

# Computer Graphics

## Lecture 8

### Antialiasing, Texture Mapping

# Today

- Texture mapping
- Antialiasing
- Antialiasing-textures

# Texture Mapping : Why needed?

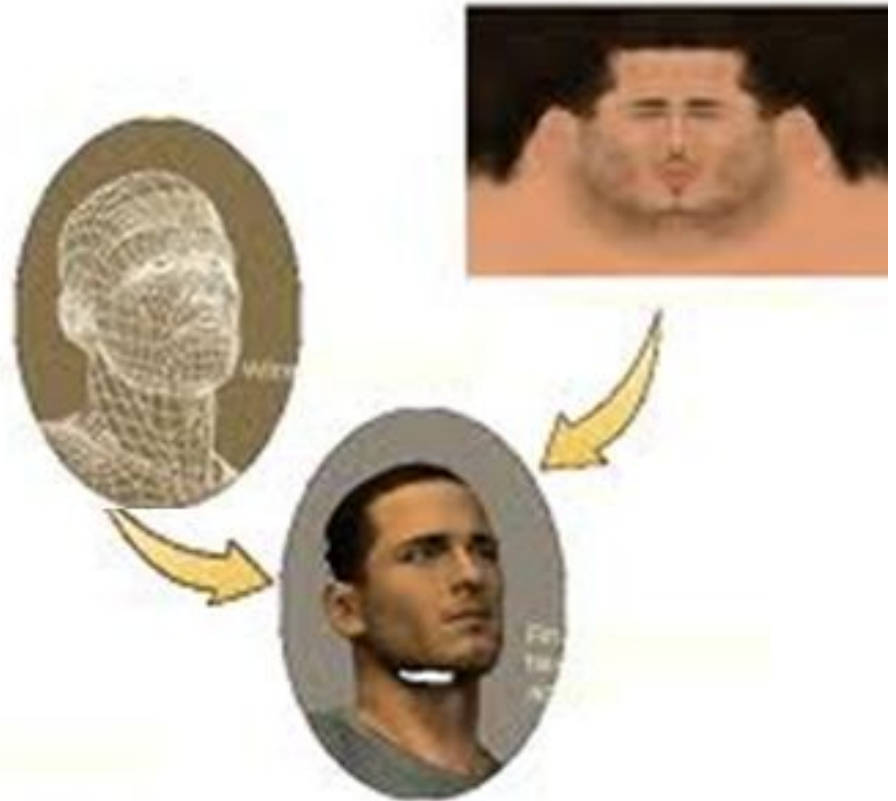
- Adding details using high resolution polygon meshes is costly
- Not very suitable for real-time applications



by Rami Ali Al-ashqar

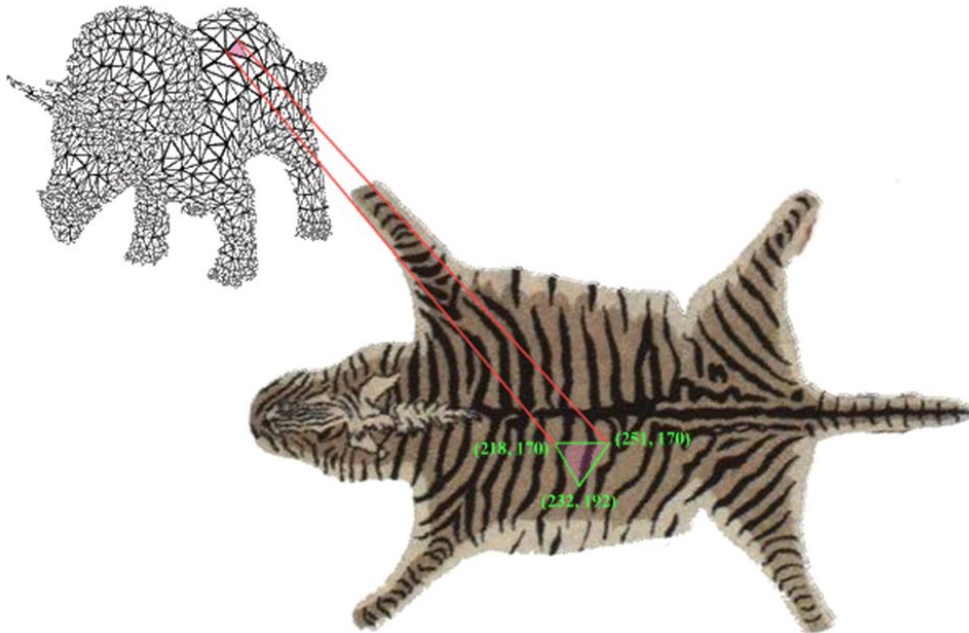
# Texture mapping.

- Method of improving surface appearance by mapping images onto the surface
- Done at the rasterization stage



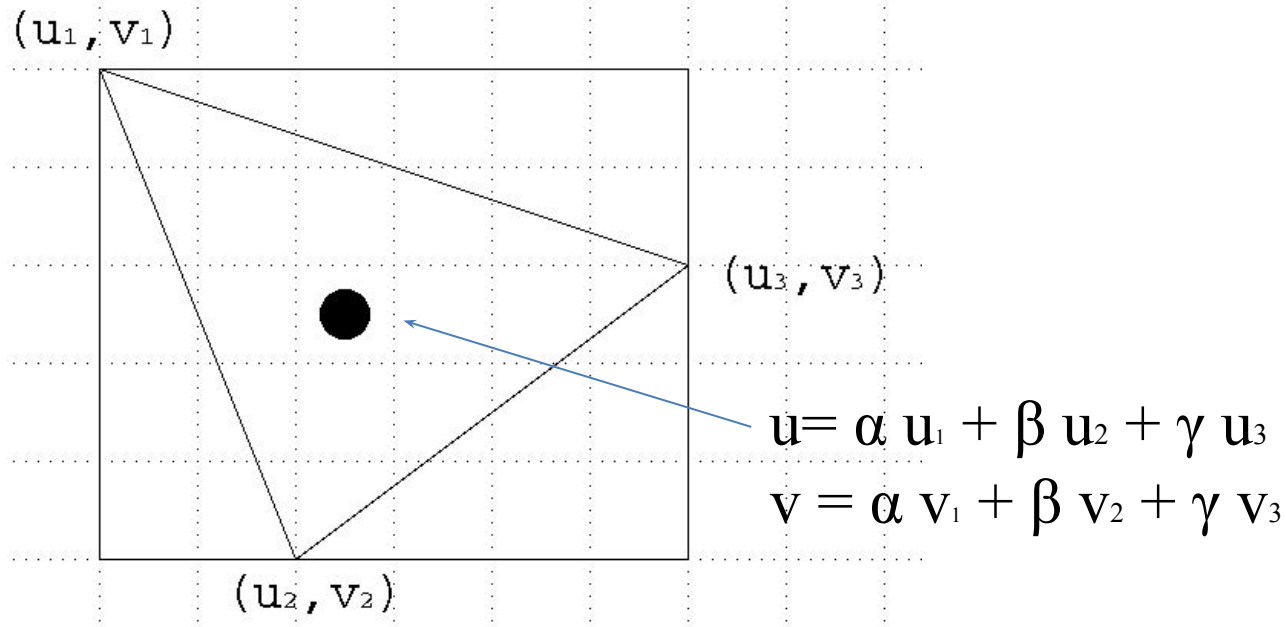
# Principles of Texture Mapping

- For each triangle in the model, establish a corresponding region in the texture image
- Image has much higher resolution than mesh
- During the rasterization, color the surface by that of the texture
- UV coordinates: the 2D coordinates of the texture map

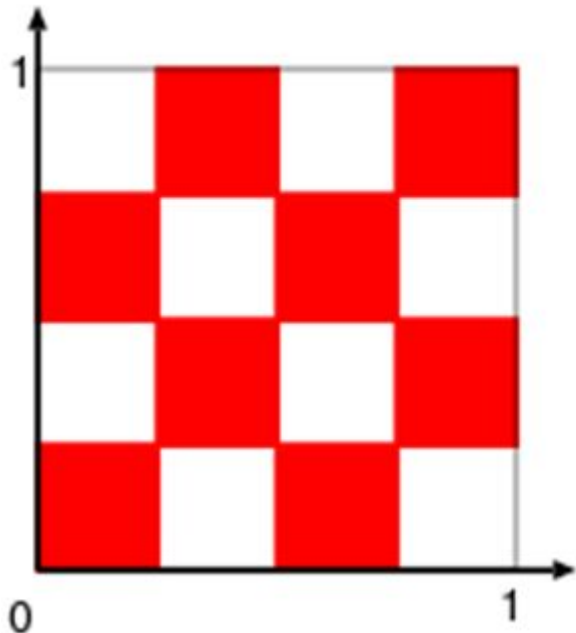


# Interpolating the uv coordinates

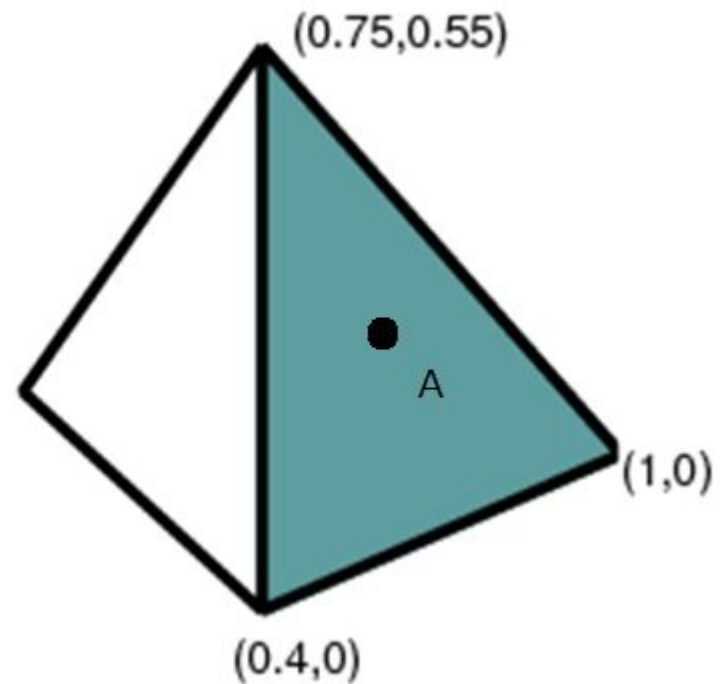
- To compute the UV inside the triangle, interpolate those at the vertices
- Done at the rasterization stage
- Use barycentric coordinates



1. What is the color of the 3 vertices of the triangle?
2. How will the textured triangle look like?



uv coordinate of triangle



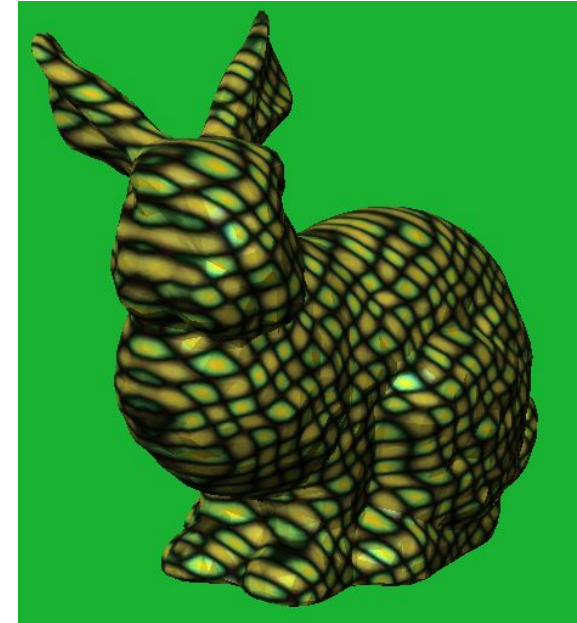
# How to Produce a UV Mapping?

- Use common mappings
  - Orthogonal, cylindrical, spherical
- Capturing the real data
  - Obtain the color as well as the 3D shape
- Manually specify the correspondence
  - Using graphical tools
  - Using automatic segmentation, unfolding



# Common Texture Coordinate Mappings

- Orthogonal
- Cylindrical
- Spherical



# Capture Real Data

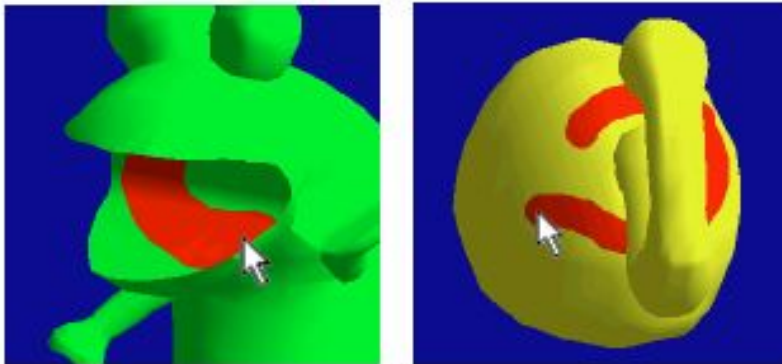
- Capture the depth as well as the color



<http://www.3dface.org/media/images.html>

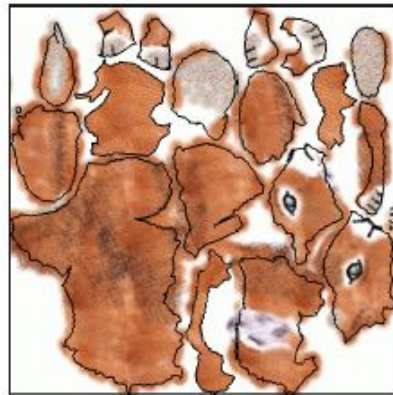
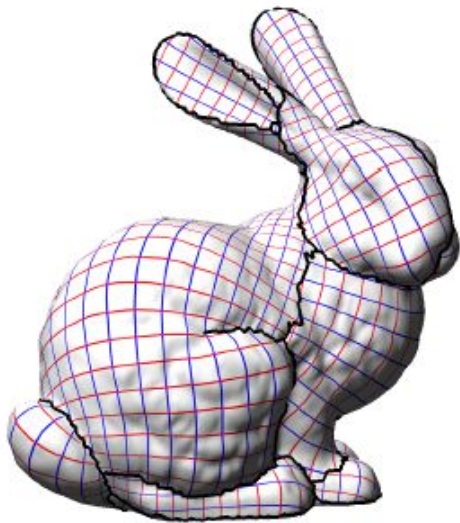
# Manually specifying the mapping

- There are good tools to map the texture to the surface, for example,
  - Directly painting on 3D geometry
  - Manually align unfolded data on the image



# Unfolding the geometry

- Segmenting the body into a set of charts
- Unfolded onto a 2D plane
- Packing: The charts are gathered in texture space
- The artist can then draw the texture on a 2D plane



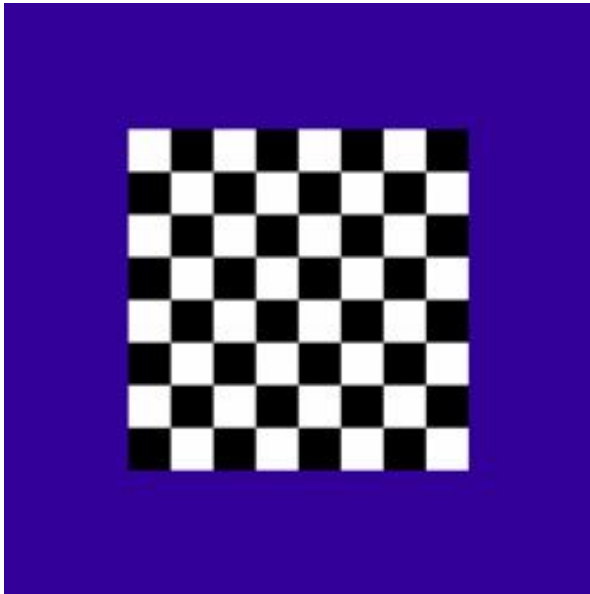
Creating a UV map on Maya

Levy et al. SIGGRAPH 2002

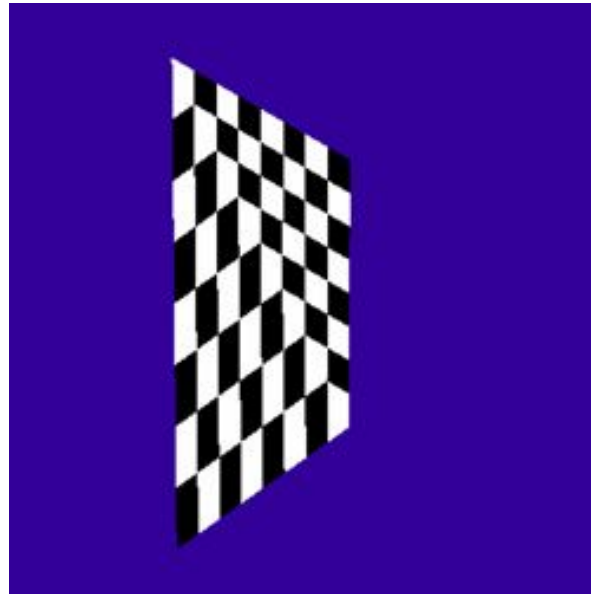
<http://www.youtube.com/watch?v=HqKDADeuTfc>

# Problems with Linear Interpolation of UV Coordinates

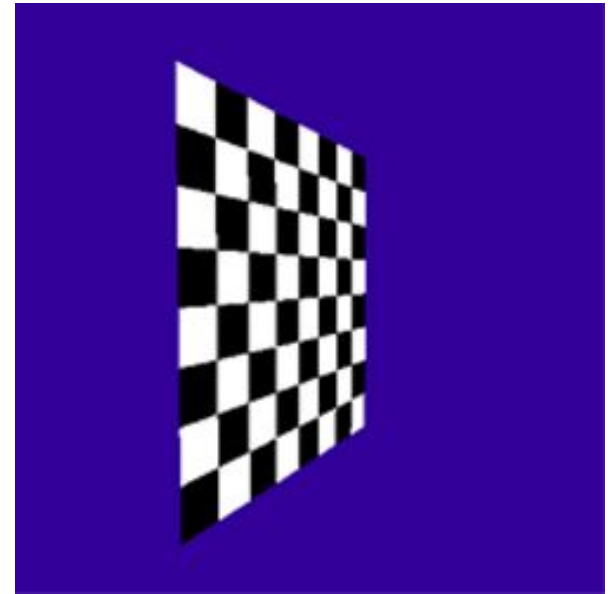
- Linear interpolation in screen space:



texture source



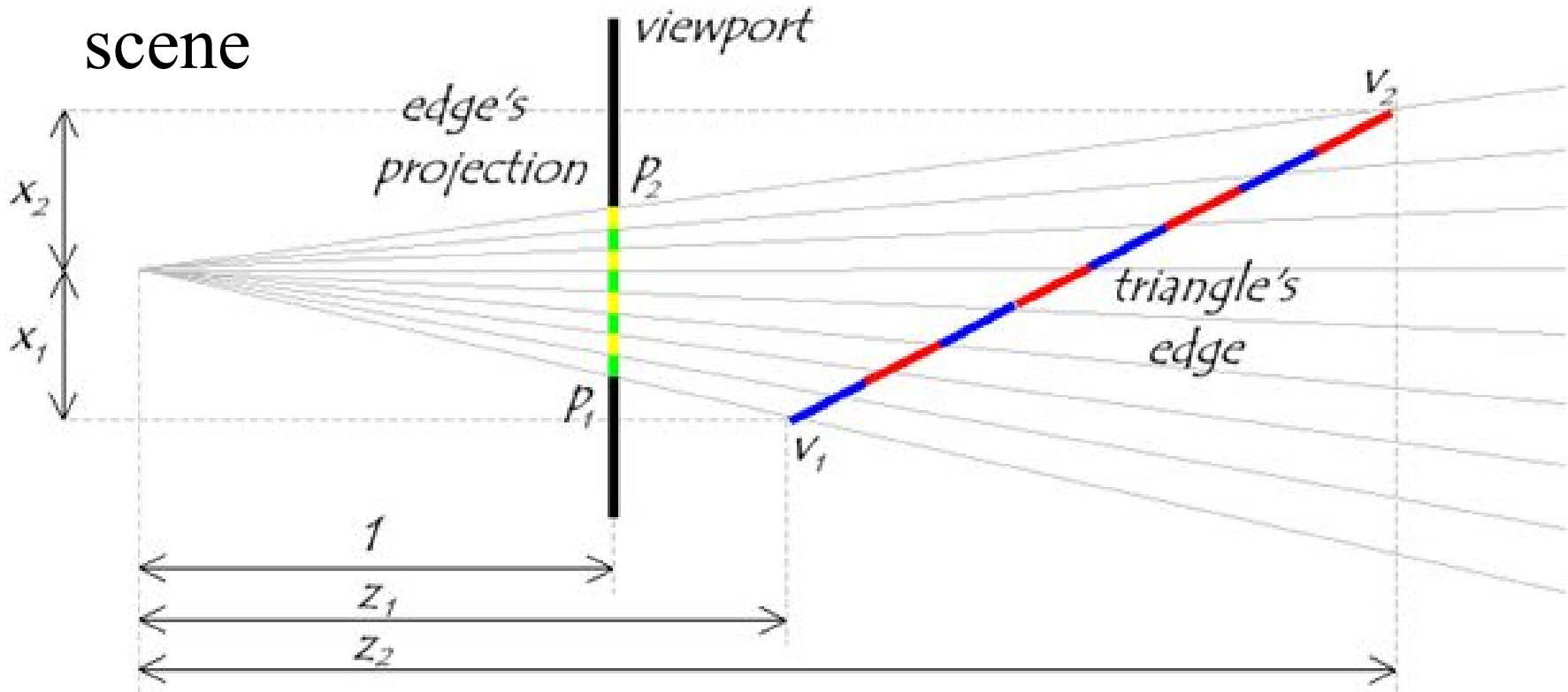
what we get|



what we want

# Why Does it Happen?

- Uniform steps on the image plane does not correspond to uniform steps in the original 3D scene

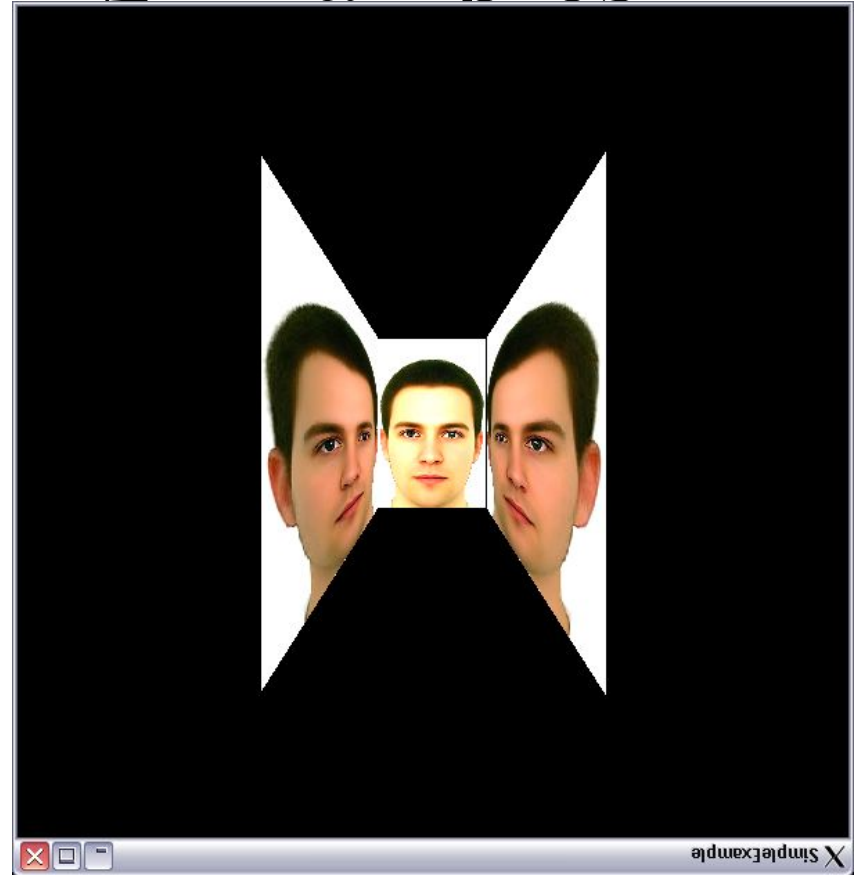


# Solution : Hyperbolic Interpolation

- $(u, v)$  cannot be linearly interpolated, but  $1/w$  and  $(u/w, v/w)$  can
- Compute  $(u, v)$  by dividing the interpolated  $(u/w, v/w)$  by the interpolated  $1/w$
- $w$  is the last component after the canonical view transformation
- A  $w$  is computed for every 3D vertex

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ w \end{bmatrix} = \begin{bmatrix} 2n & 0 & r+l & 0 \\ r-l & 0 & r-l & 0 \\ 0 & \frac{2n}{t-b} & -\frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{f+n}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Texture Mapping Examples



- Linear interpolation vs. Hyperbolic interpolation
- Two triangles per square



# Questions

1. In what case hyperbolic interpolation is needed?
2. Then, how can we solve the problem without using hyperbolic interpolation?

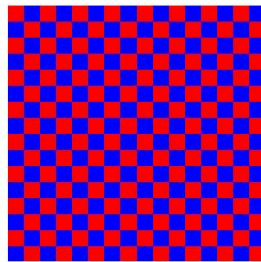
# Texture Mapping & Illumination

Texture mapping can be used to alter some or all of the constants in the illumination equation:

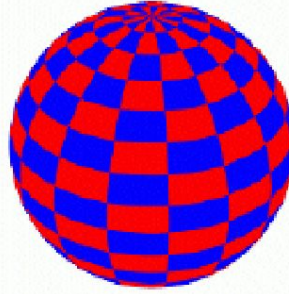
- Use the texture color as the diffuse color
  - more natural appearance



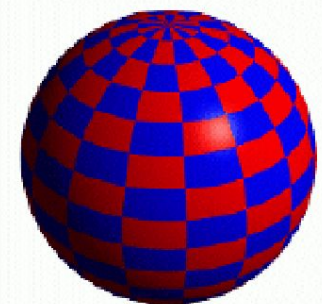
Constant Diffuse Color



Diffuse Texture Color



Texture used as Label



Texture used as Diffuse Color

# Today

- Texture mapping
- Antialiasing
- Antialiasing textures

# Anti-aliasing

- Aliasing: distortion artifacts produced when representing a high-resolution signal at a lower resolution.
- Anti-aliasing : techniques to remove aliasing



**Aliased  
polygons  
(jagged edges)**

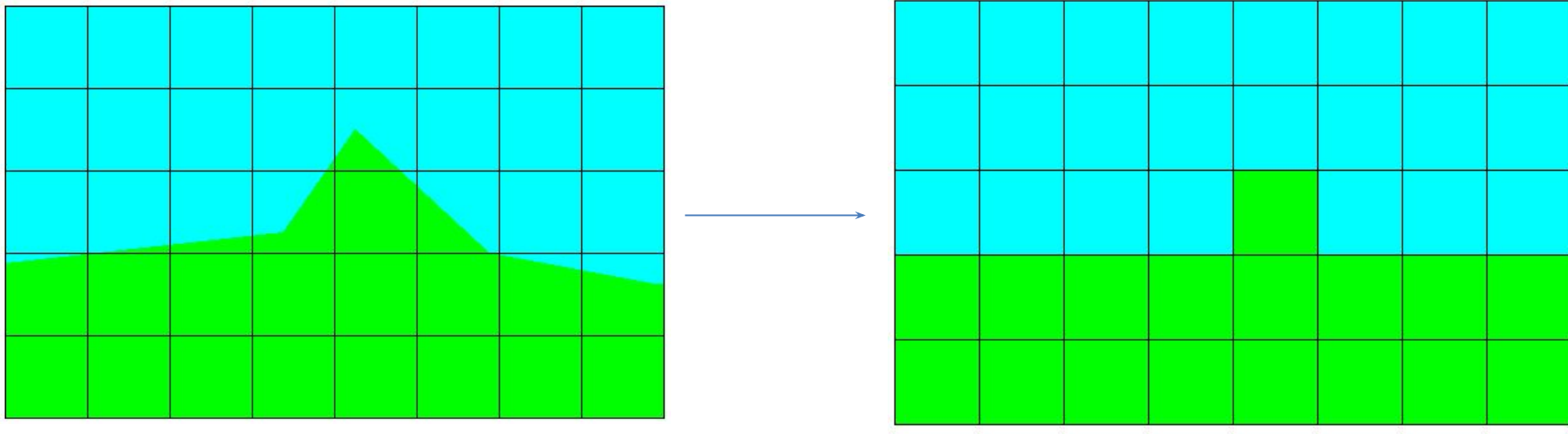


**Anti-aliased  
polygons**

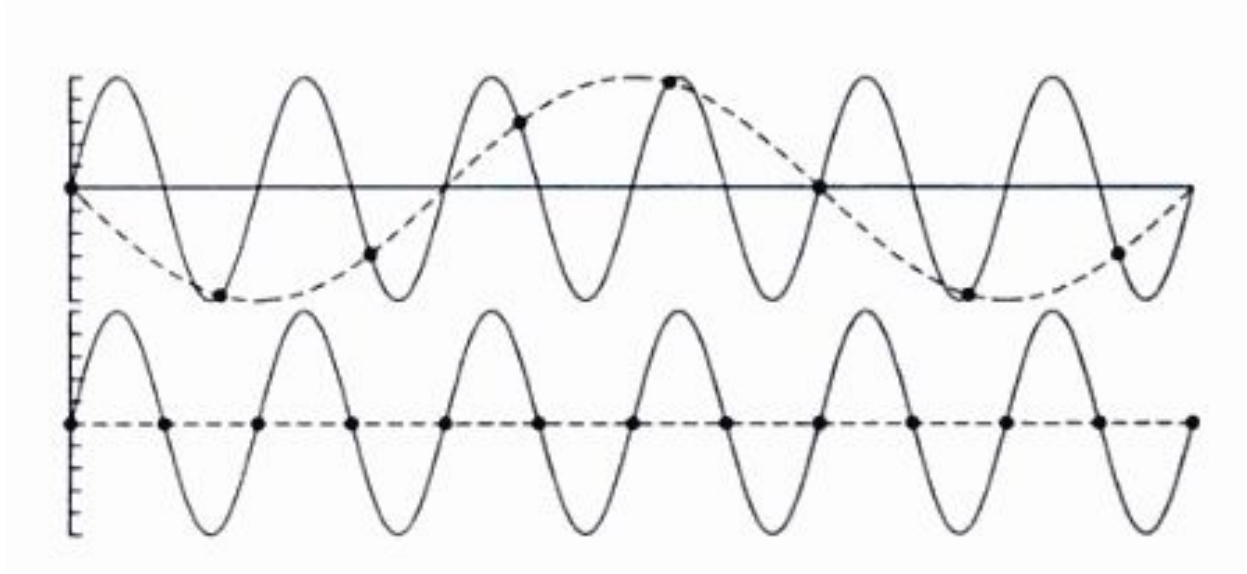
# Why Does Aliasing Happen?

Sampling frequency is too low with respect to the signal frequency

In the example below, the pixel size is too large compared to the original scene



# Nyquist Limit



- The signal frequency ( $f_{\text{signal}}$ ) should be no greater than half the sample frequency ( $f_{\text{sample}}$ )
- $f_{\text{signal}} < 0.5 f_{\text{sample}}$
- In the top,  $f_{\text{signal}} = 0.8 f_{\text{sample}}$  -> cannot reconstruct the original signal
- In the bottom  $f_{\text{signal}} = 0.5 f_{\text{sample}}$  -> the original signal can be reconstructed by slightly increasing the sampling rate

# Wagon-wheel Effect (temporal aliasing)



# Anti-aliasing by Subsampling

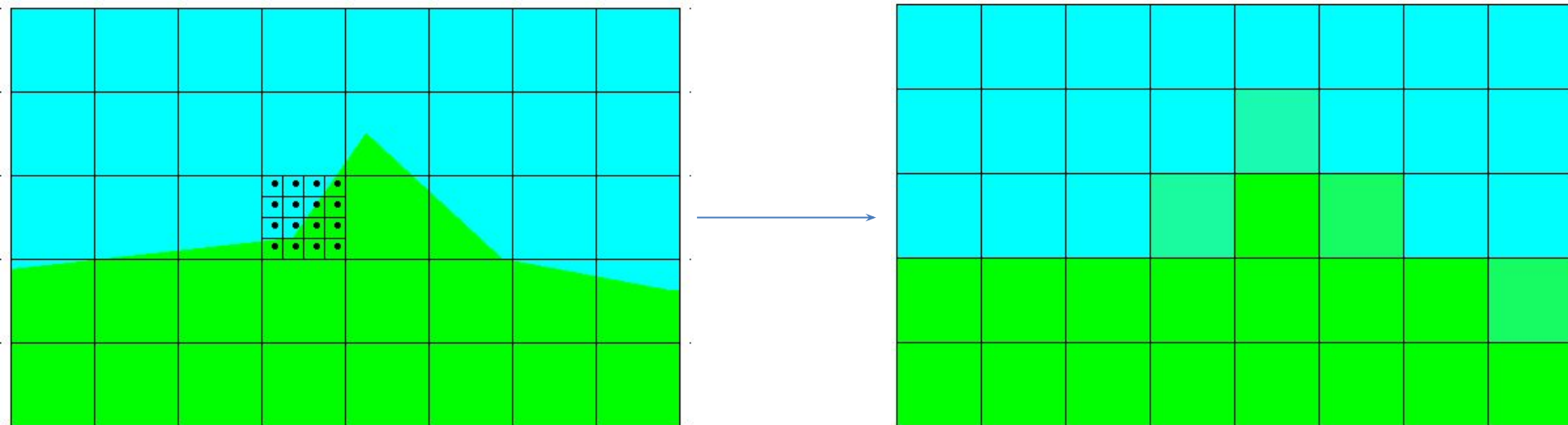
1. Each pixel is subdivided (sub-sampled) into  $n$  regions
2. Obtain the color at each subpixel
3. Compute the average color

$$p(x, y) = \sum_{i=1}^n w_i c(i, x, y)$$

$p(x, y)$  : pixel color at  $(x, y)$

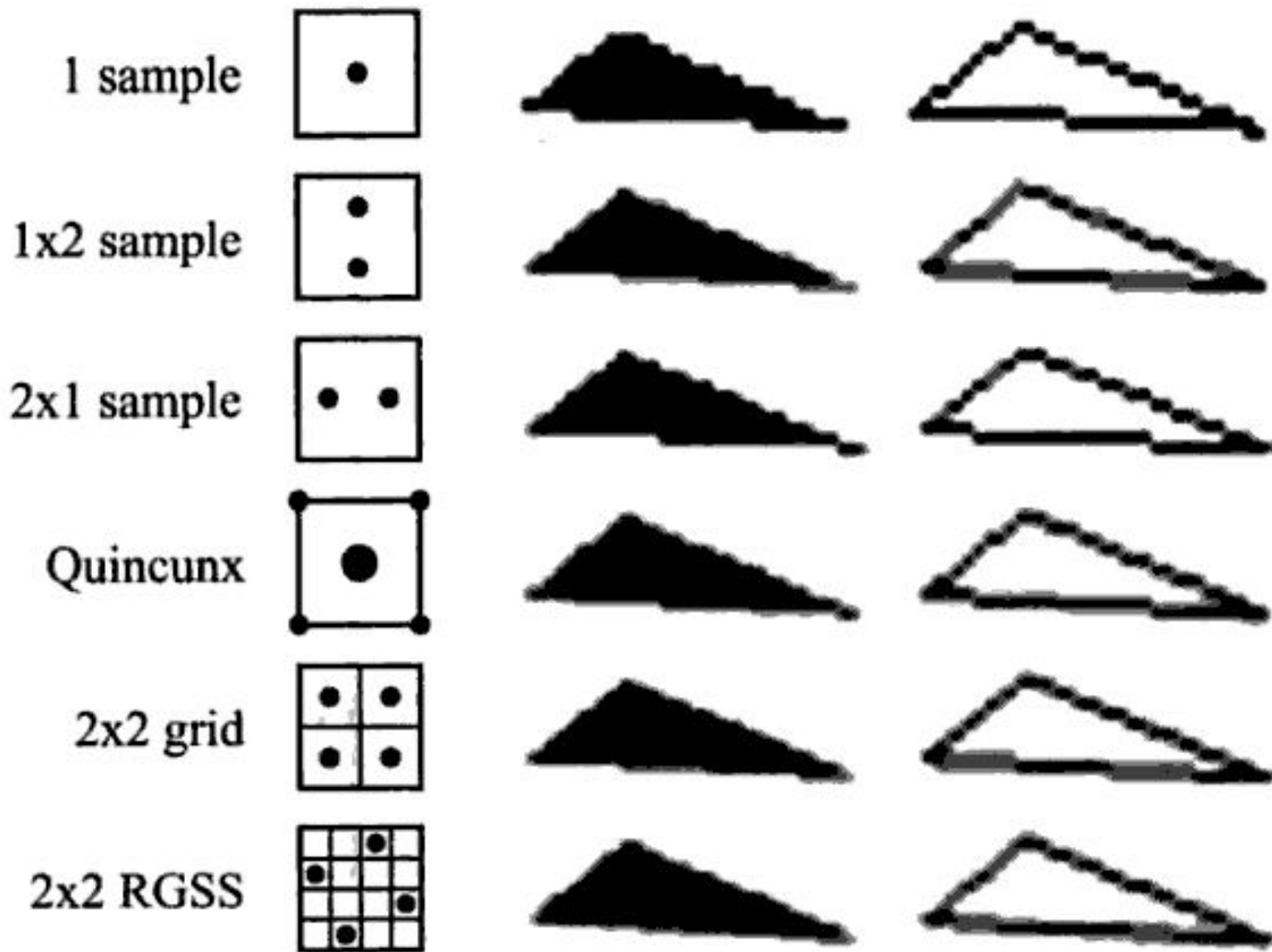
$c(i, x, y)$  : sample color

$w_i$  : weight

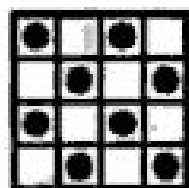




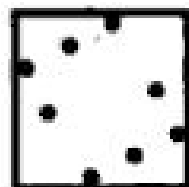
# Different Sampling Schemes



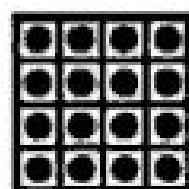
4x4 checker



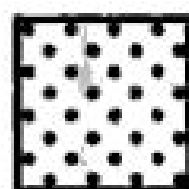
8 rooks



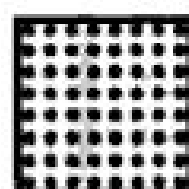
4x4 grid



8x8 checker

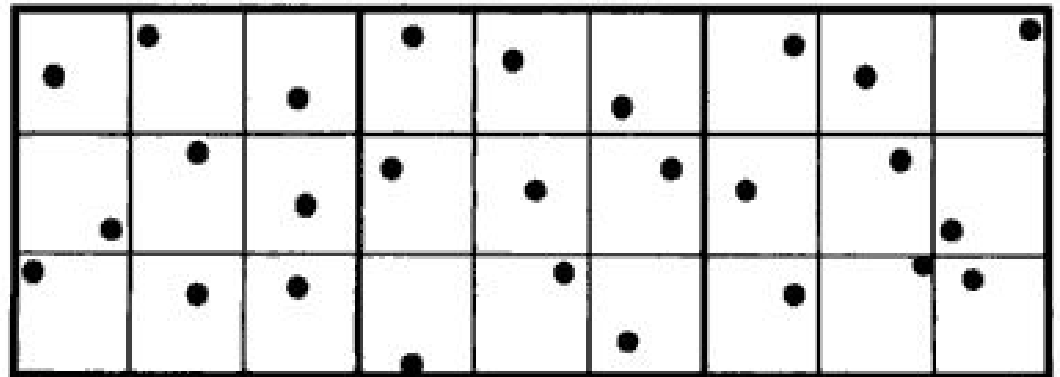


8x8 grid



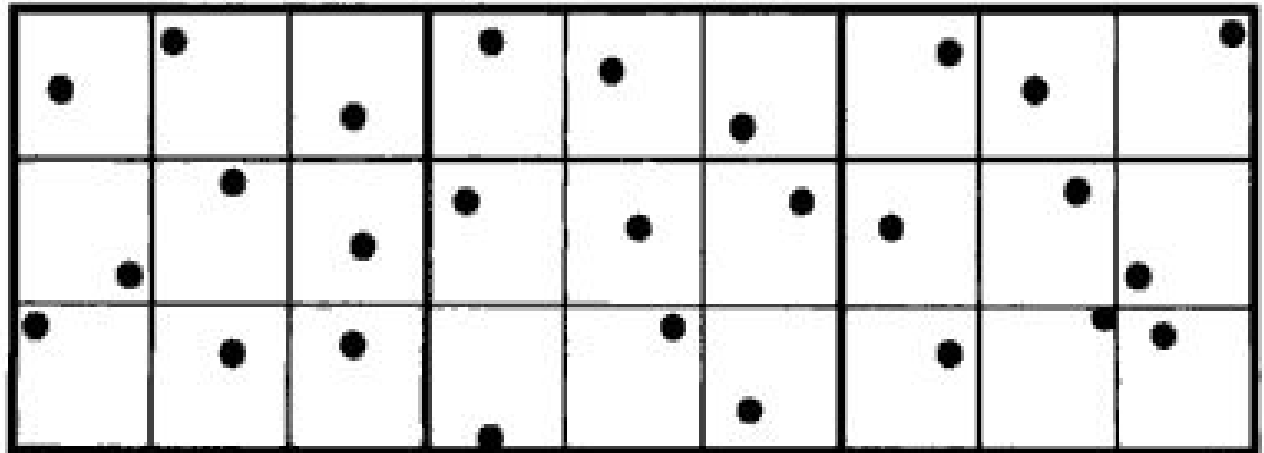
# Stochastic Sampling

- A scene can be produced of objects that are arbitrarily small
- A regular pattern of sampling will exhibit some sort of aliasing
- By irregular sampling, the higher frequencies appear in the image as noise rather than aliases
- Humans are more sensitive to aliases than noise



# Stochastic Sampling

- One approach to solve this is to randomly sample over the pixels
- Jittering : subdivide into  $n$  regions of equal size and randomly sample inside each region
- Compute the colour at the sample and average
- Can precompute a table and recycle it or simply jitter on the fly



# Comparison

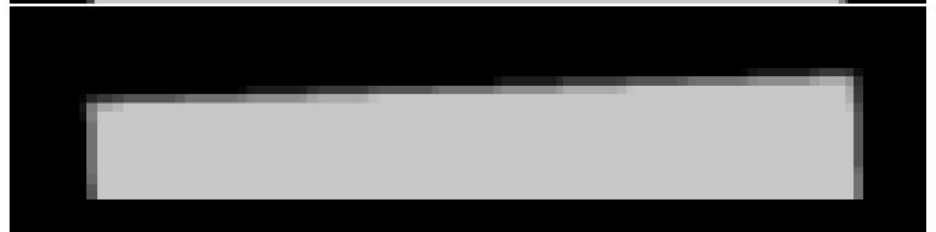
**Regular, 1x1**



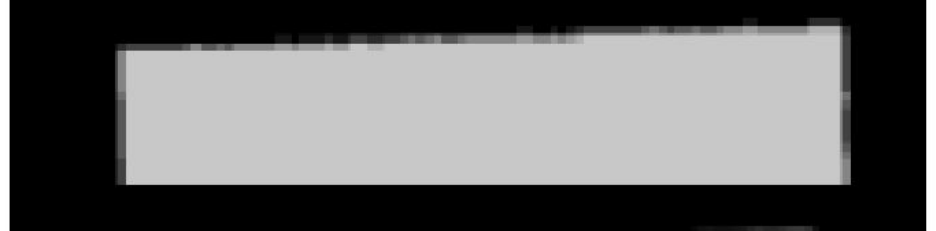
**Regular 3x3**



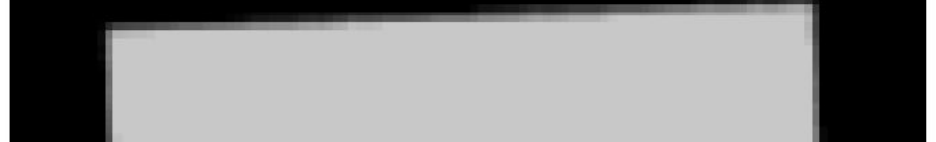
**Regular, 7x7**



**Jittered, 3x3**

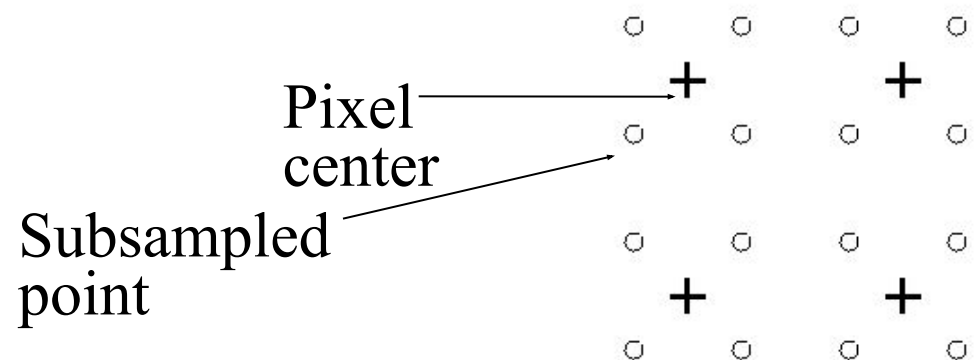


**Jittered, 7x7**



# Accumulation Buffer (A-Buffer)

- Use a buffer that has the same resolution as the original image
- To obtain a  $2 \times 2$  sampling of a scene, 4 images are made by shifting the frame buffer horizontally/vertically for half a pixel
- The results are accumulated and the final results are obtained by averaging
- We can recycle the vertex attributes



# Today

- Texture mapping
- Antialiasing
- Antialiasing textures

# Aliasing of textures

Happens when the camera is zoomed too much into the textured surface (magnification)

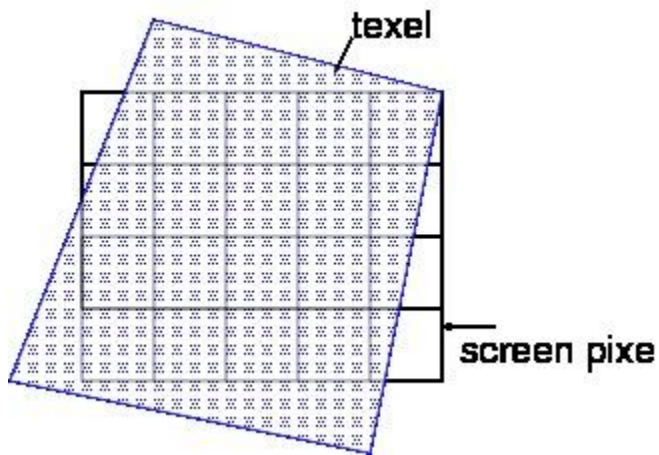
Several texels (pixels of the texture) covering a pixel's cell (minification)



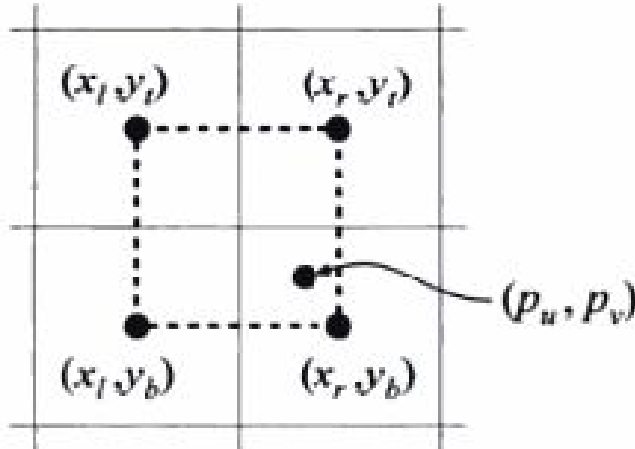


# Texture Magnification

Zooming too much into a surface with a texture  
One texel covering many pixels



# Bilinear Interpolation

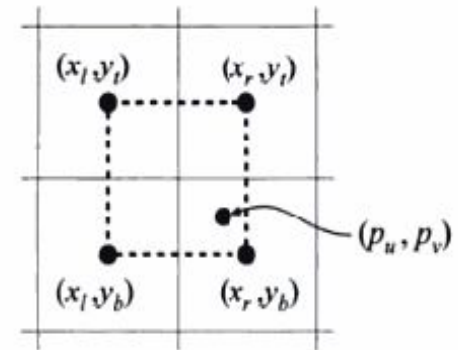


- Mapping the pixel centre to the uv coordinates
- Computing the pixel colour by interpolating the surrounding texel values

# Bilinear Interpolation - 2

- For  $(p_u, p_v)$ , compute its  $(u, v)$  coordinates by barycentric coordinates
- $u = p_u - (\text{int})p_u$ ,  $v = p_v - (\text{int})p_v$

$$\mathbf{b}(p_u, p_v) = (1 - u')(1 - v')\mathbf{t}(x_l, y_b) + u'(1 - v')\mathbf{t}(x_r, y_b) \\ + (1 - u')v'\mathbf{t}(x_l, y_t) + u'v'\mathbf{t}(x_r, y_t).$$



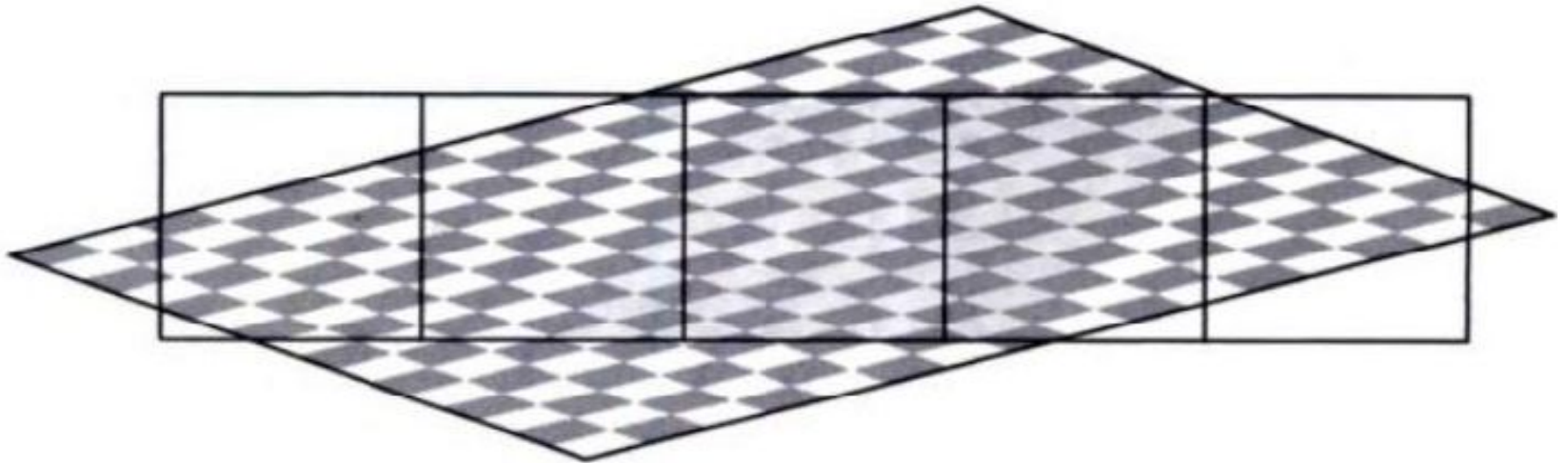
- $(p_u, p_v)$  : the pixel centre mapped into the texture space
- $\mathbf{b}(p_u, p_v)$  : the colour at point  $p_u, p_v$
- $\mathbf{t}(x, y)$  : the texel colour at  $(x, y)$
- $u = p_u - (\text{int})p_u$ ,  $v = p_v - (\text{int})p_v$

# Texture Minification

Many texels covering a pixel's cell

Results in aliasing (remember Nyquist limit)

The artifacts are even more noticeable when the surface moves

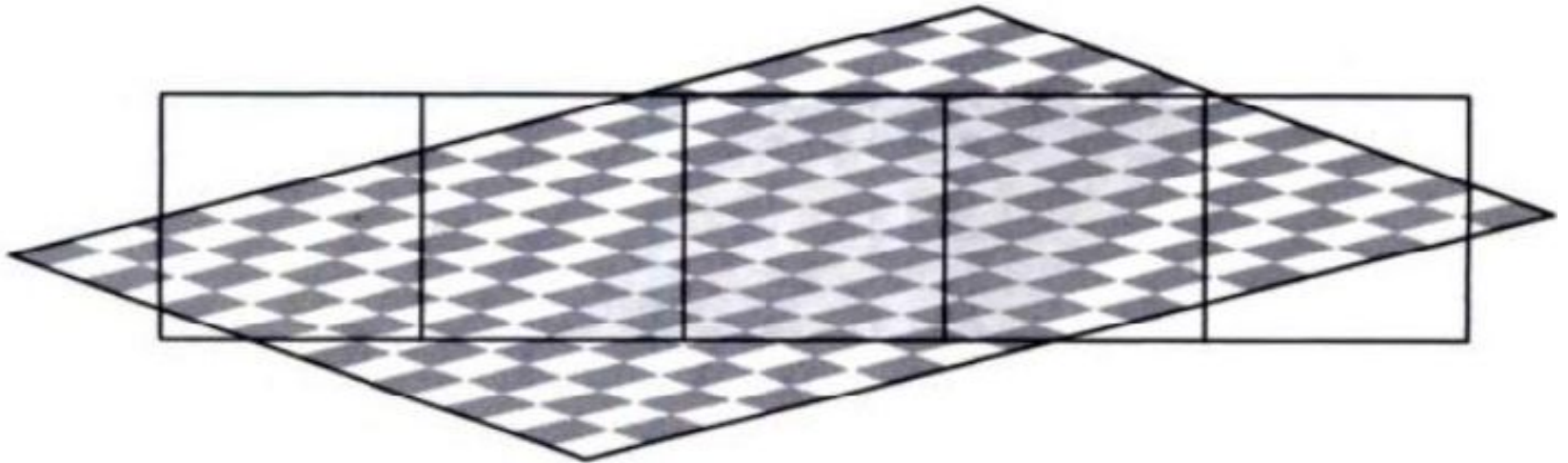


# Texture Minification (2)

We can do subsampling as before

But actually we know the texture pattern in advance

→ Mipmapping



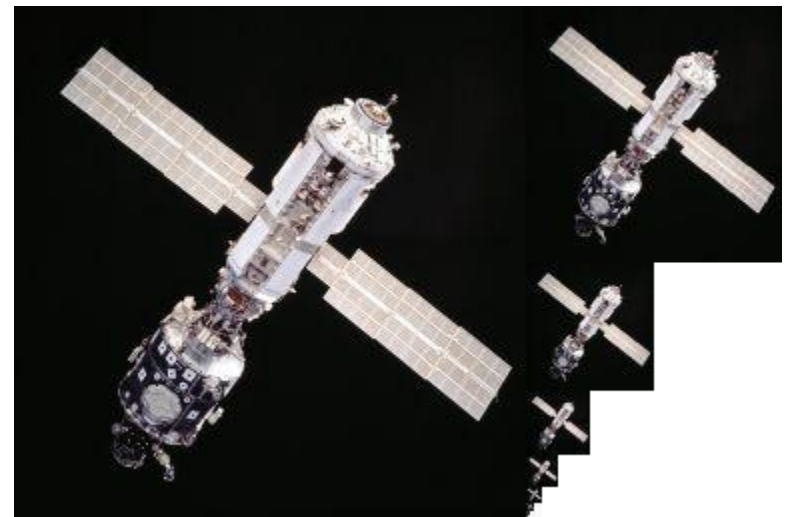
# MIP map

*Multum In Parvo = Many things in a small place*

Produce a texture of multiple resolutions

Switch the resolution according to the number of texels in one pixel

Select a level that the ratio of the texture and the pixel is 1:1



# Selecting the resolution in Mipmap

Map the pixel corners to the texture space

Find the resolution that roughly covers the mapped quadrilateral

Apply a bilinear interpolation in that resolution,

Or find the two surrounding resolutions and apply a trilinear interpolation (also along the resolution axis)

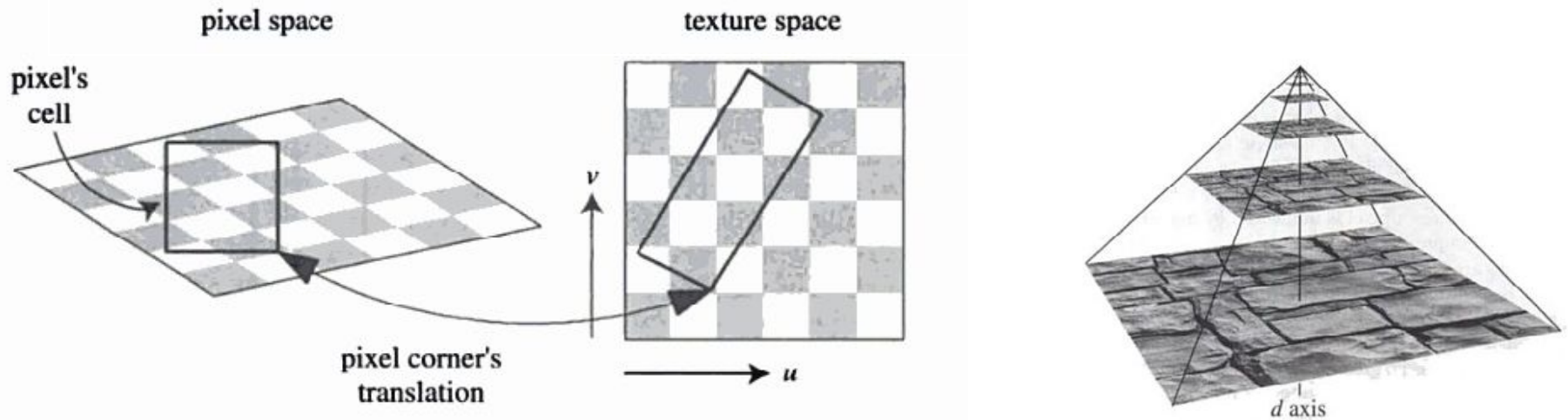
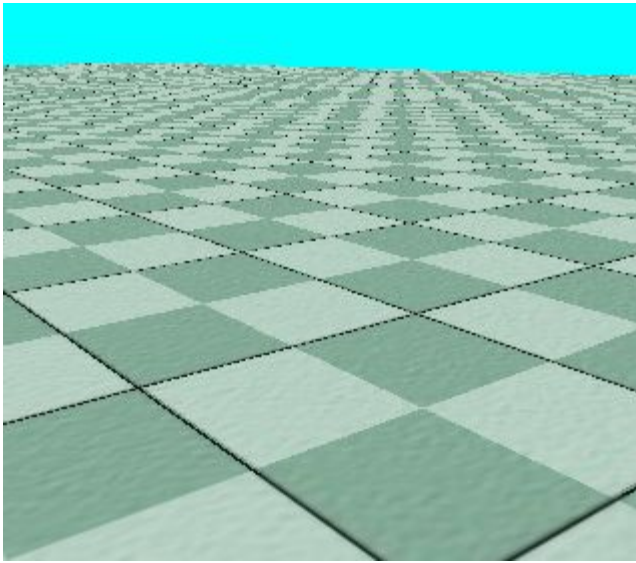


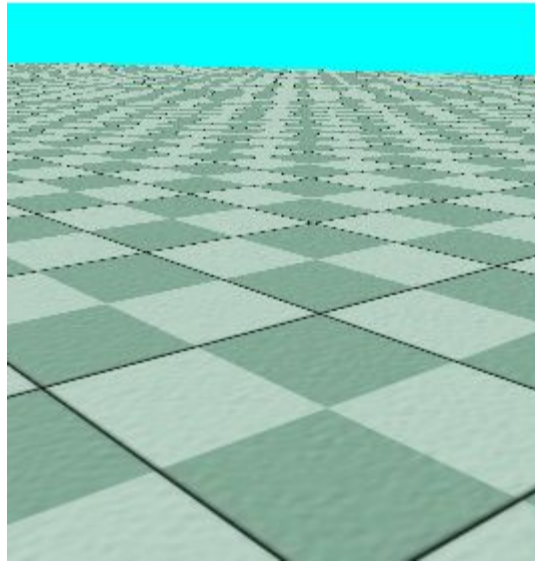
Figure 5.13. On the left is a square pixel cell and its view of a texture. On the right is the projection of the pixel cell onto the texture itself.

# Texture Minification

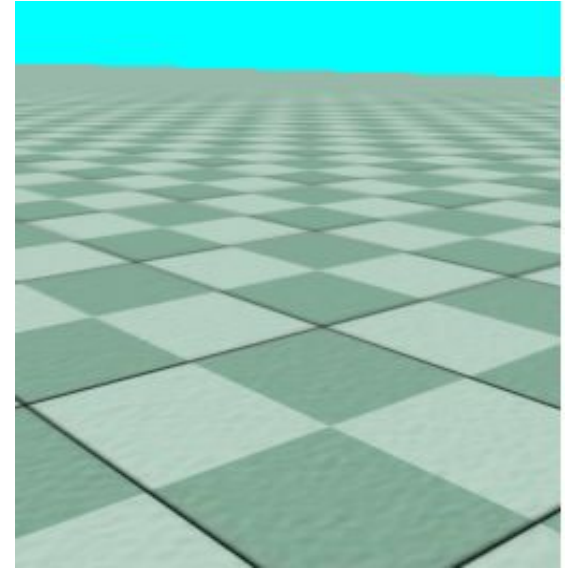
Multiple textures in a single pixel  
Solution:



Nearest neighbour



Bilinear blending



Mipmapping



# Summary

1. Texture mapping
  - a. Different ways to synthesize the texture maps
  - b. Hyperbolic interpolation
2. Antialiasing
  - a. Nyquist limit
  - b. Subsampling
  - c. texture maps
    - i. magnification : bilinear interpolation
    - ii. minification : mipmapping

# Reading List

Shirley et al. Chapter 11.

Akenine-Moller Chapter 5.6, 6.2

[http://books.google.co.uk/books?id=V1k1V9Ra1FoC&pg=PA418&dq=real-time+rendering+texture+mapping&hl=ja&sa=X&ei=ErdXUtCBBsjA0QWazYHoAw&redir\\_esc=y#v=onepage&q=real-time%20rendering%20texture%20mapping&f=false](http://books.google.co.uk/books?id=V1k1V9Ra1FoC&pg=PA418&dq=real-time+rendering+texture+mapping&hl=ja&sa=X&ei=ErdXUtCBBsjA0QWazYHoAw&redir_esc=y#v=onepage&q=real-time%20rendering%20texture%20mapping&f=false)