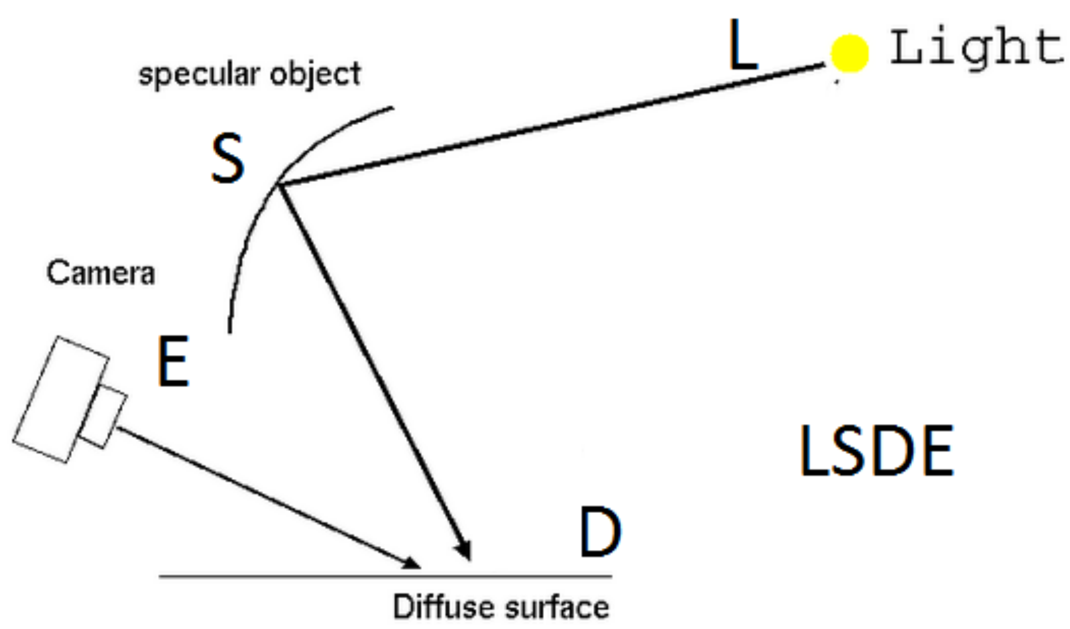
Caustics





Light transport notations

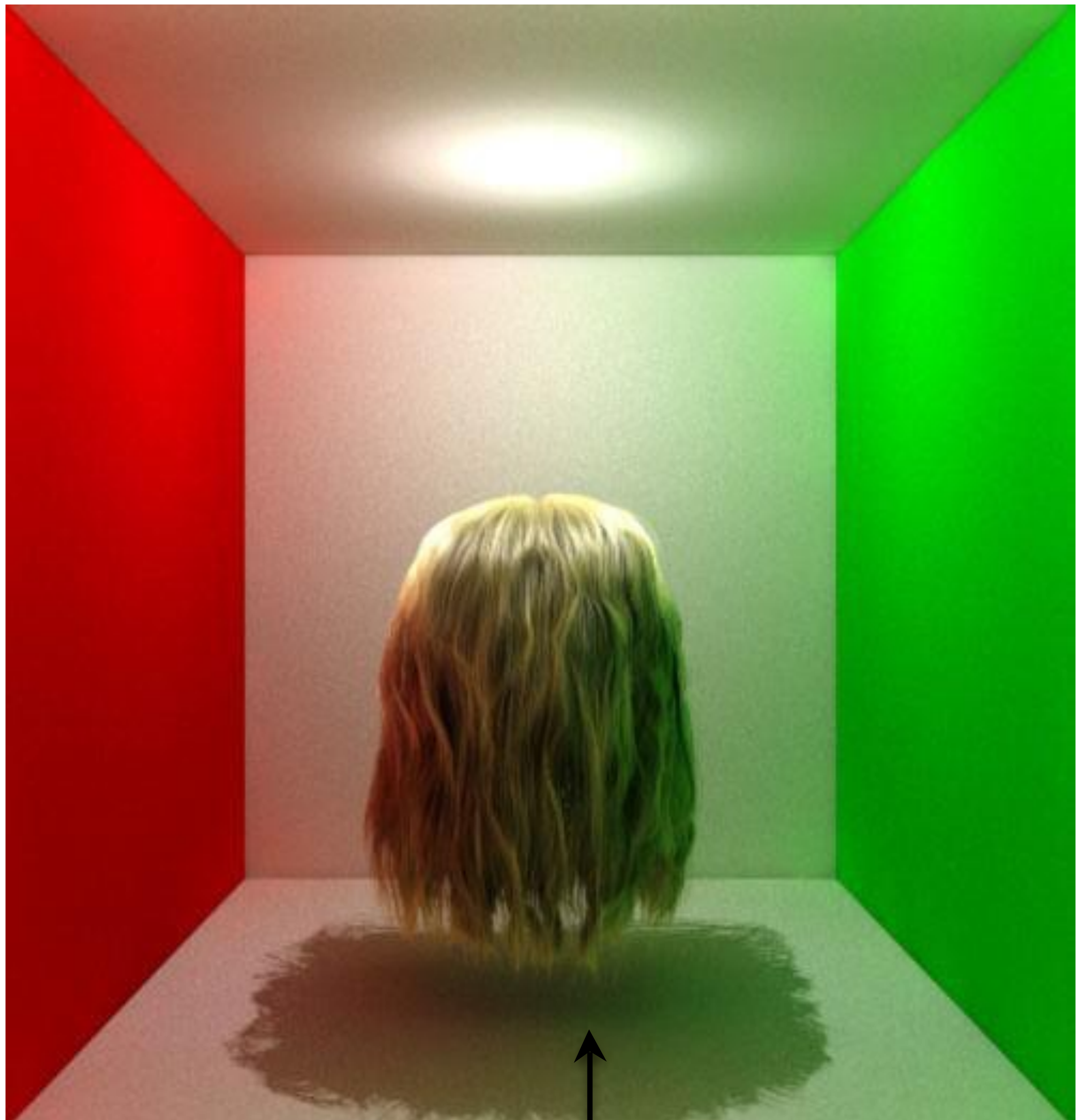
- It is useful to be able to describe the path that light takes through a scene:
  - L light source
  - E the eye
  - S specular reflection or refraction
  - D diffuse reflection



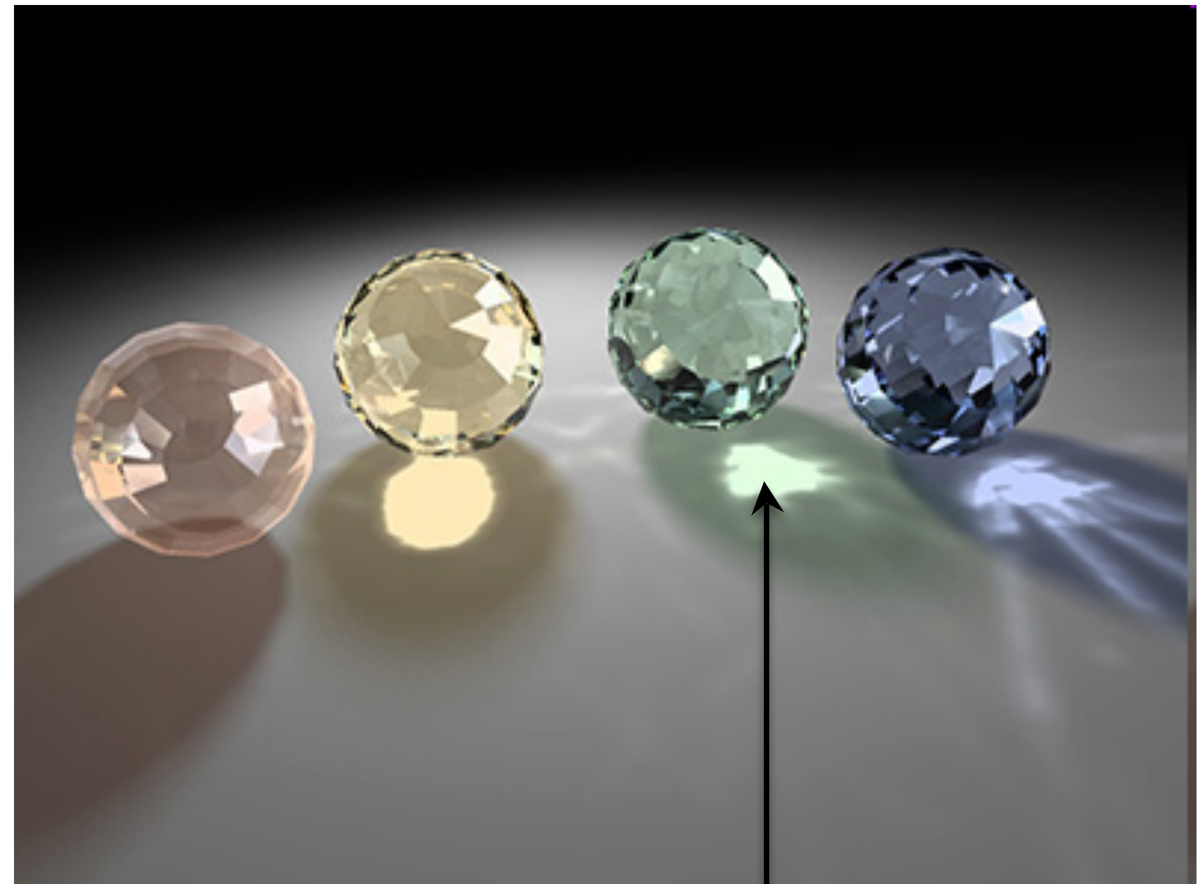
Light transport notations

- Regular expressions
  - (k)+ : one or more of event k
  - (k)* : zero or more of event k
  - (k)? : zero or one of event k
  - (k|k') : event k or k'

Examples



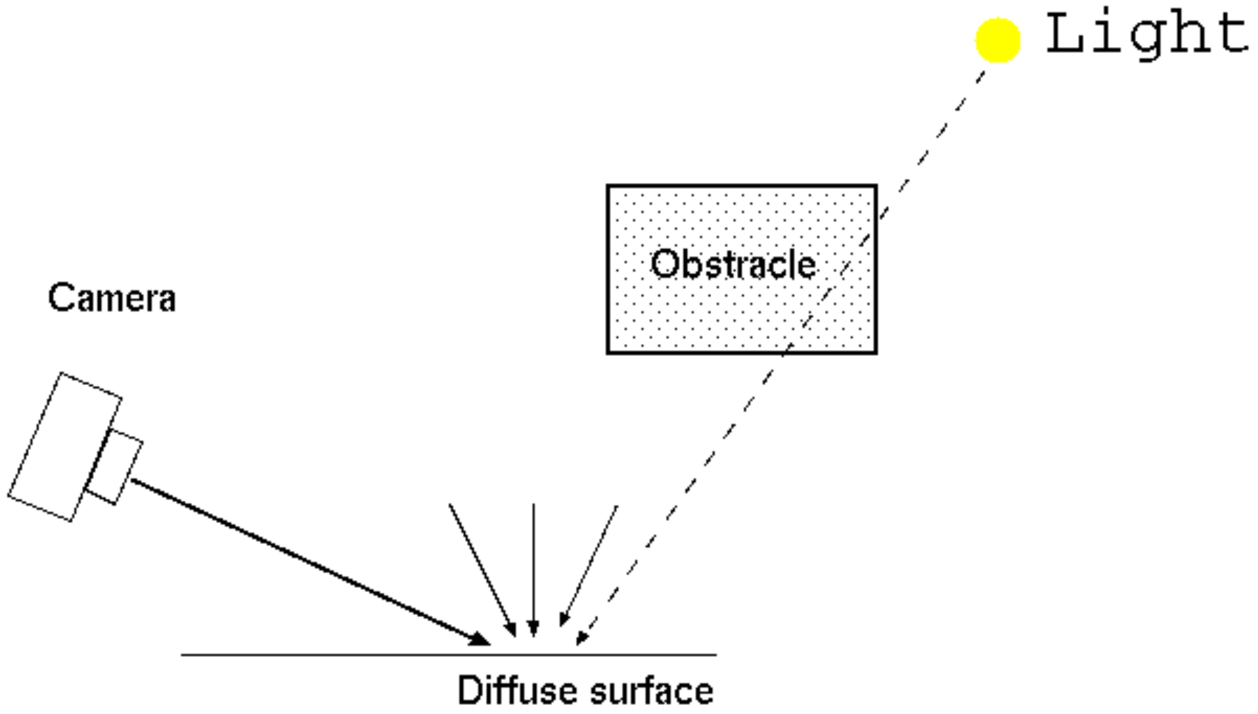
LDDE



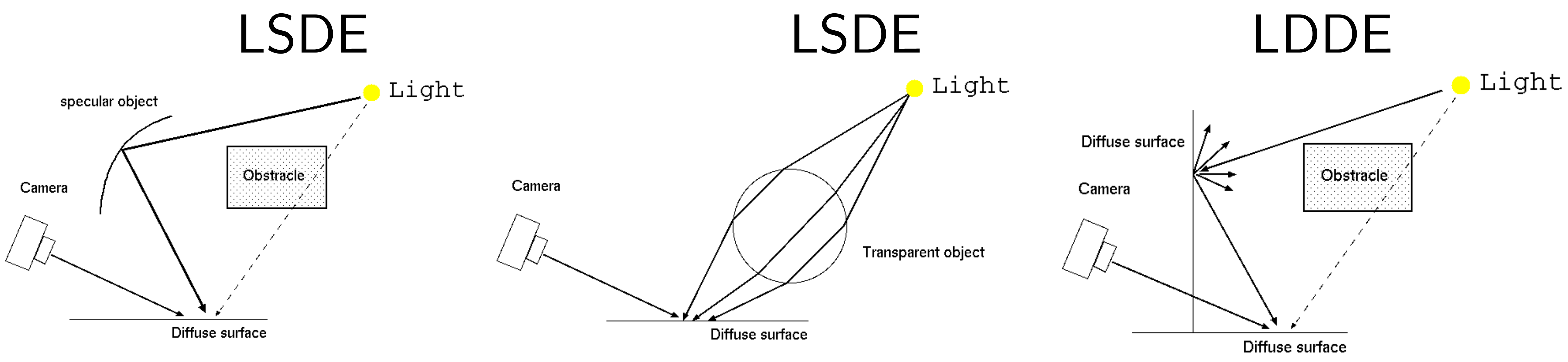
LSDE

Ray Tracing: review

- Shadow ray, reflection ray, etc.
- We calculate local illumination at diffuse surfaces using the direct lighting
- We do not know where the indirect (ambient) light illuminating diffuse surfaces comes from



Indirect lighting by ray-tracing

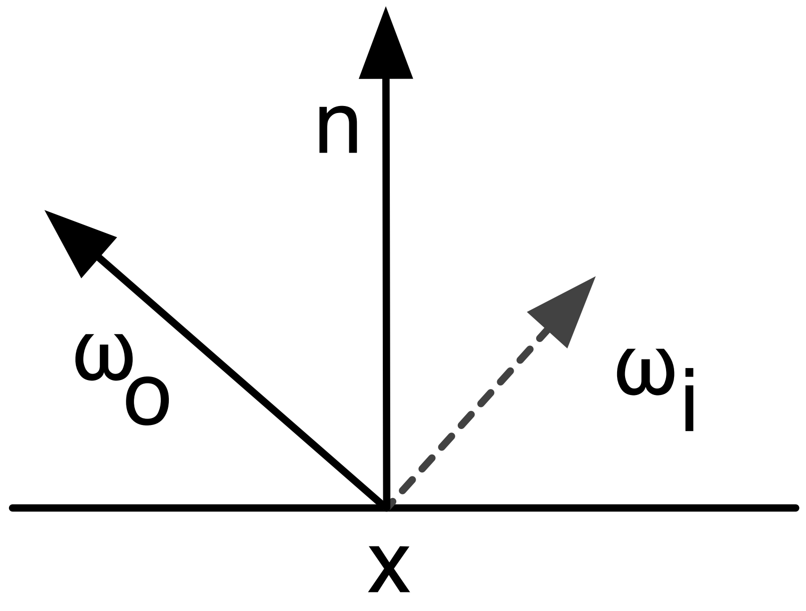


- Caustics and colour bleeding are produced by indirect light – how can we simulate such effects in the ray tracing framework?

Rendering equation

$$L_o(x, \omega_o) = L_e(x, \omega_o) + \int_\Omega L_i(x, \omega_i) f_r(x, \omega_i, \omega_o)(\omega_i \cdot \boldsymbol{n}_x) d\omega_i$$

- $L_o(x, \omega_o)$ outgoing radiance at point x in direction ω_o
- L_e emitted radiance
- $L_i(x, \omega_i)$ incoming radiance from direction ω_i
- f_r is the Bidirectional Reflectance Distribution Function (BRDF) of the surface
- $\boldsymbol{n}_x$ is the surface normal at point x
- Ω is the hemisphere of incoming directions at point x



Monte Carlo integration

To estimate an integral for some function f,

$$I = \int_{\Omega} f(x)dx,$$

we can generate N uniform random samples within Ω, $\xi_1, \ldots, \xi_N$ and then approximate I as:

$$I \approx V \frac{1}{N} \sum_i f(\xi_i),$$

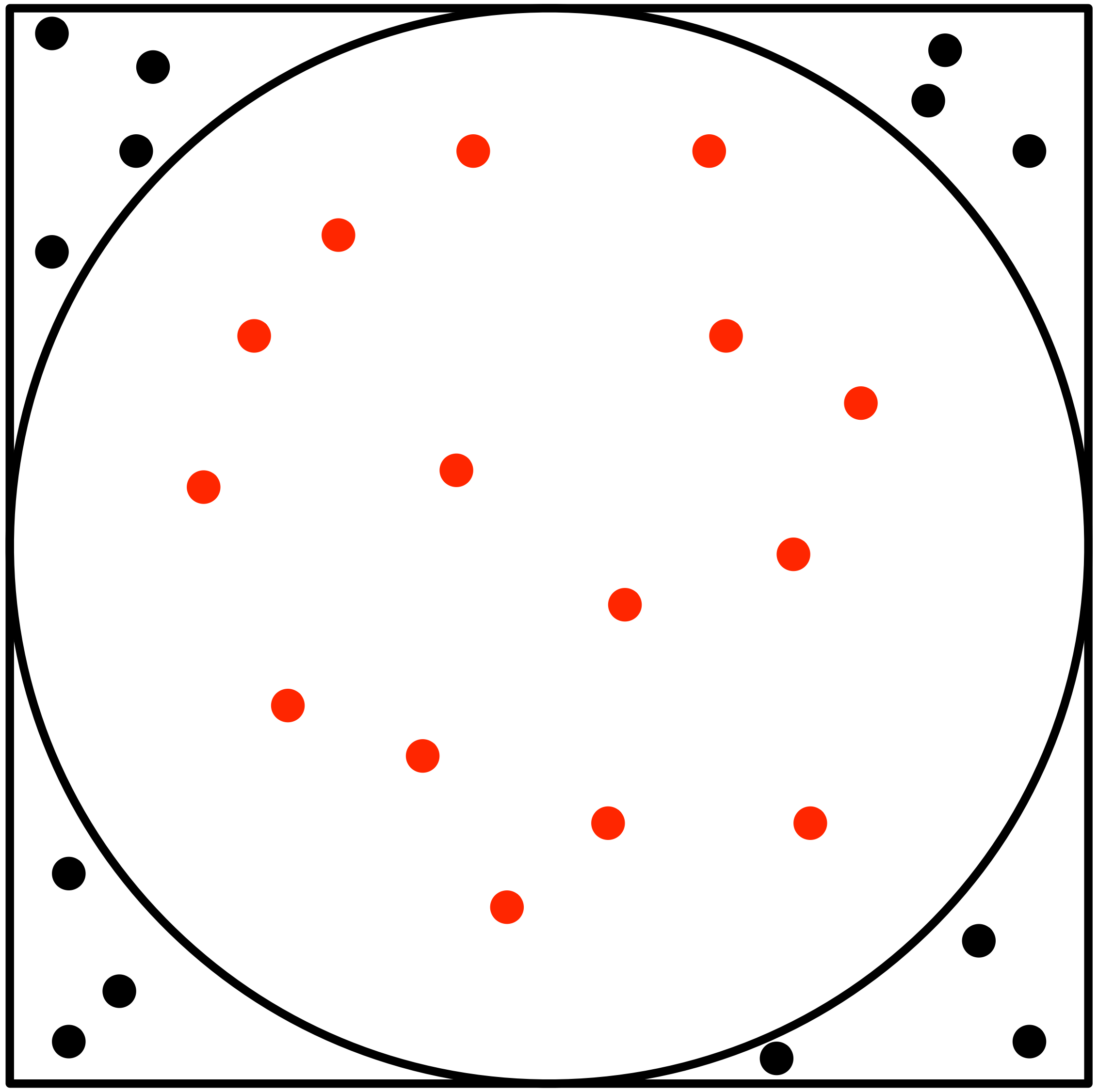
where $V = \int_{\Omega} dx$. Then

$$\lim_{N \to \infty} V \frac{1}{N} \sum_{i=1}^{N} f(\xi_i) = I.$$

Monte Carlo integration example

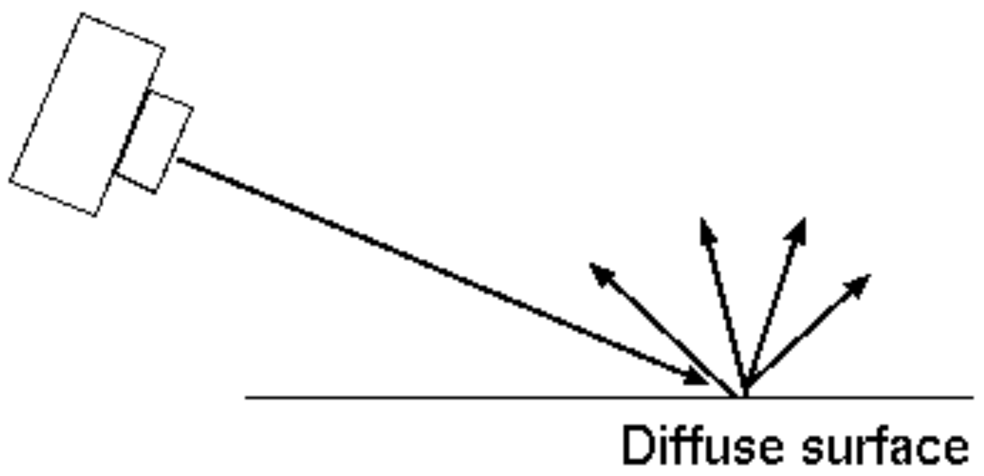
For the region
$D = (-1 \leq x \leq 1, -1 \leq y \leq 1)$,
we define the function $f(x, y)$:

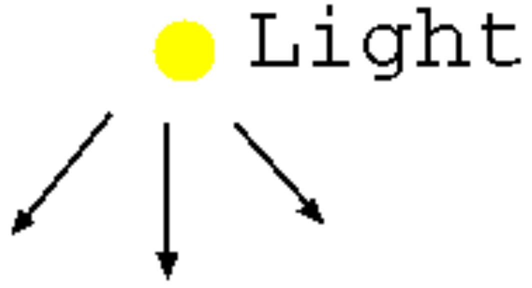
- $f(x, y) = \begin{cases} 1 & \text{if } x^2 + y^2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$
- $\int_D f(x, y) dxdy = \pi$
- $\pi \approx 4 \frac{1}{N} \sum_i f(x_i, y_i)$



Two ways to simulate indirect light

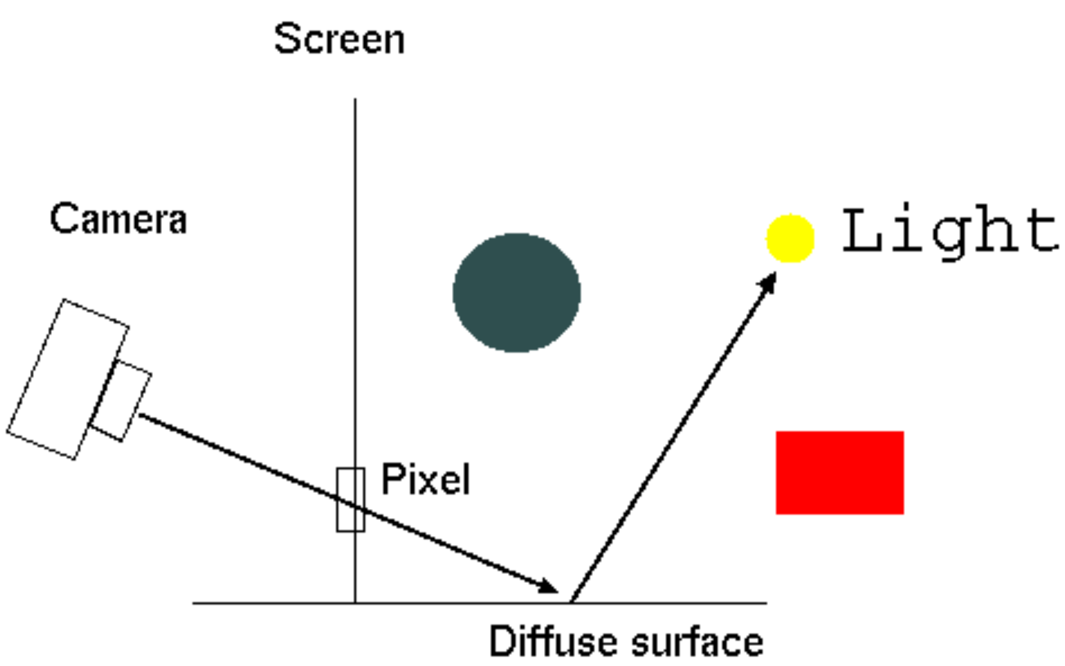
- Launch tracing rays in random directions at diffuse surfaces: Path tracing
- Shoot rays that represent the path of light from the light source: Bidirectional Path Tracing, Photon Mapping

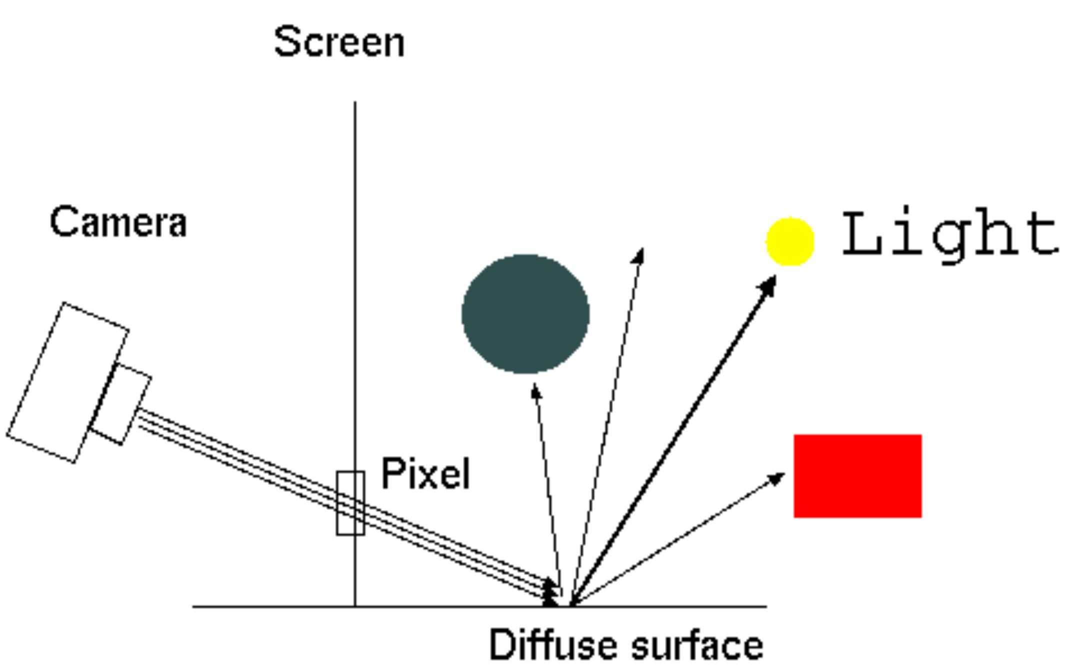




Path Tracing

- An enhancement of the ordinary ray-tracing scheme
- When hitting a diffuse surface, pick one direction at random, and find the colour of the incoming light
- Trace many paths per pixel (100-10000 per pixel)
- by Kajiya, SIGGRAPH 86





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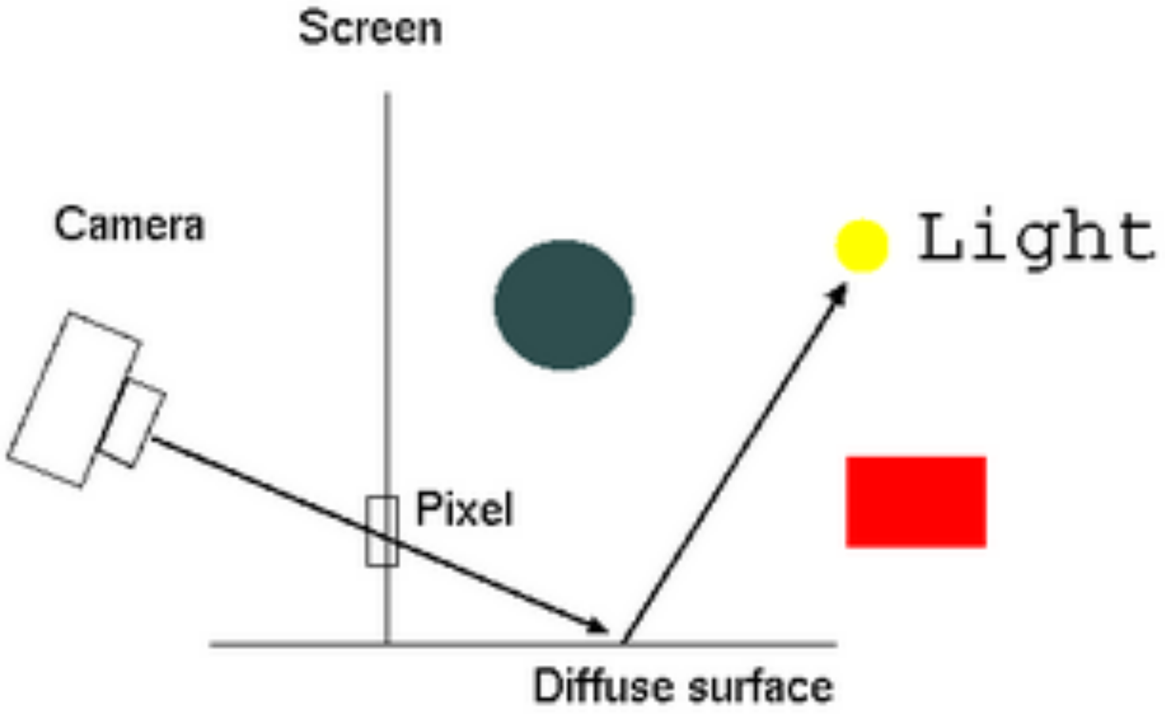
Original Ray Tracing Algorithm

- Trace (ray)
  - -Find the intersection of the ray and the scene
  - -Compute the shadow ray : Color=Color_ambient
  - -Do the local illumination : Color += Color_local (not shadowed)
  - -If specular compute the reflection vector R
    - Color += Trace(R)
  - -If refractive compute the refractive vector T
    - Color += Trace(T)



Path Tracing Algorithm

- Trace (ray)

  -Find the intersection of the ray and the scene

  -Compute the shadow ray : Color=Color_ambient

  -Do the local illumination : Color += Color_local (not shadowed)

  -If specular compute the reflection vector R

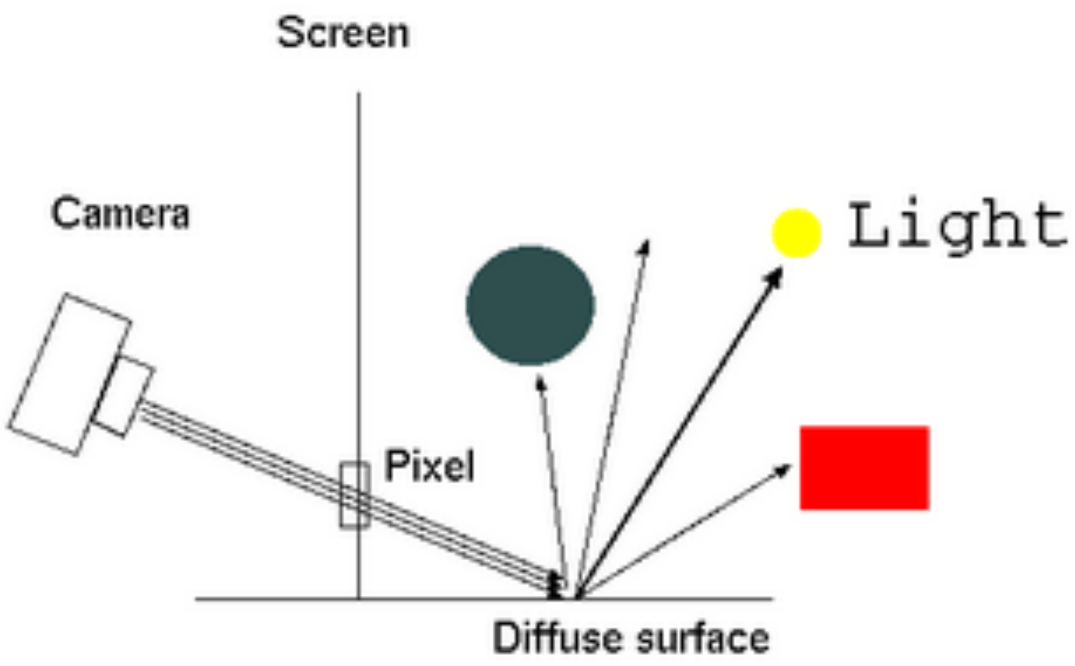
  - Color += Trace(R)

  -If refractive compute the refractive vector T

  - Color += Trace(T)

  -Else if diffuse compute a
  random vector R'

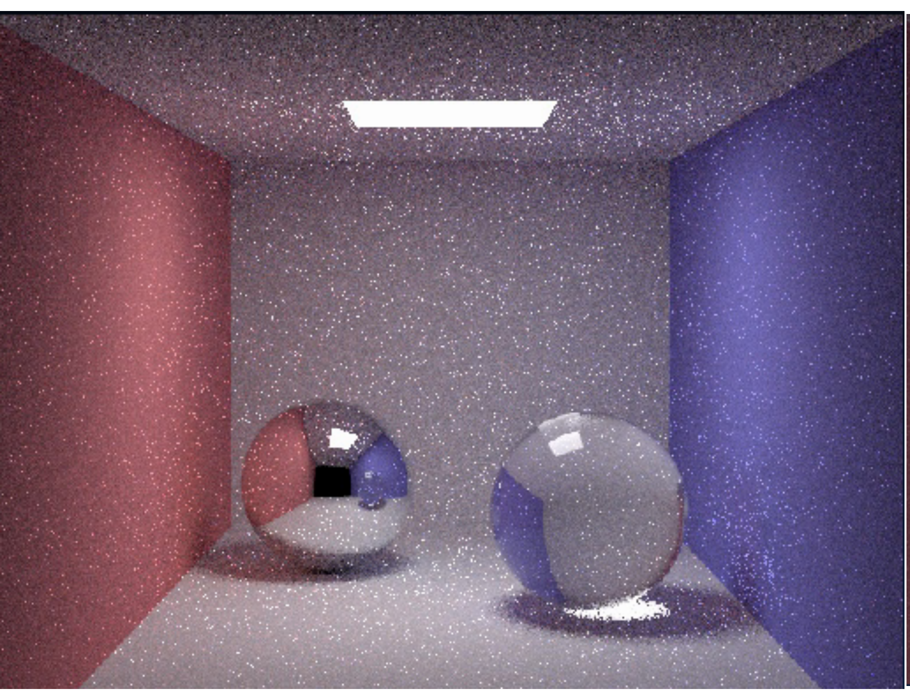
  - Color += Trace(R')

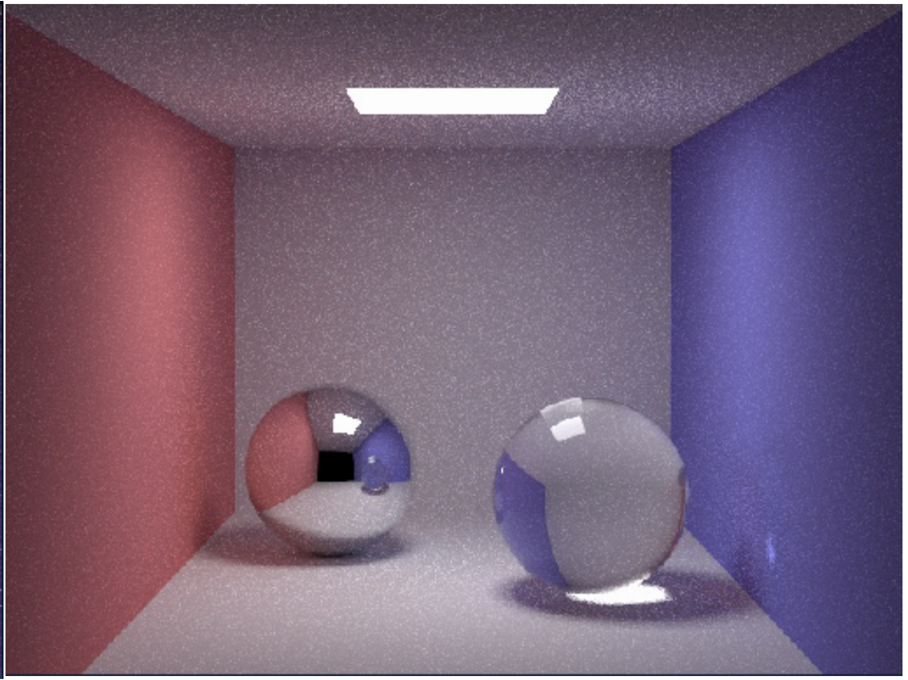


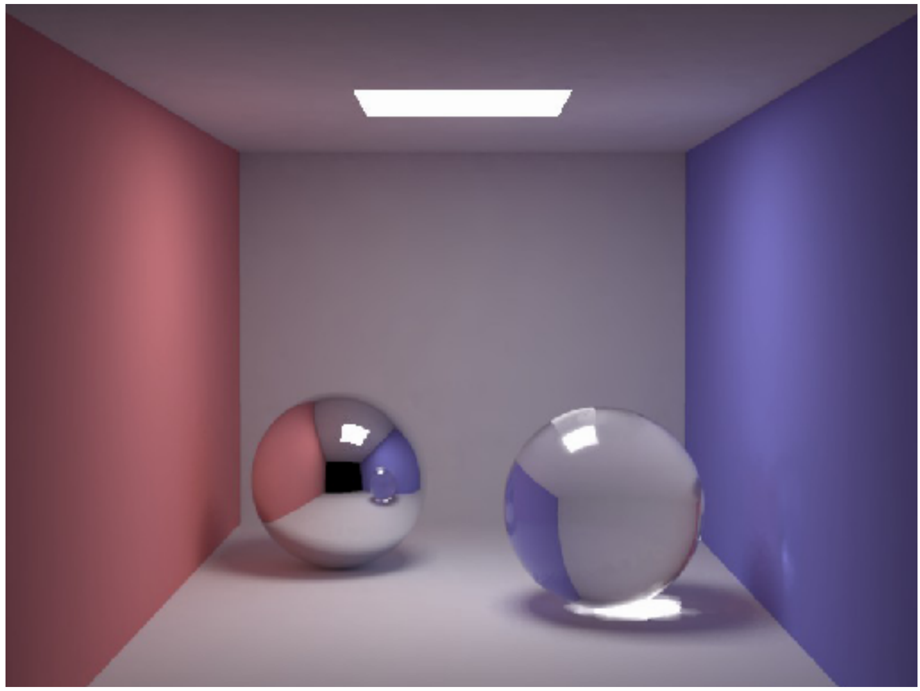
Path tracing: problems

- Variance in the pixel colours, appearing as noise
- Need many samples for precise results

-Requires 1000~10000 samples per pixel for good results







10 paths/pixel 100 paths/pixel 1000 paths/pixel

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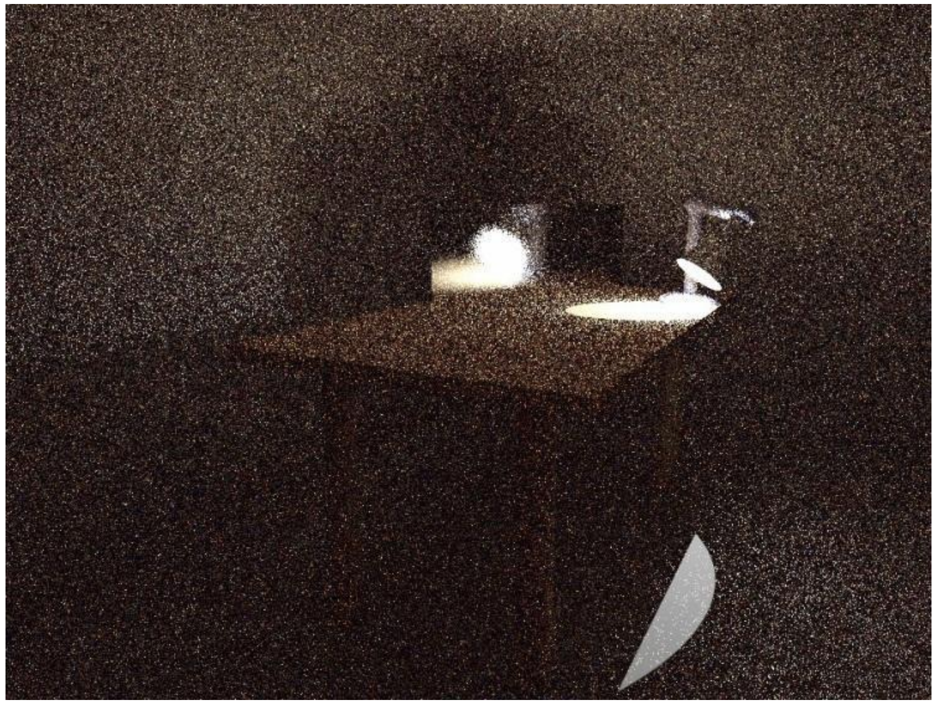
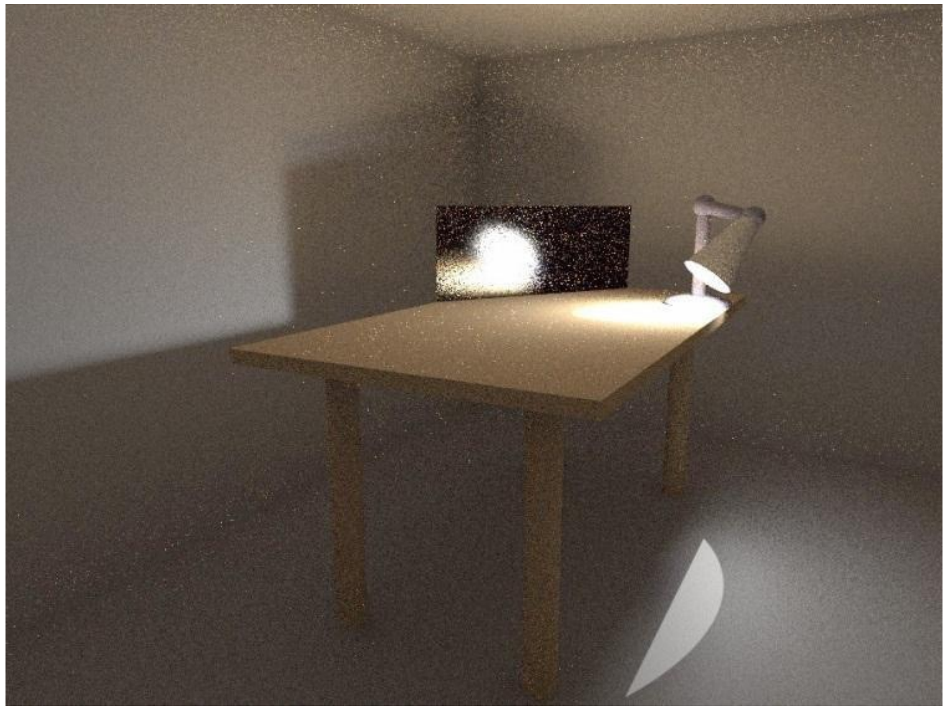
Path tracing: problems

- Some lights are difficult to reach from the camera - such as those produced by spot lights
- For such lights, we cannot simulate indirect light well
- Results in a very dim image with high variance





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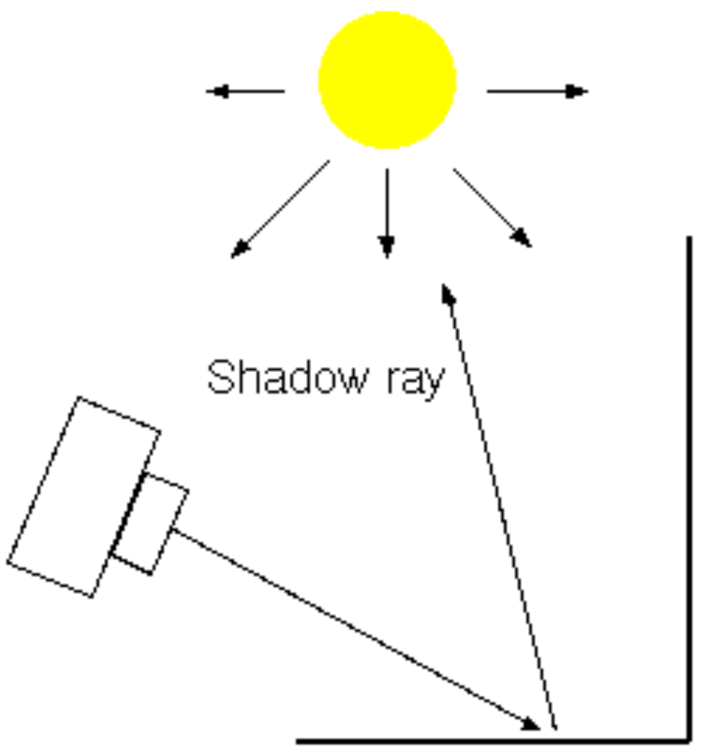
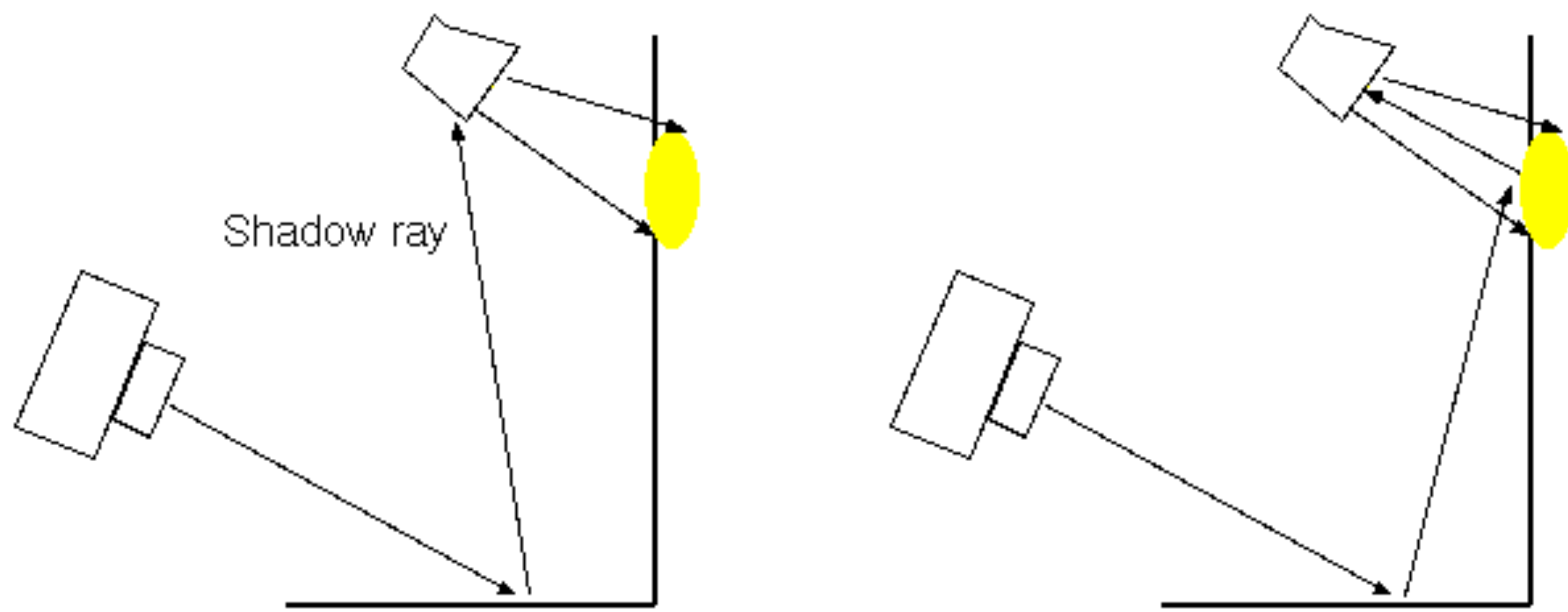
Why? Shadow rays are always occluded

- For the pixel to be lit, the path must be lucky enough to reach the light source



Point light



Spot light

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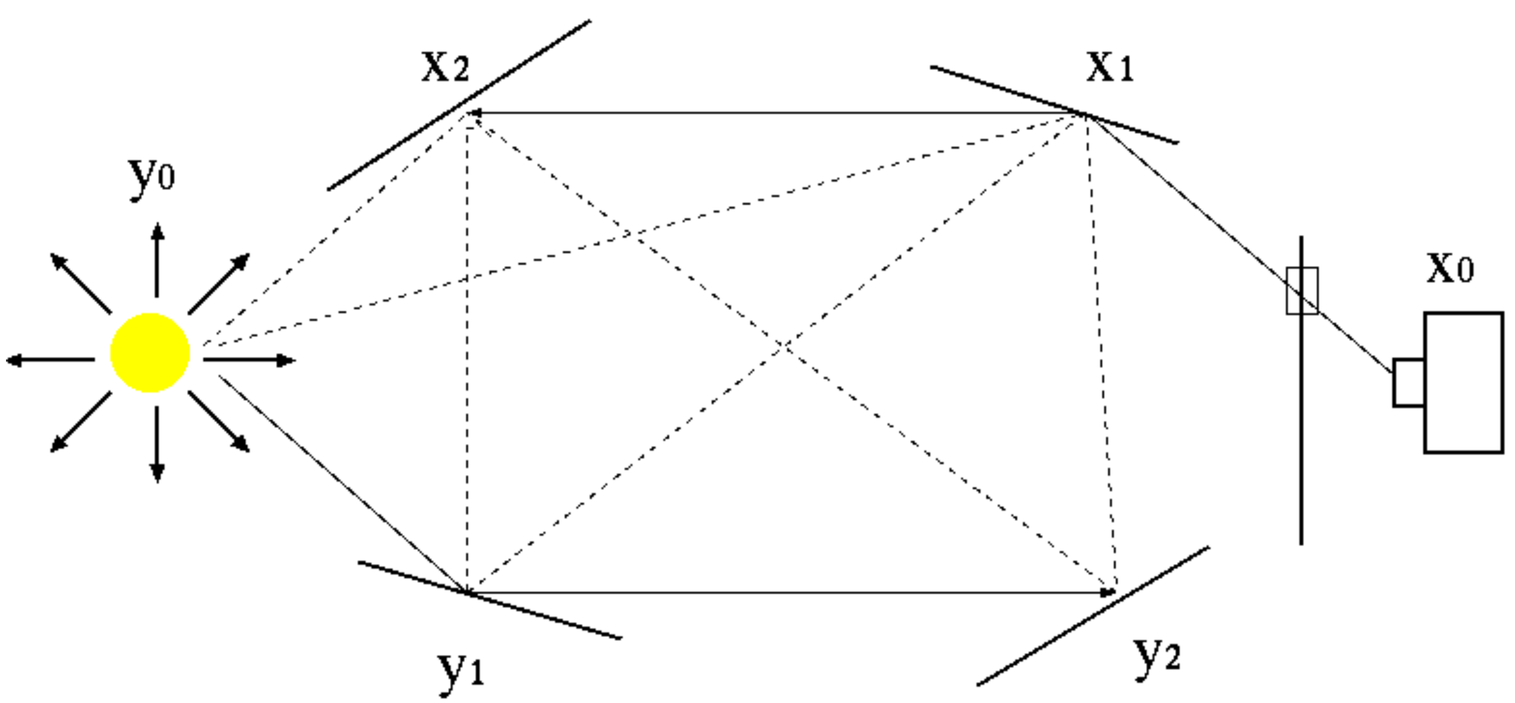
Bidirectional path tracing

- Compute a light path y0,y1,...,yn
- Compute an eye path x0,x1,...,xm
- The colour of the fragment at x1 is

-The amount of light reaching x1 from y0,...,yn and reflecting towards x0 plus

-The amount of light reaching x1 from x2 and reflecting towards x0



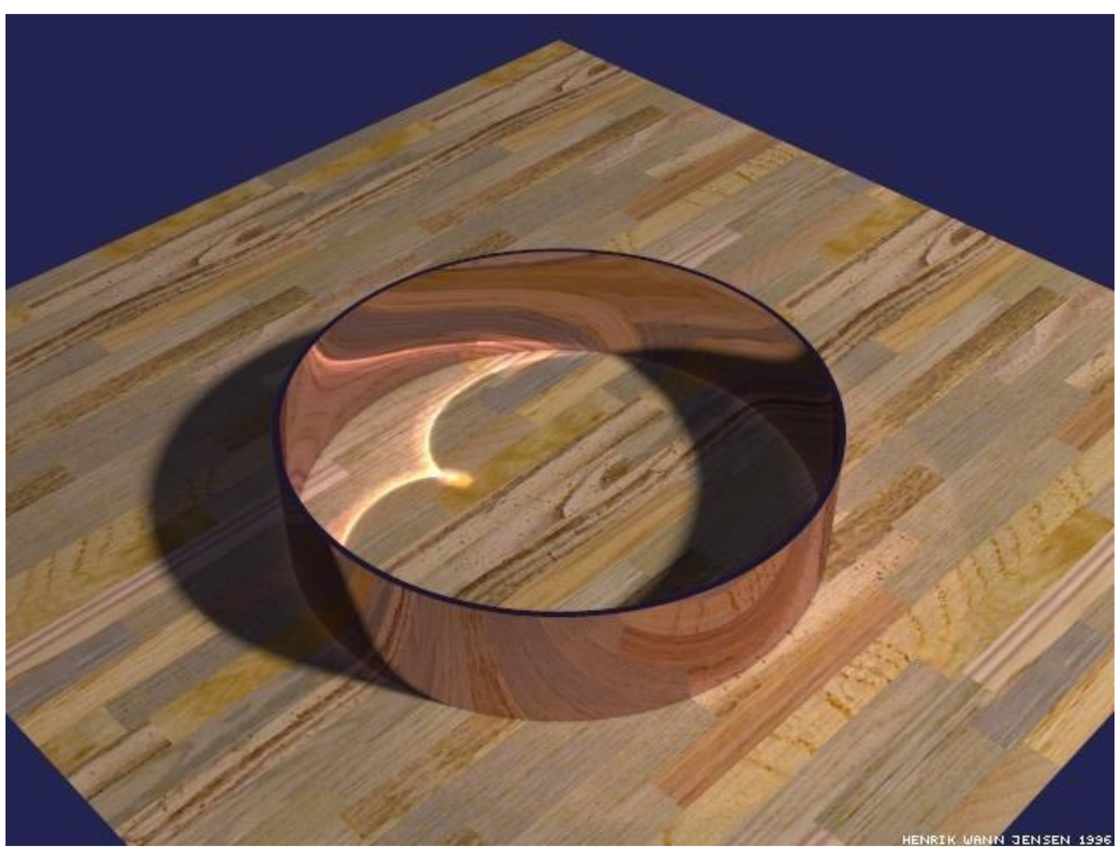
Comparison

(a) Bidirectional path tracing with 25 samples per pixel

(b) Standard path tracing with 56 samples per pixel (the same computation time as (a))

Benefits of bidirectional method

- Caustics
  - Easier to produce by tracing from the light source
- When the light sources are not easy to reach from the eye



Metropolis-Hastings algorithm

If we want to sample from some probability density function $f(x)$, we can generate a Markov Chain whose stationary distribution is $f(x)$ using the following algorithm:

for $i = 1 \ldots N$ **do**
  Generate $x' \sim q(x \to \cdot)$;
  Generate $t \sim \text{Uniform}(0, 1)$;
  $a \leftarrow \min\left(1, \frac{f(x')q(x' \to x)}{f(x)q(x \to x')}\right)$;
  if $t < a$ **then**
    $x \leftarrow x'$;
  end
end

where $q(x \to x')$ is a proposal distribution that generates random moves from the current state of the chain.

Metropolis light transport

- Bidirectional mutation:
    - Delete a subpath and sample a new one
- Perturbation:
    - Move intersection points within a subpath
- If a proposal is not valid, reject immediately

Top: Bidirectional path tracing, Bottom: Metropolis light transport. Same computation time as path tracing.

Summary for Monte Carlo Ray Tracing

- An approach that simulates the light reflection at diffuse surfaces
- Can simulate indirect lighting
- Results are subject to variance
  - Requires a lot of samples per pixel to reduce the noise
  - Bidirectional methods can reduce the noise

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Overview

- Global illumination and light transport
- Monte-Carlo integration
- Monte-Carlo Ray Tracing
  -Path Tracing
  -Bidirectional Path Tracing
- **Photon Mapping**

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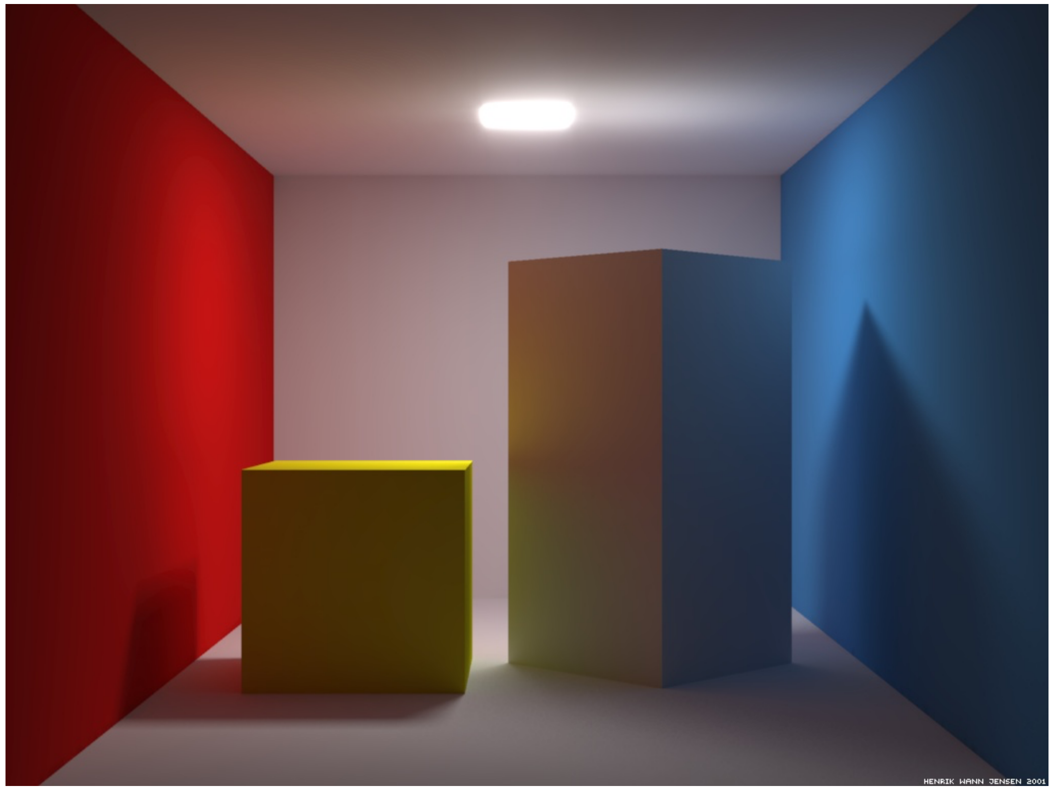
Photon Mapping

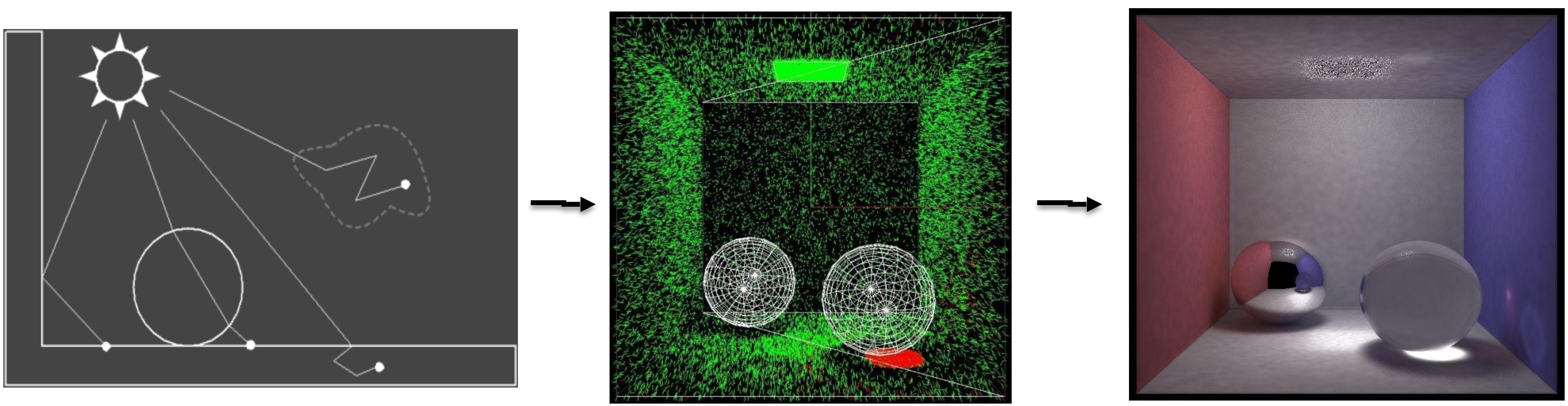
- A fast, global illumination algorithm based on Monte-Carlo method
- A stochastic approach that estimates the radiance from a limited number of samples



http://www.youtube.com/watch?v=wqWRVcsIcAQ

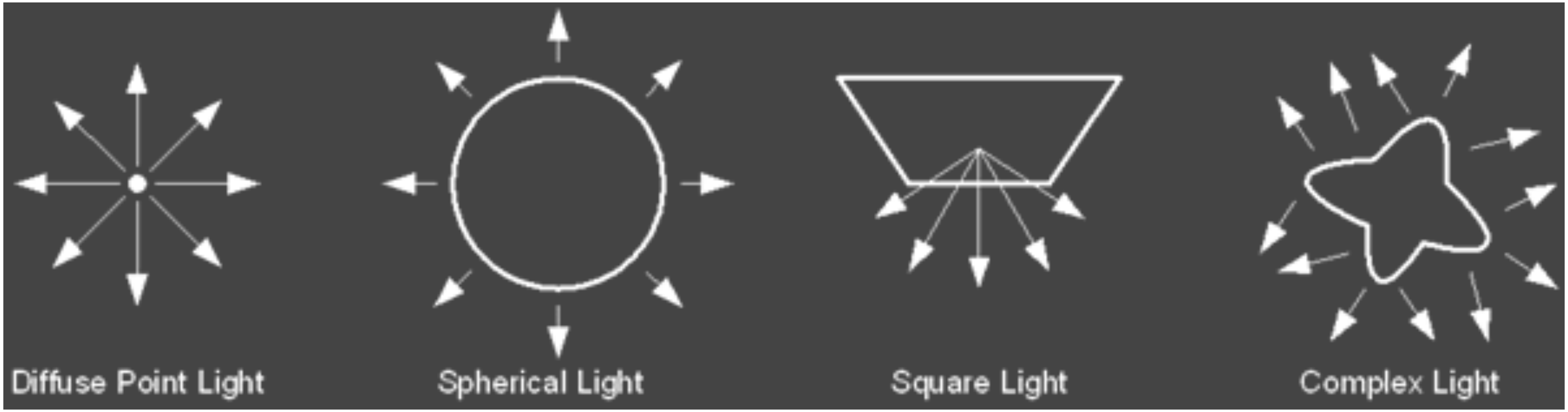
Photon Mapping

- A two pass global illumination algorithm
  - First Pass - photon tracing:
    - Casting photons from the light source
    - Storing photon positions in the "photon map",
  - Second Pass – rendering (radiance estimate):
    - the shading of pixels is estimated from the photon map



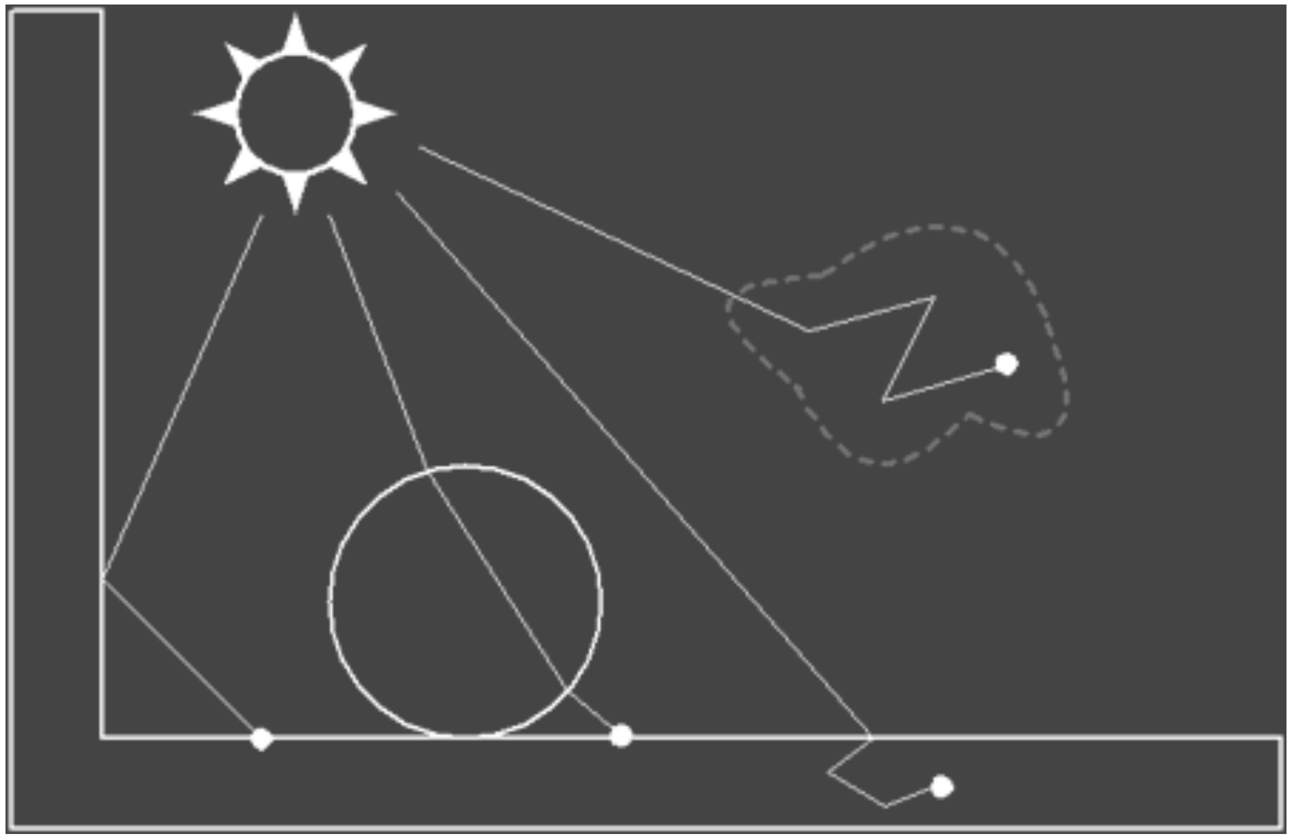
Photon emission

- A photon's life begins at the light source.
- Different types of light sources
- Brighter lights emit more photons



Photon scattering

- Emitted photons are scattered through a scene and are eventually absorbed or lost
- When a photon hits a surface we can decide how much of its energy is absorbed, reflected and refracted based on the surface's material properties



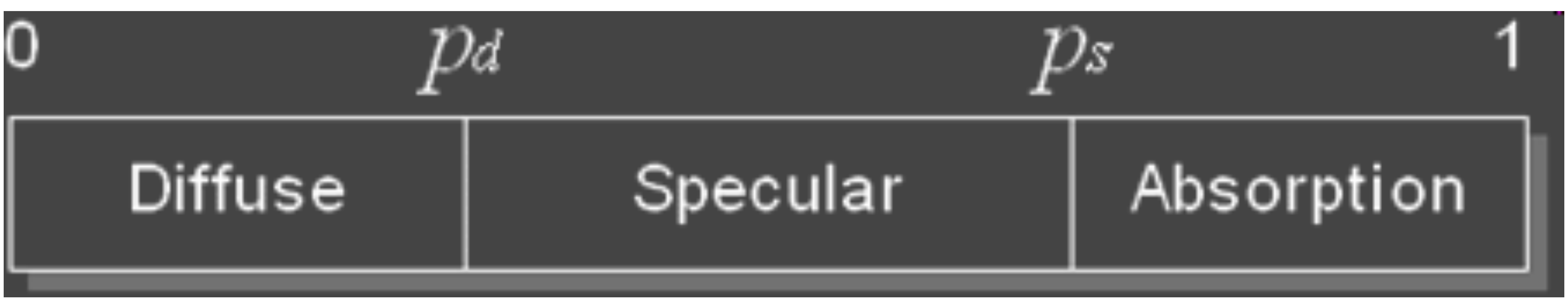
What happens when photons hit surfaces?

• Photons are reflected or absorbed. There are two ways to determine this:

- ⋆ Attenuate the power and reflect the photon
  - For arbitrary BRDFs
- ⋆ Use Russian Roulette techniques
  - Decide stochastically whether the photon is reflected or absorbed based on the probability of reflection, and do not attenuate power if it is reflected.

Russian Roulette

- If the surface is diffuse and specular, a Monte Carlo technique called Russian Roulette is used to probabilistically decide whether photons are reflected, refracted or absorbed.
- Produce a random number between 0 and 1
- Determine whether to transmit, absorb or reflect in a specular or diffusive manner, according to the value



Probability of reflection and absorption

- Probability of reflection

$$P_r = max(d_r + s_r, d_g + s_g, d_b + s_b)$$

- Probability of diffuse reflection

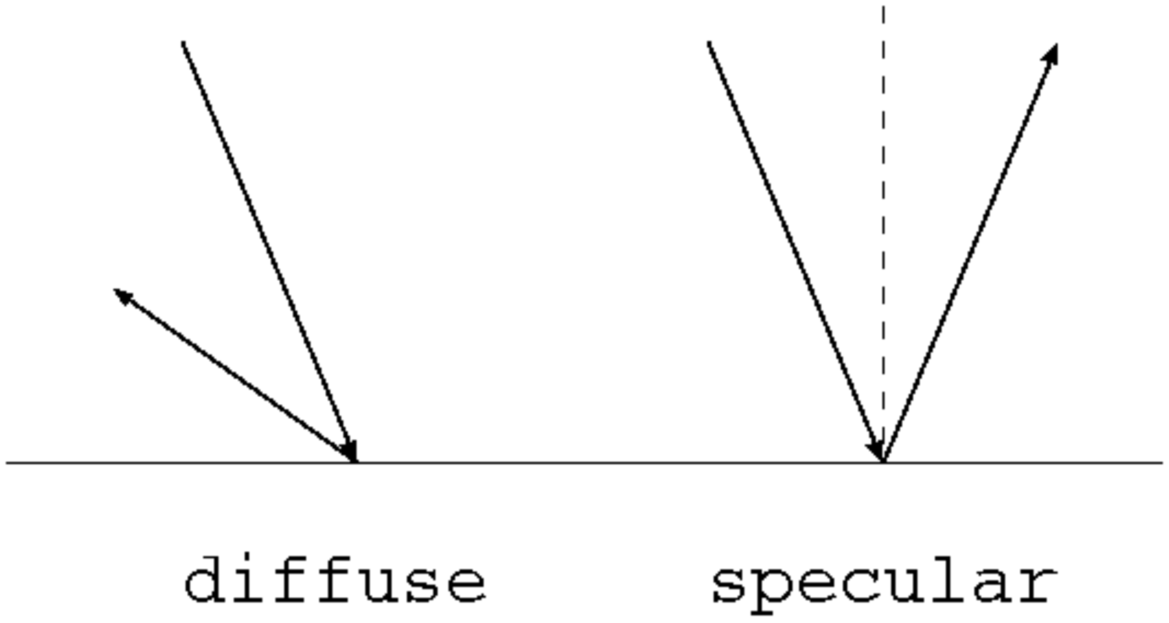
$$P_d = \frac{d_r + d_g + d_b}{d_r + d_g + d_b + s_r + s_g + s_b} P_r \, .$$

- Probability of specular reflection

$$P_s = \frac{s_r + s_g + s_b}{d_r + d_g + d_b + s_r + s_g + s_b} P_r = P_r - P_d \, .$$

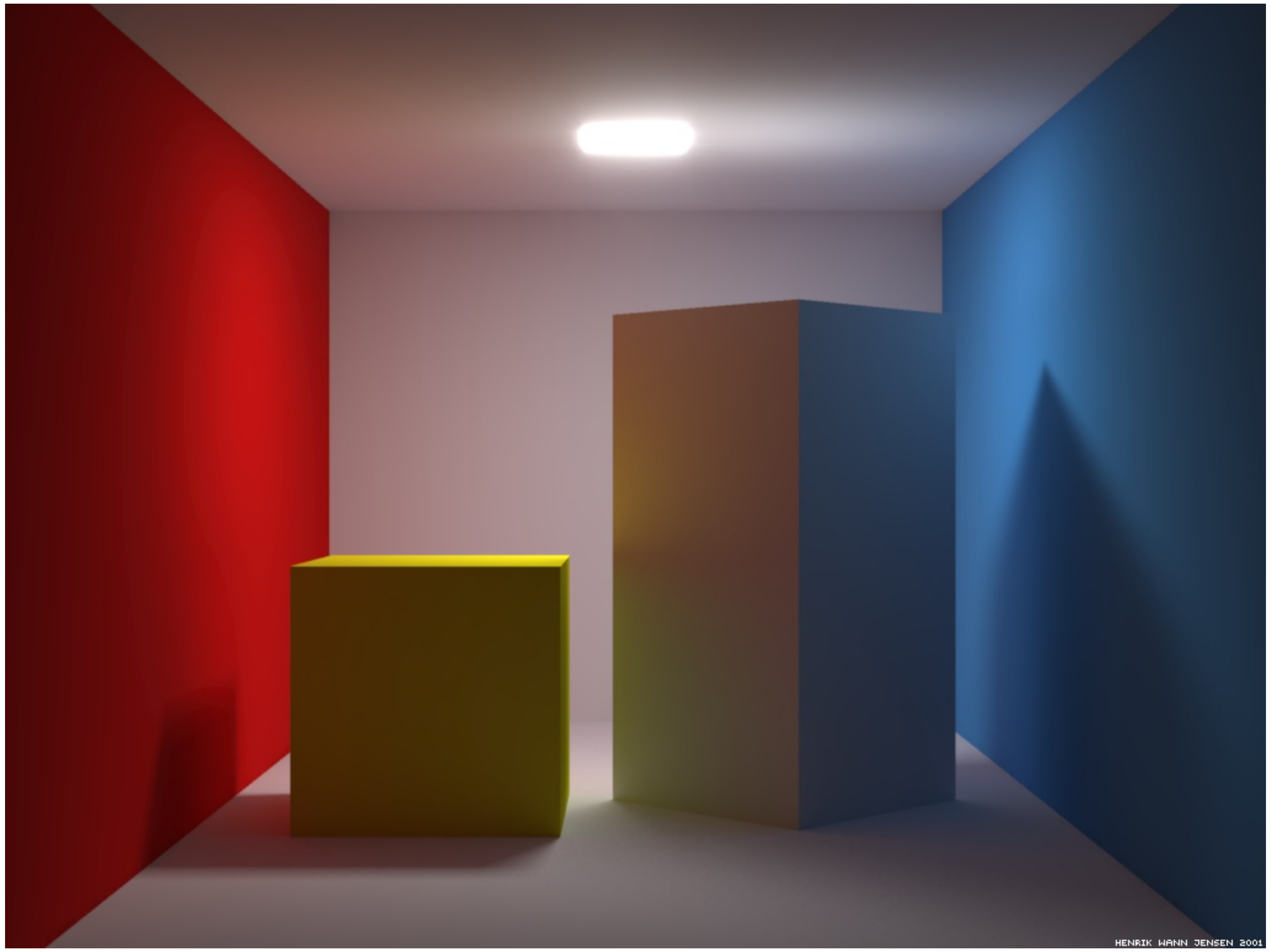
Diffuse and specular reflection

- If the photon is to make a diffuse reflection, randomly determine the direction
- If the photon is to make a specular reflection, reflect in the mirror direction



Power attenuation

- The colour of the light must change after specular/diffuse reflection
- This is essential for producing effects like colour bleeding



Power after reflectance

The power after reflection Φ_{ref} for incident photon with power Φ_i is

Specular reflection:

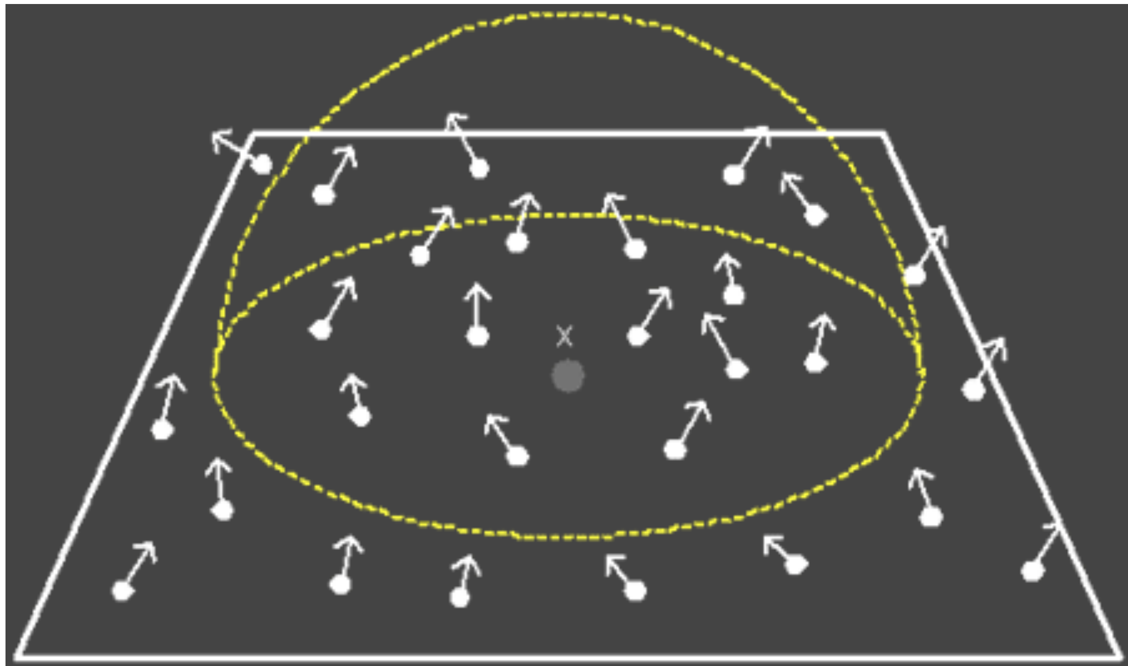
- $\Phi_{ref,r} = \frac{\Phi_{i,r} s_r}{P_s}$
- $\Phi_{ref,g} = \frac{\Phi_{i,g} s_g}{P_s}$
- $\Phi_{ref,b} = \frac{\Phi_{i,b} s_b}{P_s}$

Diffuse reflection:

- $\Phi_{ref,r} = \frac{\Phi_{i,r} d_r}{P_d}$
- $\Phi_{ref,g} = \frac{\Phi_{i,g} d_g}{P_d}$
- $\Phi_{ref,b} = \frac{\Phi_{i,b} d_b}{P_d}$

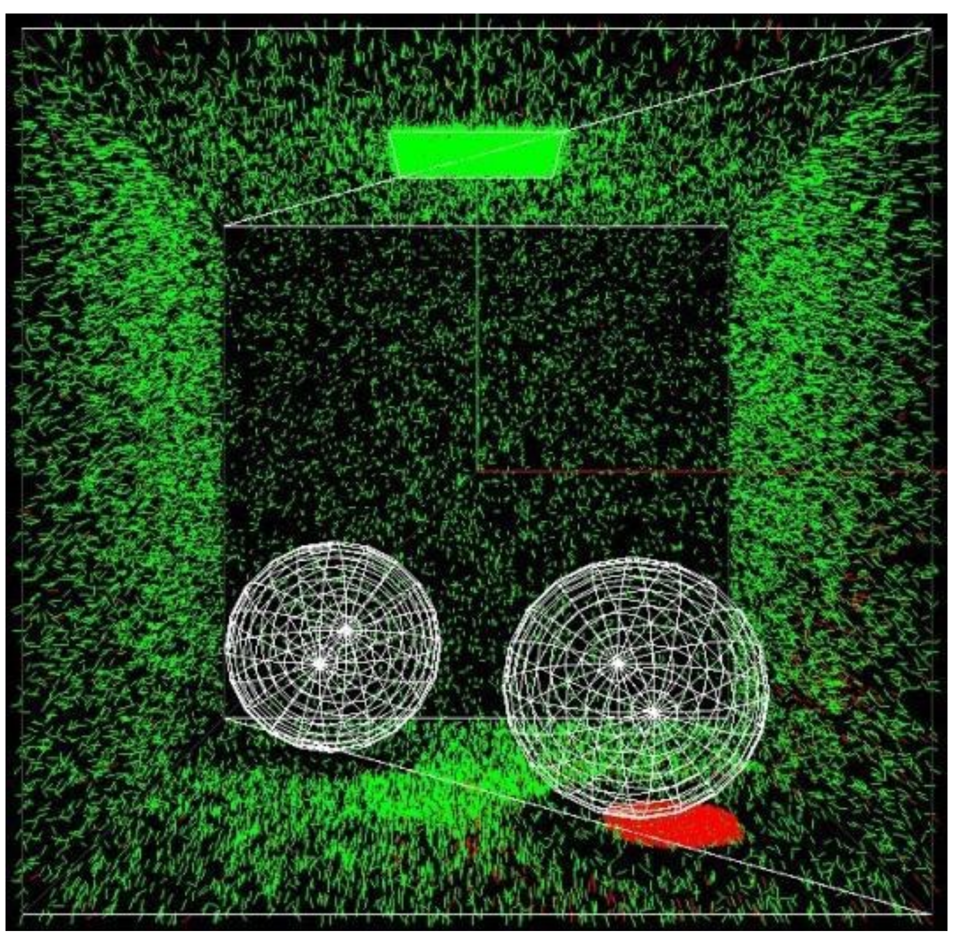
Photon Map

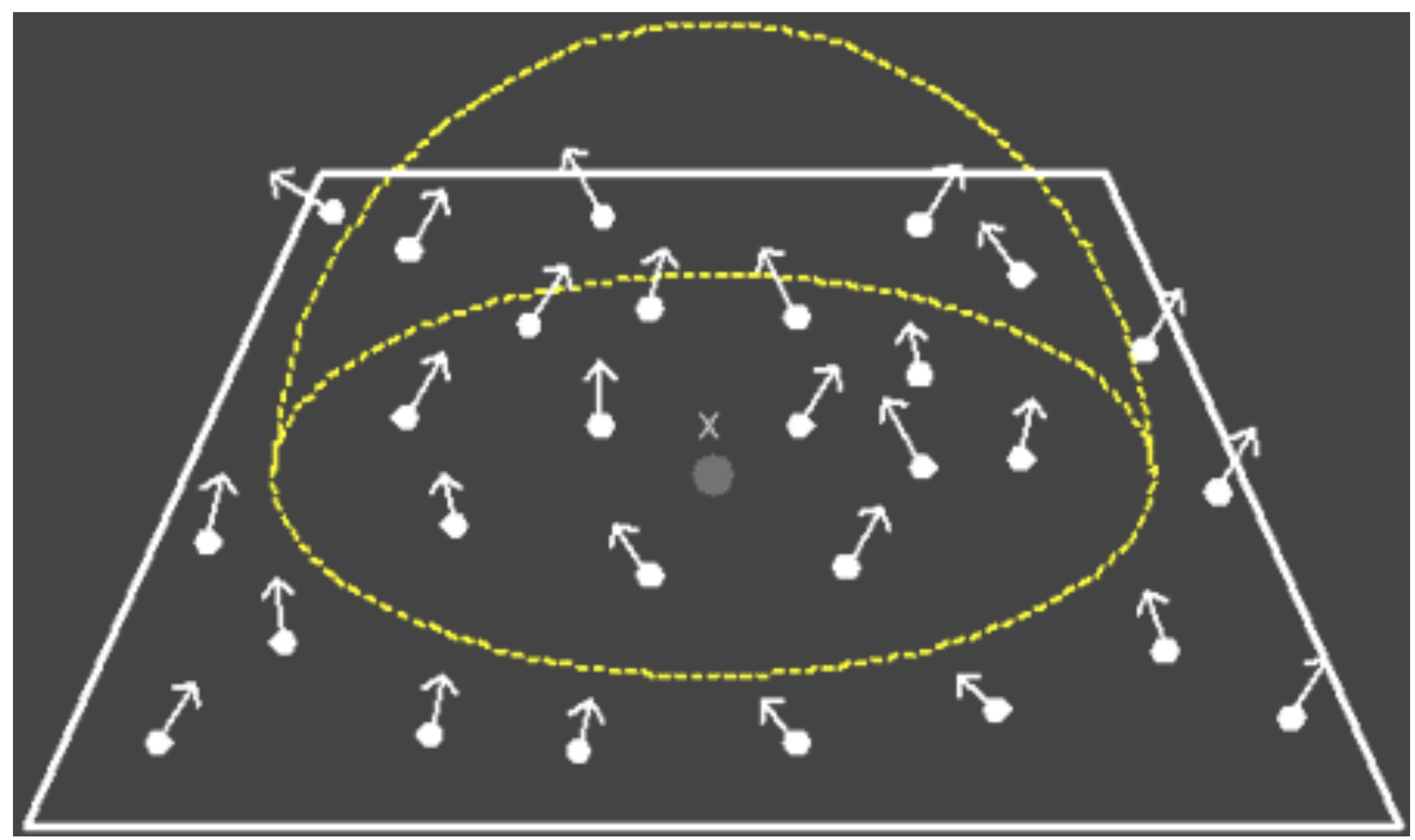
- When a photon makes a diffuse bounce, or is absorbed at the surface, the ray intersection is stored in memory
  - -3D coordinates on the surface
  - -Color intensity
  - -Incident direction
- The data structure is called Photon Map
- The photon data is not recorded for specular reflections



Second Pass – Rendering

- Finally, a traditional ray tracing procedure is performed by shooting rays from the camera
- At the location the ray hits the scene, a sphere is created and enlarged until it includes N photons





Radiance Estimation

- The radiance estimate can be written by the following equation

$$L_r(x,\vec{\omega}) = \sum_{p=1}^{N} f_r(x,\overrightarrow{\omega_p},\vec{\omega}) \frac{\Delta\Phi_p(x,\overrightarrow{\omega_p})}{\Delta A}$$

x : location the ray hits the scene

$\vec{\omega}$: direction towards the camera

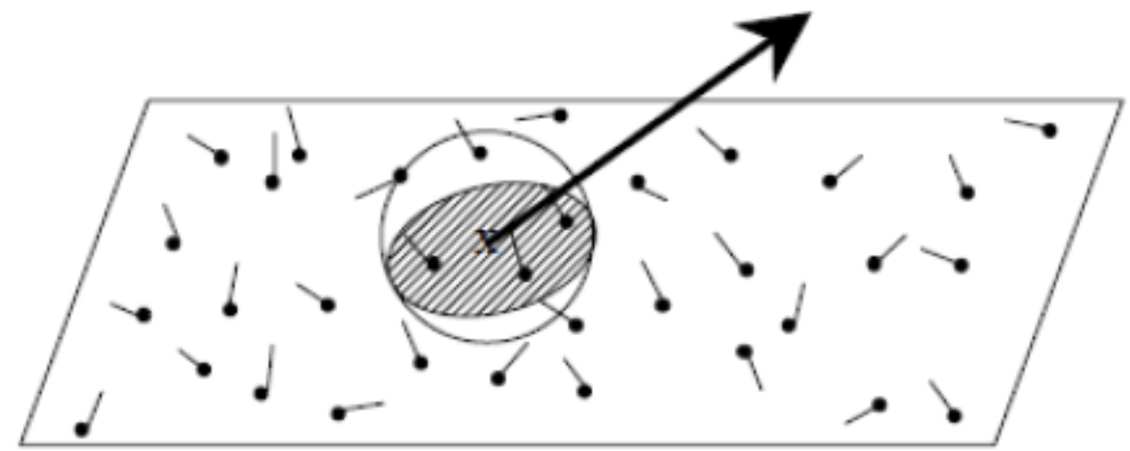
$\overrightarrow{\omega_p}$: incident vector of photon p

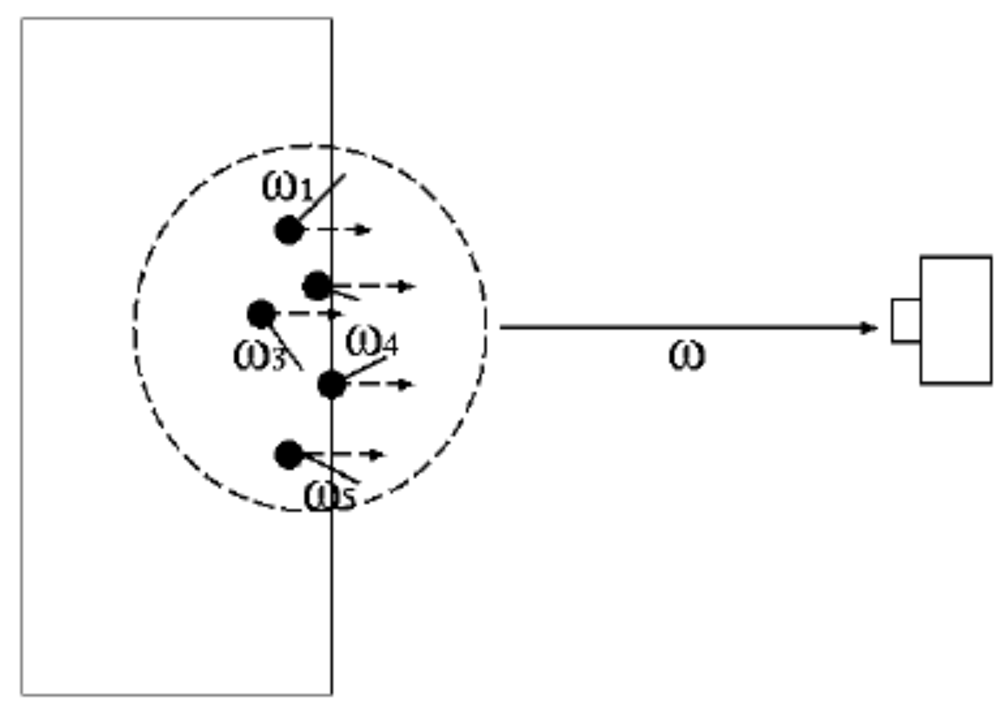
f_r : BRDF

N : the number of photons

$\Delta\Phi_p$: power of photon p

ΔA : Area of the circle πr^2





Radiance Estimation

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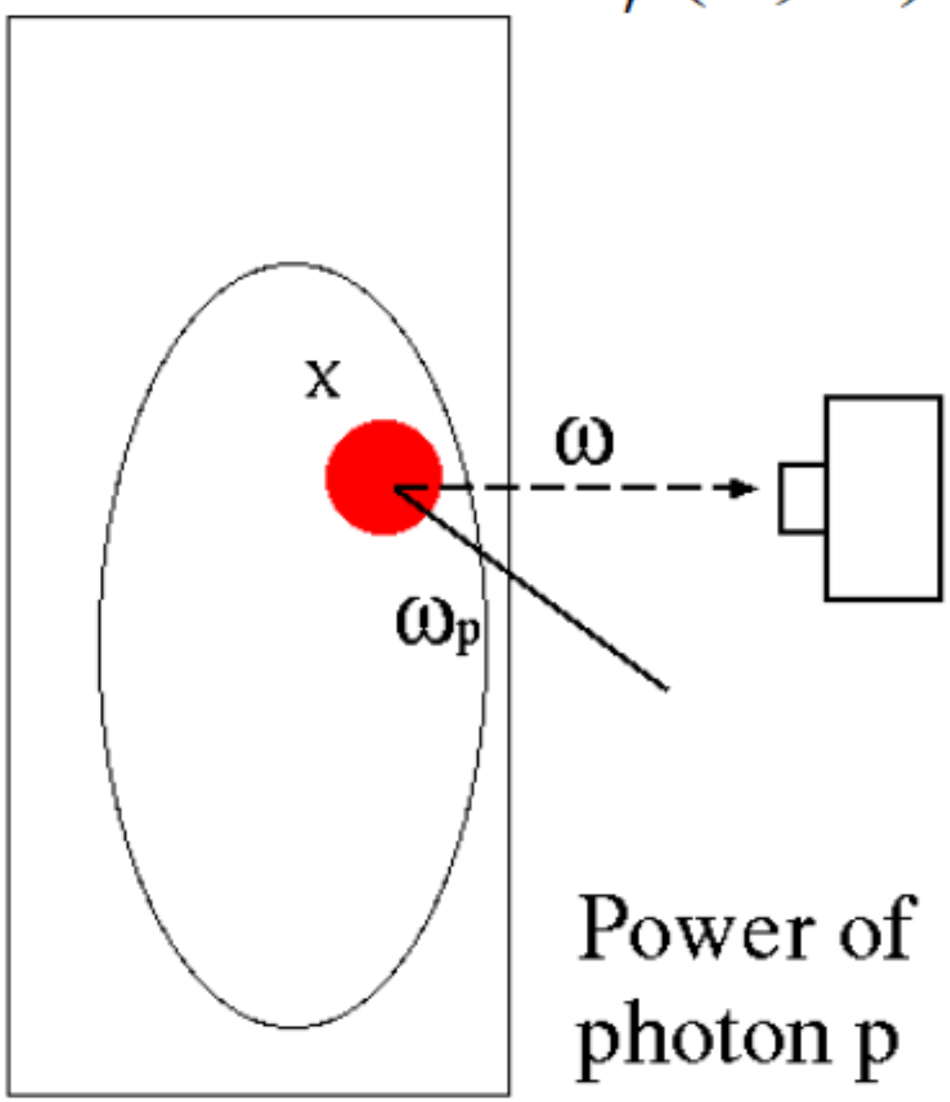
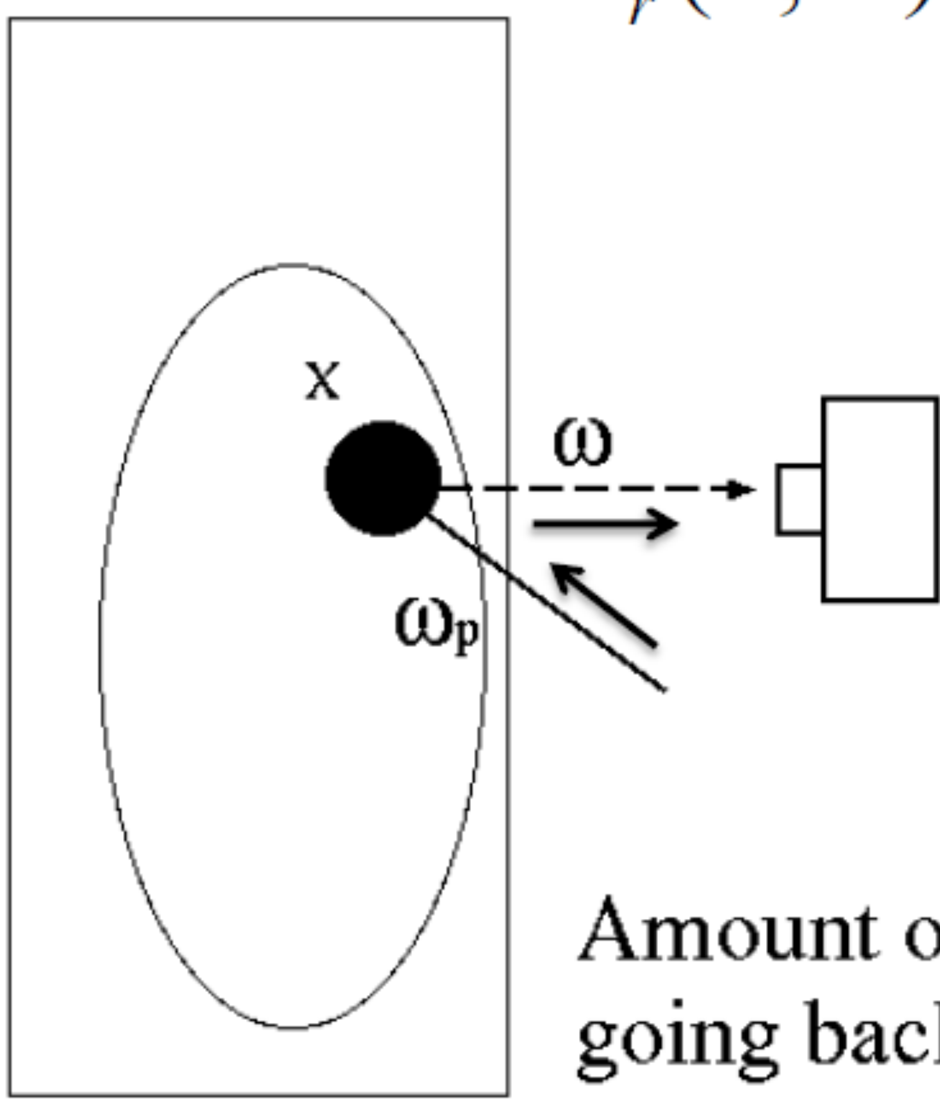
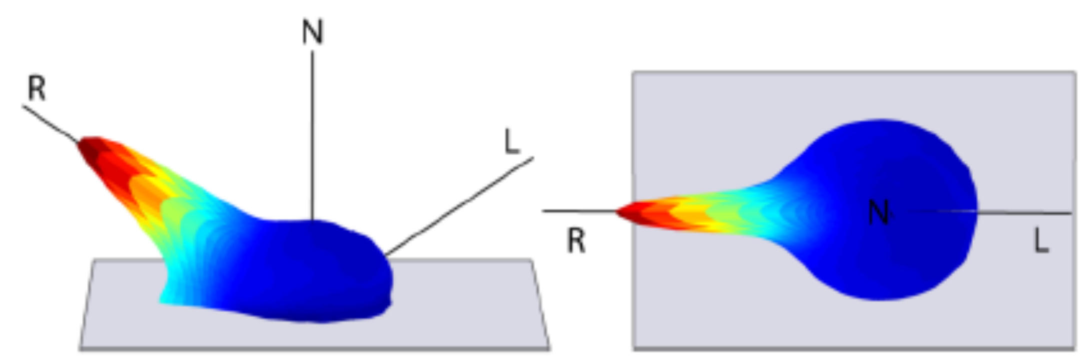


Power of photon p

Radiance Estimation

$$L_r(x,\vec{\omega}) = \sum_{p=1}^{N} f_r(x,\vec{\omega_p},\vec{\omega}) \frac{\Delta\Phi_p(x,\vec{\omega_p})}{\Delta A}$$

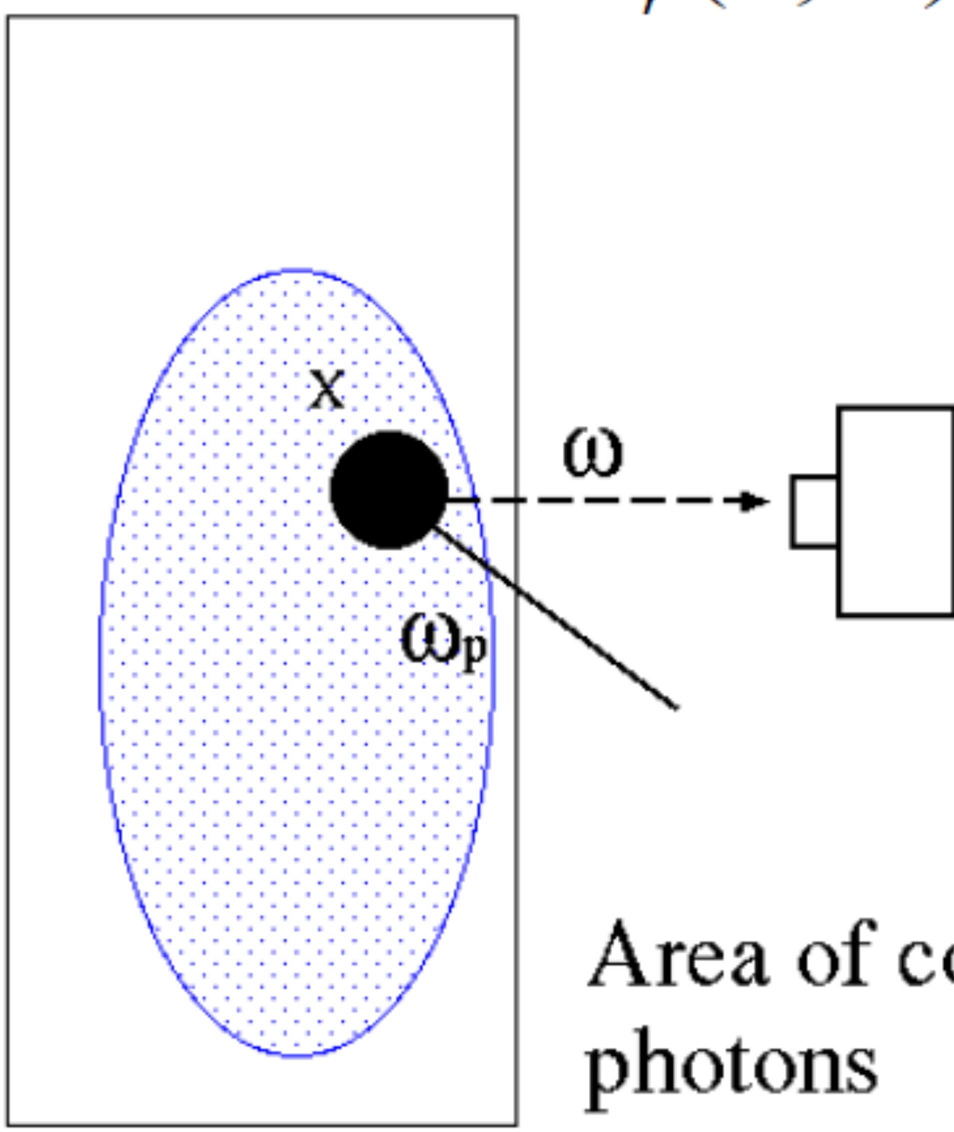




Amount of light coming in from ω_p going back toward ω

Radiance Estimation

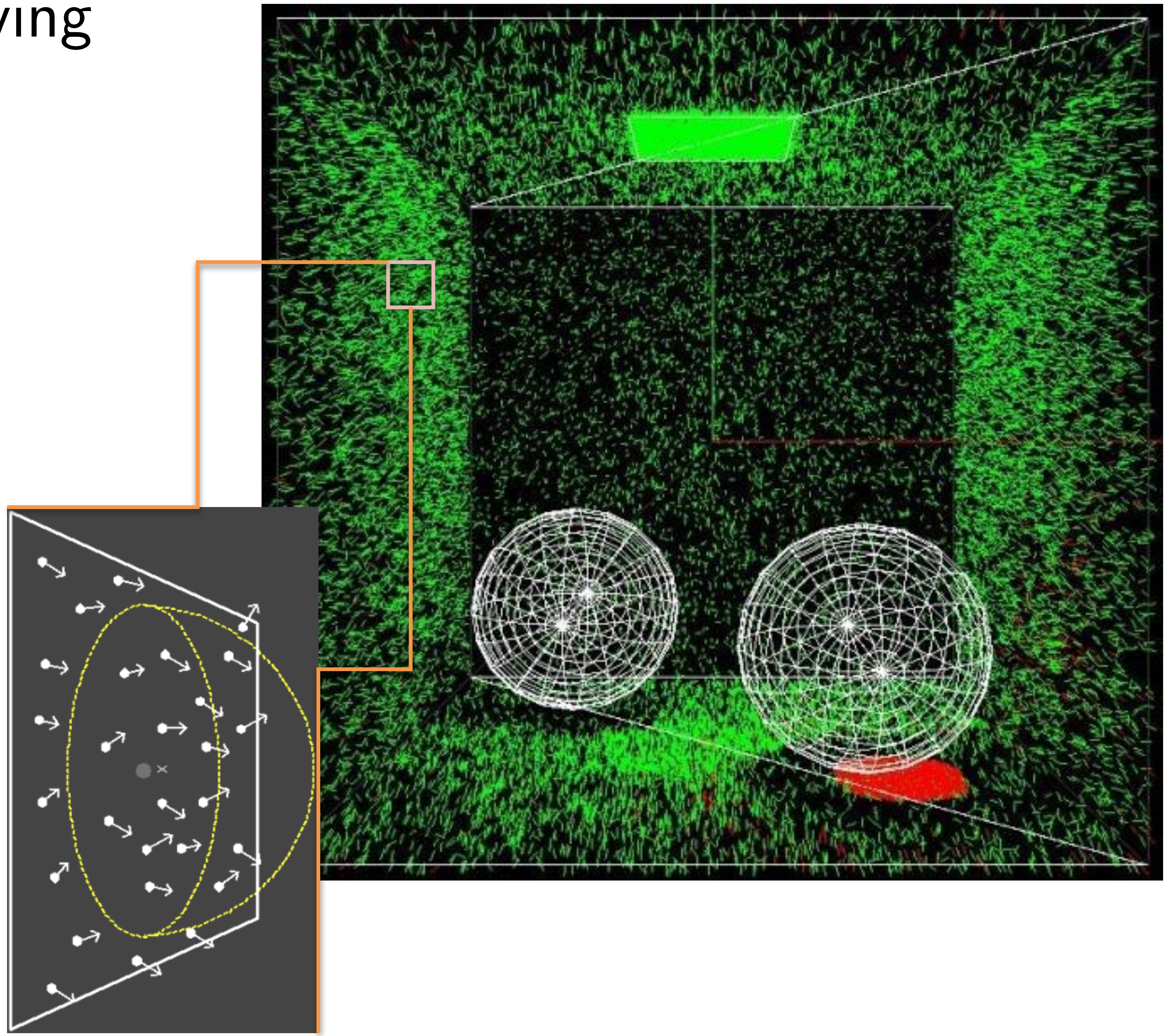
$$L_r(x,\vec{\omega}) = \sum_{p=1}^{N} f_r(x,\overrightarrow{\omega_p},\vec{\omega}) \frac{\Delta\Phi_p(x,\overrightarrow{\omega_p})}{\Delta A}$$



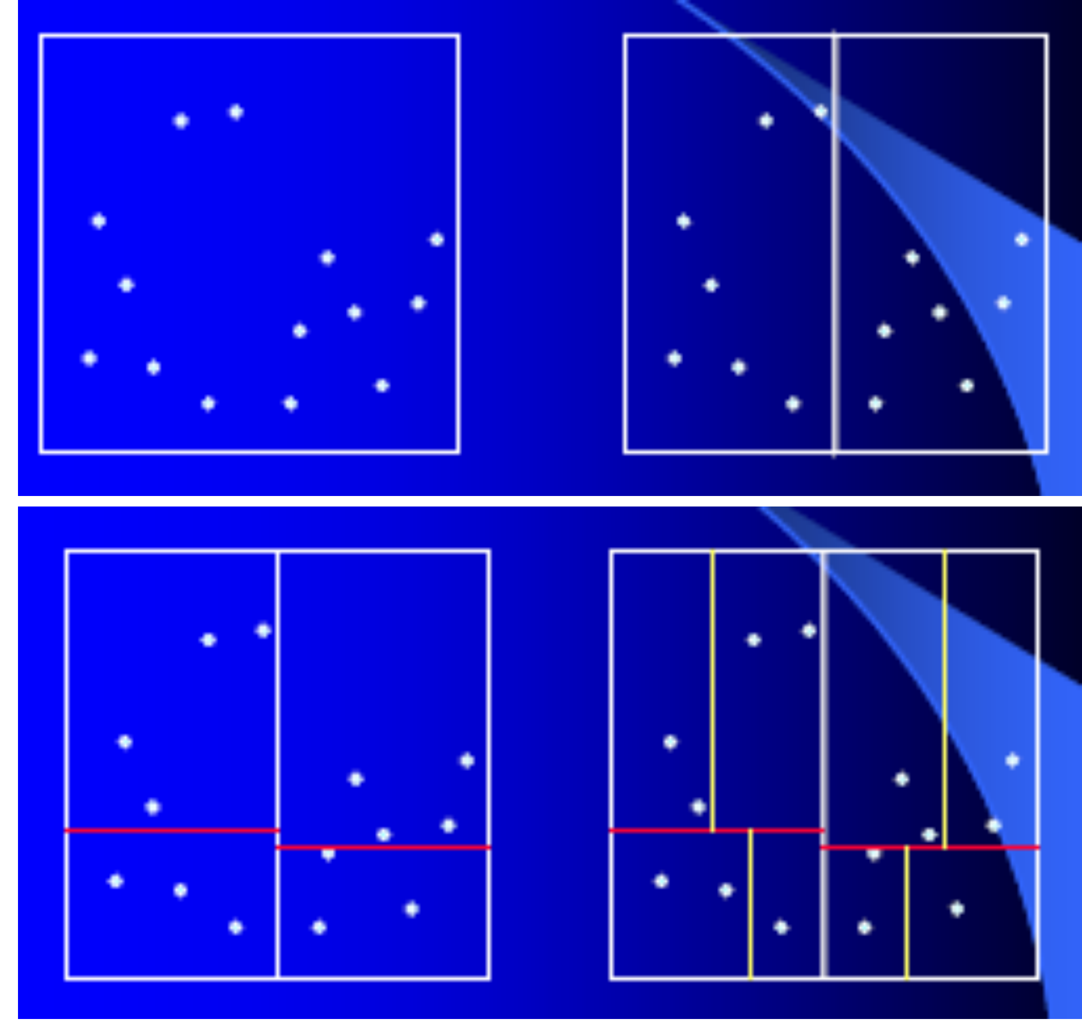
Area of collecting photons

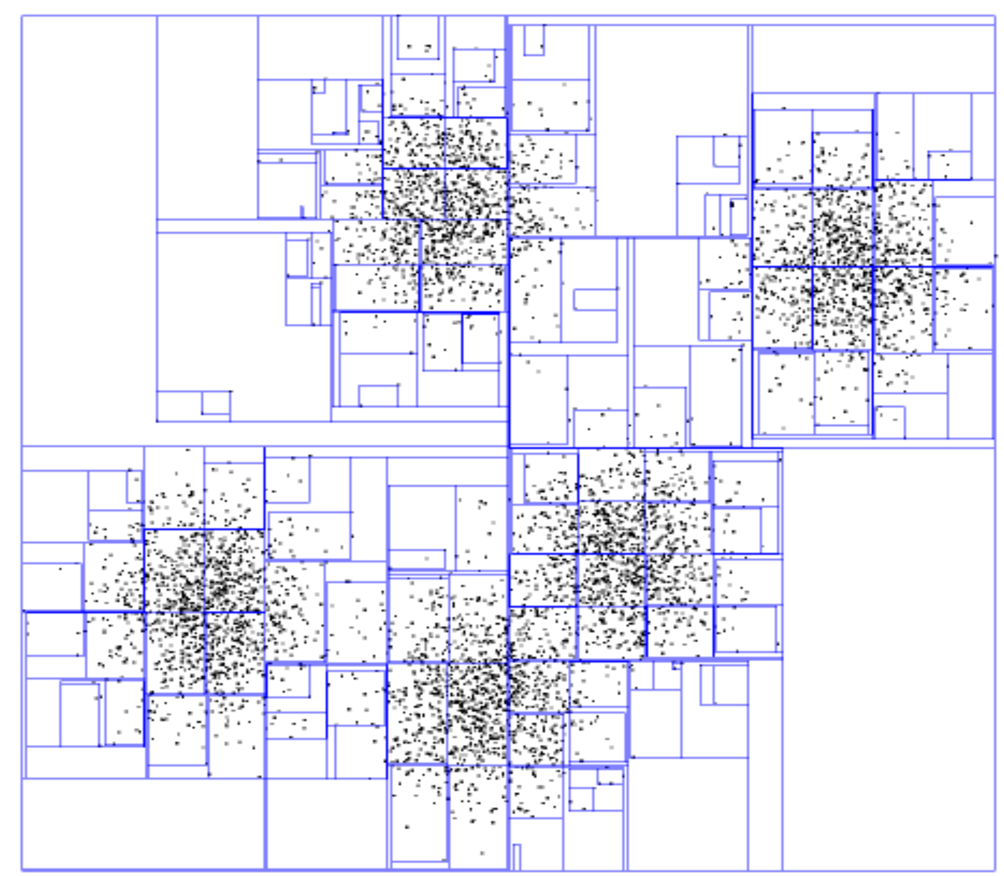
Data structure for photon data

- We need an efficient data structure for retrieving photon maps when colouring the pixels
  - KD-tree
  - Spatial Hash



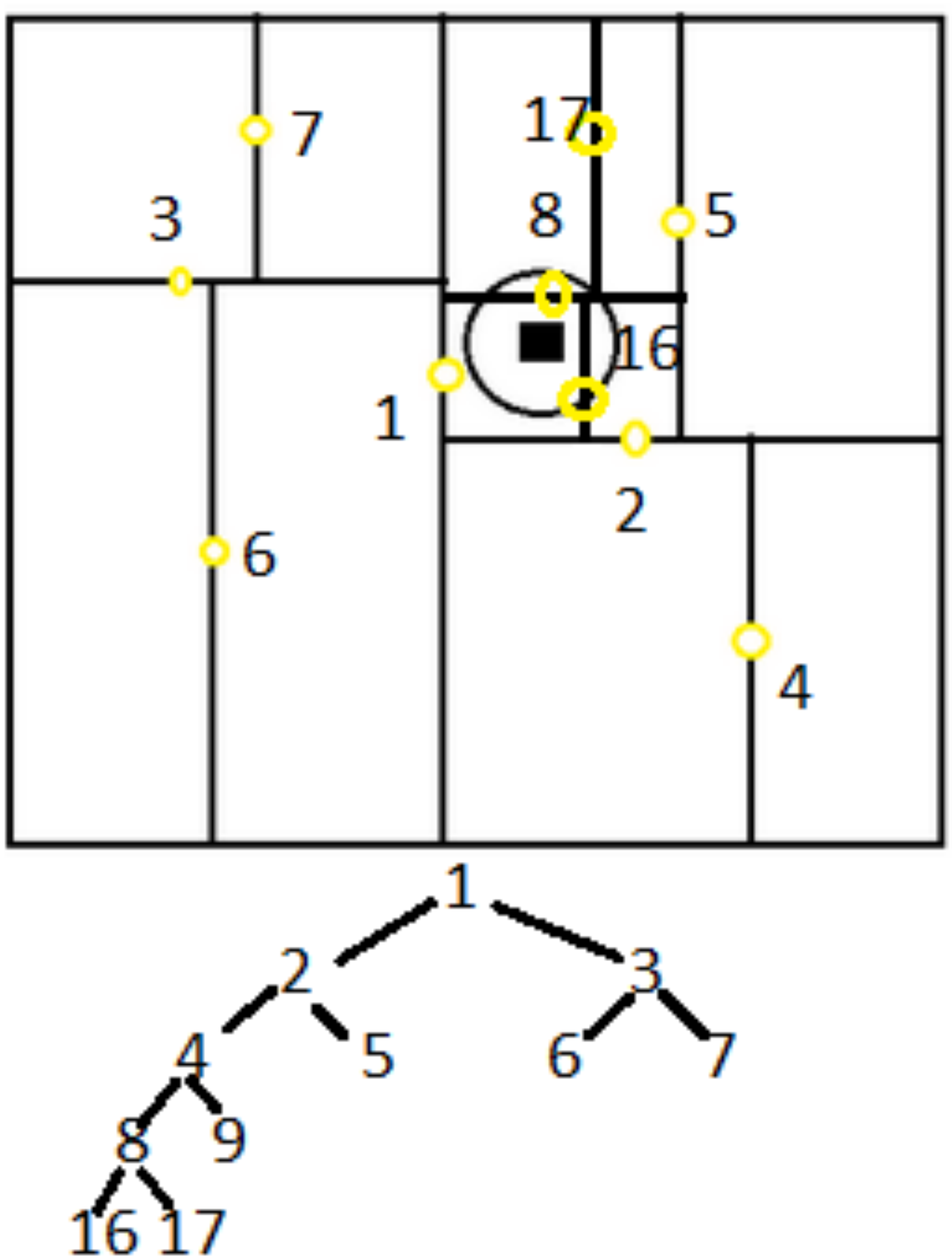
Storing photons: kd-tree





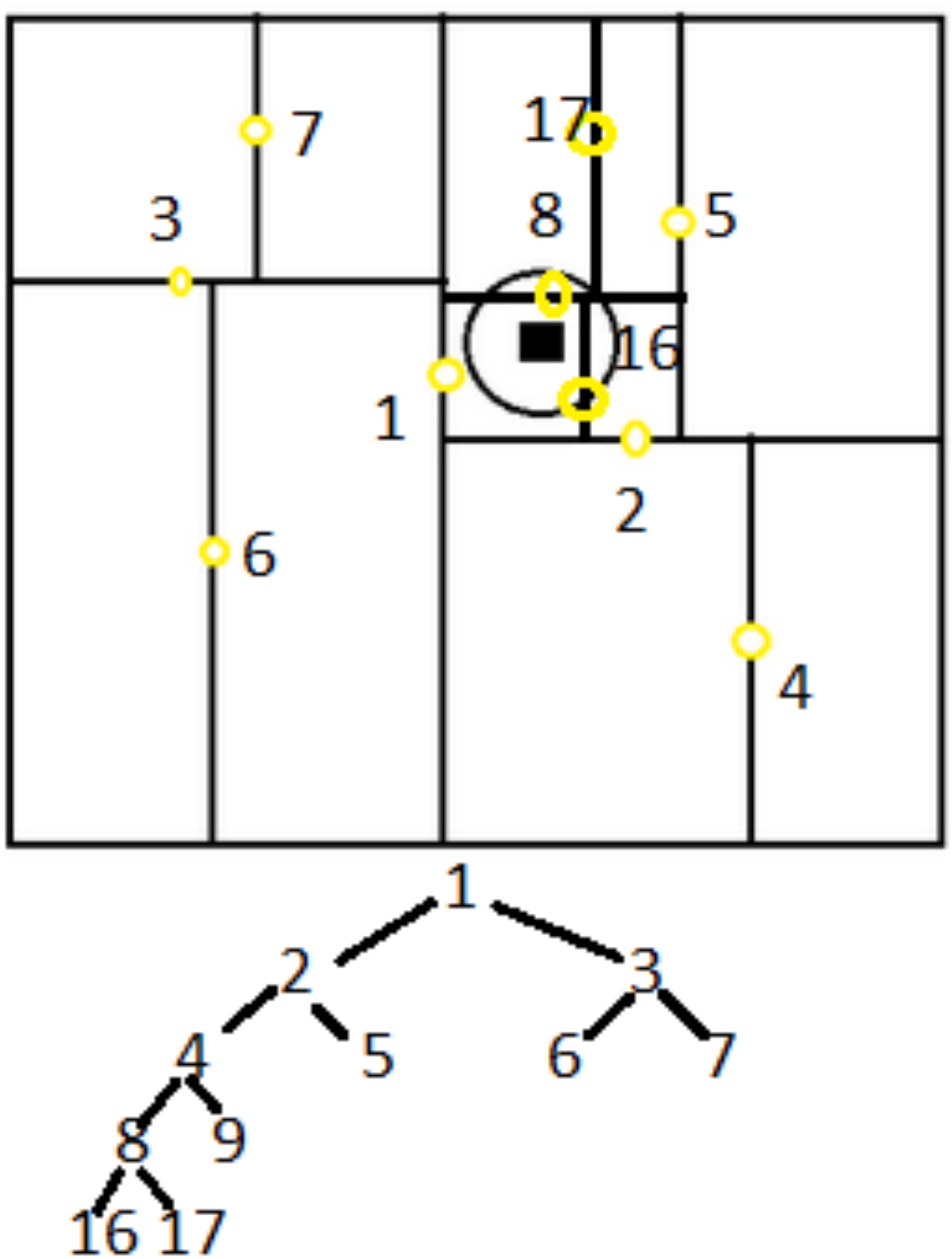
- An efficient hierarchical data structure for saving spatial data
- Procedure to produce it:
  - divide the samples at the median along current axis (e.g. x,y or z)
  - The median sample becomes the parent node, and the samples on either side become the child nodes
  - Further subdivide the child trees on the next axis (rotating through x,y,z)
- Can efficiently find the neighbours when rendering the scene

Query for N-nearest neighbouring photons



- Given a point X, we traverse the tree to find the nearest N points to X
- Start from the root, check if the bounding circle is totally within one side or not
- If it is, then you do not have to search the other side

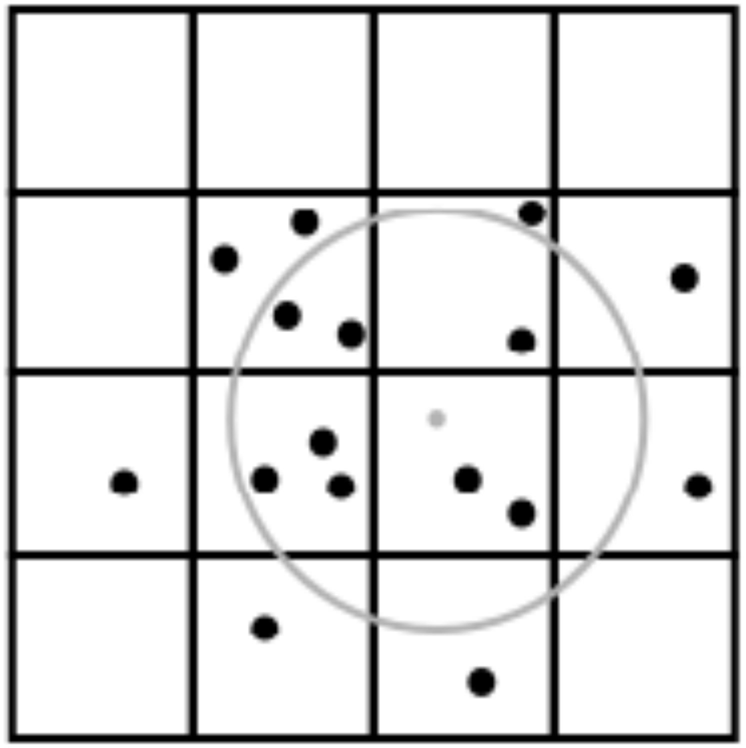
Query for N-nearest neighbouring photons



- If the photon is within bounding volume, you add it into the heap
- Descend to the children (only if they are within the bounding distance)
- The heap is sorted so that the farthest photon is on the top.
- Only the top N photons are kept in the heap.

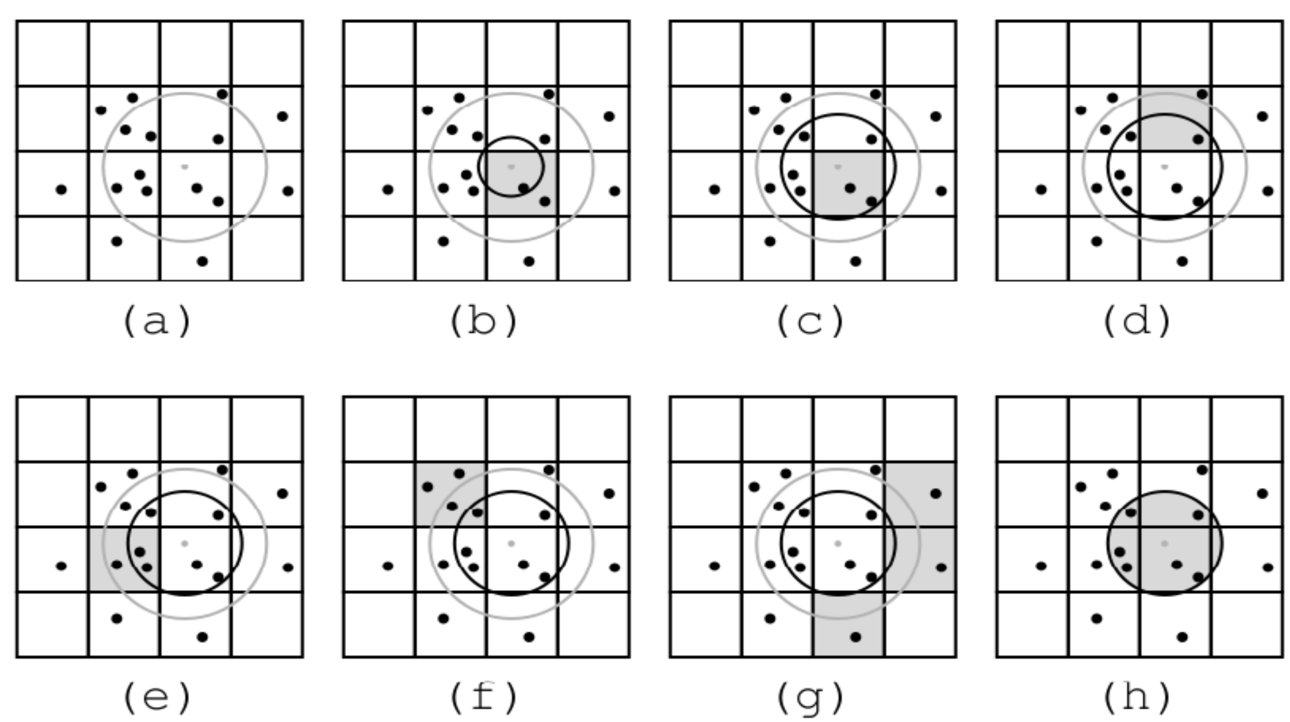
Storing photons: spacial hashing

- A uniform 3D grid based hashing system
- Create a hash function that maps each grid region to a list that stores the photons in that region
- Scan the photons in the list to find those close to the sample point



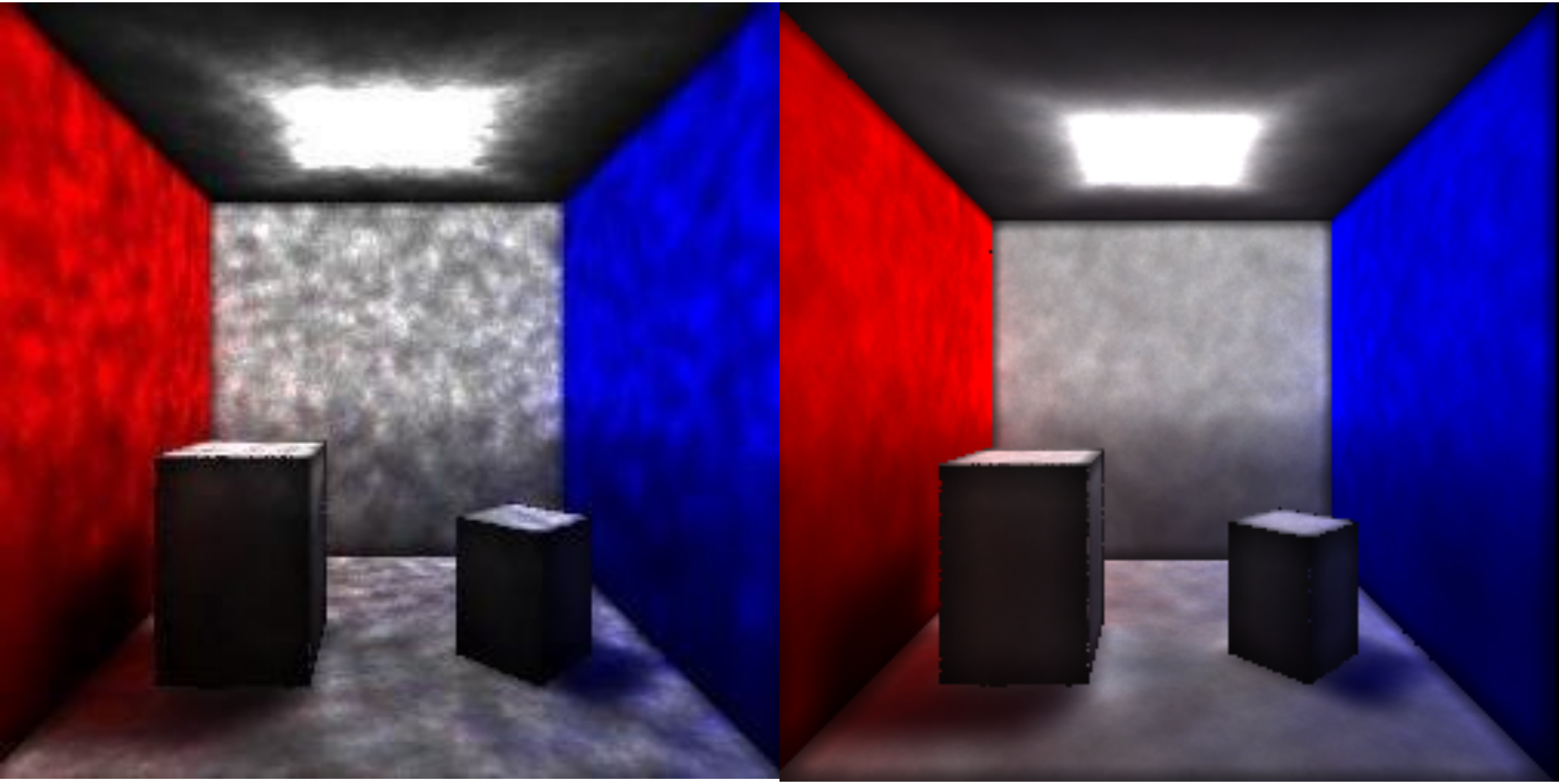
Nearest neighbour search

- Decide the maximum radius of search
- Examine the distance between the sample point and the photons in the grid
- Gradually increase the radius, search in all the reachable grids until we reach the photon count
- Suitable for hardware implementation
- “Photon Mapping on Programmable Graphics Hardware”, Proceedings of the ACM SIGGRAPH/EUROGRAPHICS Conference on Graphics Hardware, pp. 41-50, 2003

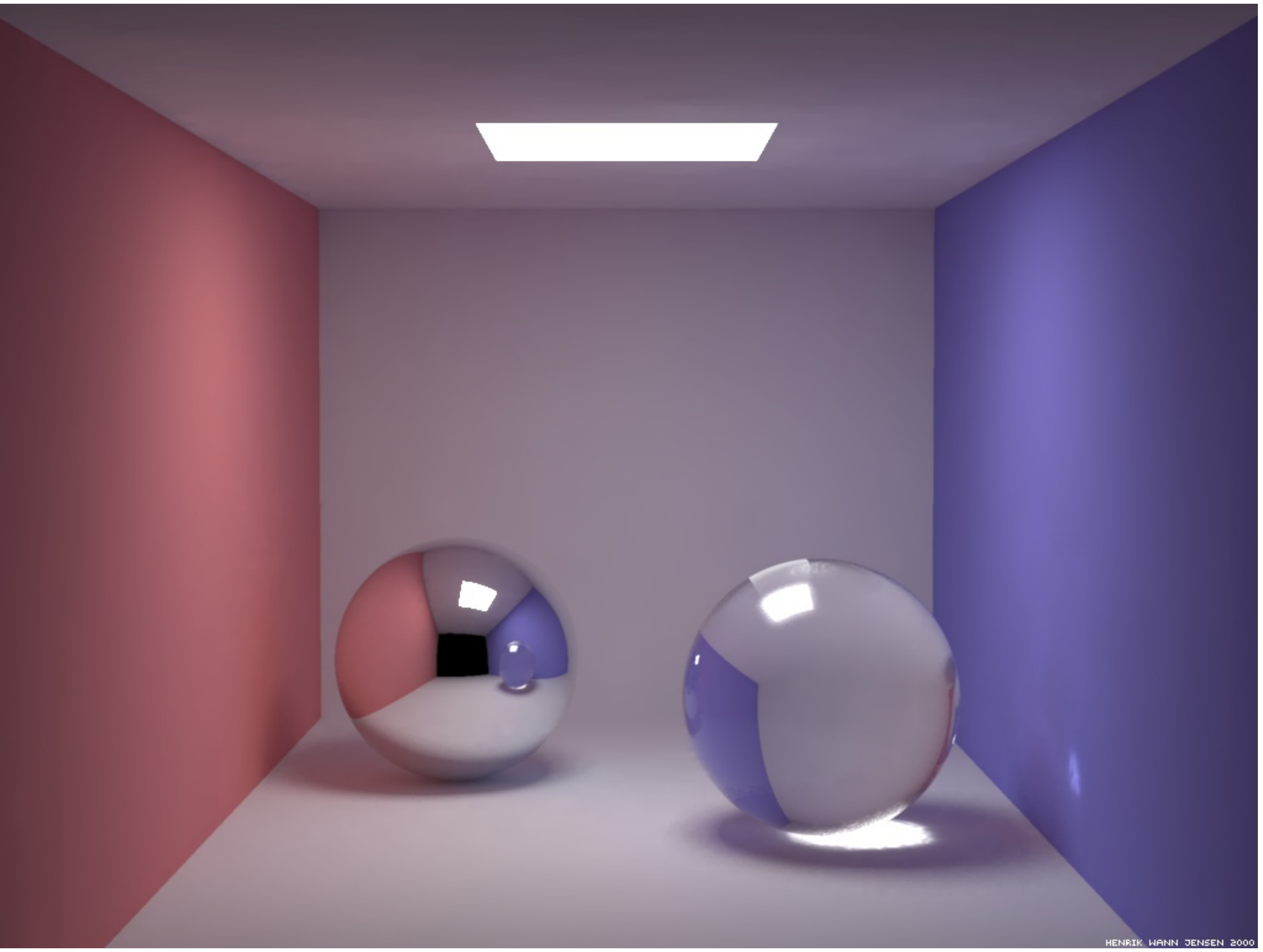


Precision

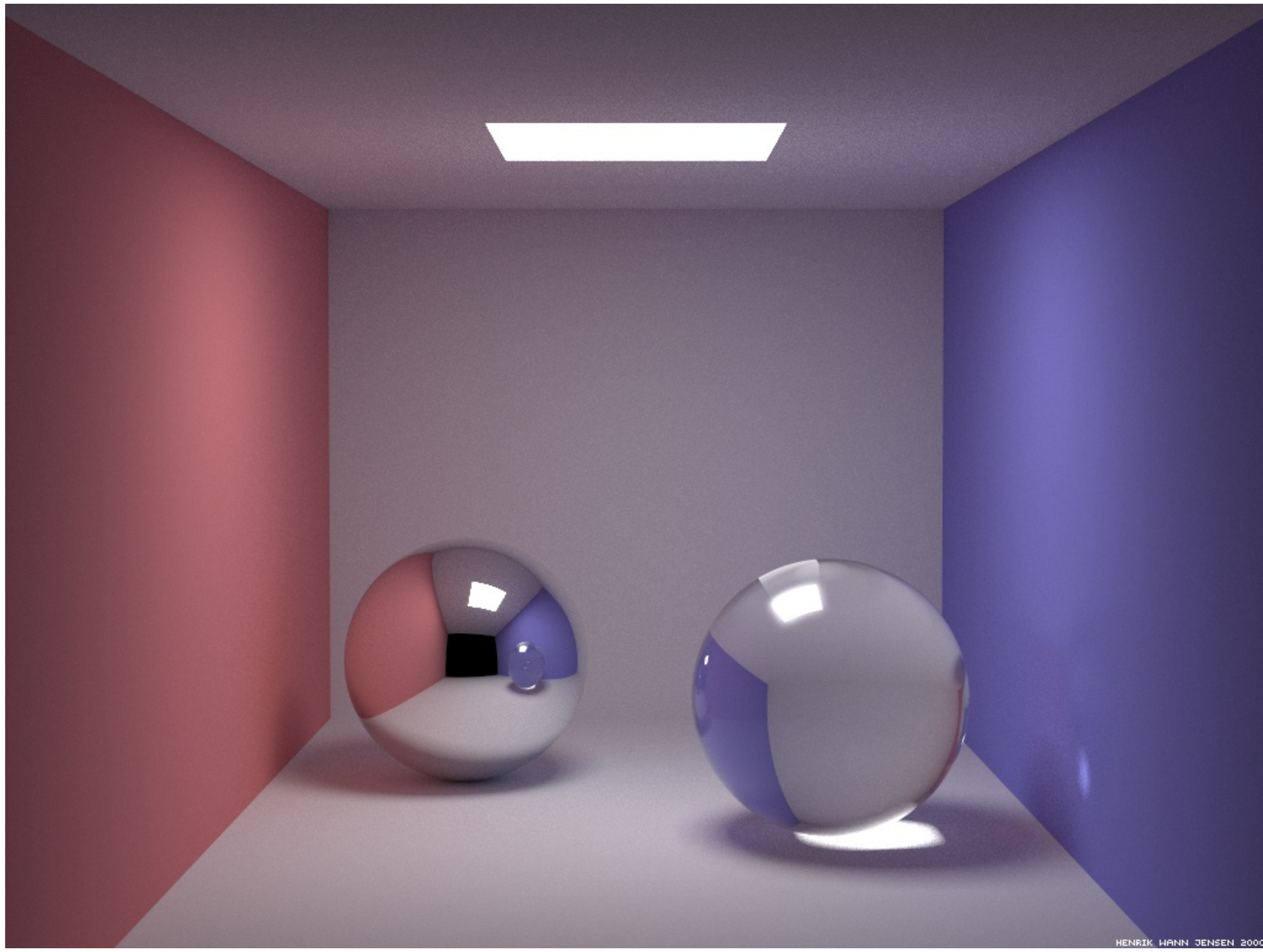
- The precision of the final results depends on
  - the number of photons emitted
  - the number of photons counted for calculating the radiance



- 10000 photons and 50 samples(left), and 500000 photons and 500 samples (right)



Photon mapping: 250000 photons, 15 seconds



Path tracing

Summary

- Monte Carlo Ray Tracing
  -Accurate but requires a lot of samples per pixel
  -Suffers from noise which is due to variance
  -Bidirectional method can reduce the variance

- Photon Mapping
  -A stochastic approach that estimates the radiance from a limited number of photons
  -Requires less samples compared to path tracing

References

★ Shirley Chapter 24 (Global illumination)

★ A Practical Guide to Global Illumination using Photon Maps
- Siggraph 2000 Course 8
- http://graphics.stanford.edu/courses/cs348b-01/course8.pdf

- http://graphics.stanford.edu/papers/metro/

- Realistic Image Synthesis Using Photon Mapping by Henrik Wann Jensen, AK Peters, 2001.

- Global Illumination using Photon Maps (Rendering Techniques '96) Henrik Wann Jensen (http://graphics.ucsd.edu/~henrik/)