Spline

- A long flexible strips of metal used by draftspersons to lay out the surfaces of airplanes, cars and ships
- Ducks weights attached to the splines were used to pull the spline in different directions
- The metal splines had second order continuity

B-Splines (for basis splines)

- B-Splines
  - Another polynomial curve for modelling curves and surfaces
  - Consists of curve segments whose polynomial coefficients only depend on just a few control points
    - Local control
    - Segments joined at knots

B-Splines

- The curve does not necessarily pass through the control points
- The shape is constrained to the convex hull made by the control points
- Uniform cubic b-splines has $C_2$ continuity
  - Higher than Hermite or Bezier curves

Basis Functions

- We can create a long curve using many knots and B-splines
- The unweighted cubic B-Splines have been shown for clarity.
- These are weighted and summed to produce a curve of the desired shape

Generating a curve

Opposite we see an example of a shape to be generated.

Here we see the curve again with the weighted B-Splines which generated the required shape.
Cubic B-splines with uniform knot-vector is the most commonly used form of B-splines. The domain of the function is defined by point \( t_0, \ldots, m \) and knot \( t_j, j = 0, \ldots, k + m \). The domain of the function \( tk-l \leq t \leq km + l \) is defined by point \( t_k \) and \( t_{m+1} \). For each \( i \geq 4 \), there is a knot between \( Q_{ij} \) and \( Q_{i+1,j} \) at \( t = t_i \). Initial points at \( t_0 \) and \( t_{m+1} \) are also knots. The following illustrates an example with control points set \( P_0 \ldots P_9 \).
A Bspline of order \( k \) is a parametric curve composed of a linear combination of basis B-splines \( B_{i,n} \).

\[
p(t) = \sum_{i=0}^{m} P_i B_{i,n}(t)
\]

The control points \( P_i \), \( i = 0, \ldots, m \)

The B-splines can be defined by

\[
B_{i,2}(t) = \begin{cases} 1, & 1 \leq t \leq 3 \\ 0, & \text{otherwise} \end{cases}
\]

C0 continuous

http://www.ibiblio.org/e-notes/Splines/Basis.htm

The shape of the basis functions

B0: linear basis functions
Order = 2, degree = 1
C0 continuous

http://www.ibiblio.org/e-notes/Splines/Basis.htm

The shape of the basis functions

B3: Cubic basis functions
Order = 4, degree = 3
C2 continuous

http://www.ibiblio.org/e-notes/Splines/Basis.htm

Uniform / non-uniform B-splines

- Uniform B-splines
  - The knots are equidistant / non-equidistant
  - The previous examples were uniform B-splines
    \( t_0, t_1, t_2, \ldots, t_m \) were equidistant, same interval

- Parametric interval between knots does not have to be equal.
  \[ \longrightarrow \text{Non-uniform B-splines} \]

Non-uniform B-splines.

- Blending functions no longer the same for each interval.
- Advantages
  - Continuity at selected control points can be reduced to \( C_1 \) or lower – allows us to interpolate a control point without side-effects.
  - Can interpolate start and end points.
  - Easy to add extra knots and control points.
    - Good for shape modelling
Controlling the shape of the curves

- Can control the shape through
  - Control points
    - Overlapping the control points to make it pass through a specific point
  - Knots
    - Changing the continuity by increasing the multiplicity at some knot (non-uniform bsplines)

Controlling the shape through control points

First knot shown with 4 control points, and their convex hull.

First two curve segments shown with their respective convex hulls.
Centre Knot must lie in the intersection of the 2 convex hulls.

Repeated control point.

First two curve segments shown with their respective convex hulls.
The curve is forced to lie on the line that joins the 2 convex hulls.

Triple control point.

First two curve segments shown with their respective convex hulls.
Both convex hulls collapse to straight lines – all the curve must lie on these lines.

Controlling the shape through knots

- Smoothness increases with order \( k \) in \( B_{i,k} \)
  - Quadratic, \( k = 3 \), gives up to \( C_1 \) continuity.
  - Cubic, \( k = 4 \) gives up to \( C_2 \) continuity.
- However, we can lower continuity order too with Multiple Knots, i.e. \( t_i = t_{i+1} = t_{i+2} = \ldots \) Knots are coincident and so now we have non-uniform knot intervals.
- A knot with multiplicity \( p \) is continuous to the \((k-1-p)th\) derivative.
- A knot with multiplicity \( k \) has no continuity at all, i.e. the curve is broken at that knot.
B-Splines at multiple knots

- Cubic B-spline
- Multiplicities are indicated

Knot multiplicity

- Consider the uniform cubic \((n=4)\) B-spline curve, \(t=\{0,1,\ldots,13\}\), \(m=9, n=4\), 7 segments

Knot multiplicity

- Double knot at 5,
  - knot \(=\{0,1,2,3,4,5,5,6,7,8,9,10,11,12\}\)
  - 6 segments, continuity \(= 1\)

Knot multiplicity

- Triple knot at 5
  - knot \(=\{0,1,2,3,4,5,5,5,6,7,8,9,10,11\}\)
  - 5 segments

Knot multiplicity

- Quadruple knot at 5
  - 4 segments

Summary of B-Splines.

- Functions that can be manipulated by a series of control points with \(C^2\) continuity and local control.
- Don’t pass through their control points, although can be forced.
- Uniform
  - Knots are equally spaced in \(t\)
- Non-Uniform
  - Knots are unequally spaced
  - Allows addition of extra control points anywhere in the set.
Summary cont.

• Do not have to worry about the continuity at the join points
• For interactive curve modelling
  – B-Splines are very good.

2nd Practical

• Use OpenGL to draw the teapot
• You must extend your code in the first assignment
• Bonus marks for making it nice
  – Bump mapping
  – Texture mapping
  – Or whatever

Deadline: 10th December

Reading for this lecture

• Foley at al., Chapter 11, sections 11.2.3, 11.2.4, 11.2.9, 11.2.10, 11.3 and 11.5.
• Introductory text, Chapter 9, sections 9.2.4, 9.2.5, 9.2.7, 9.2.8 and 9.3.