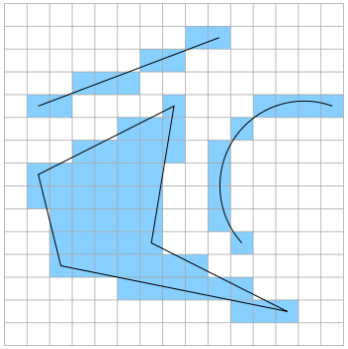
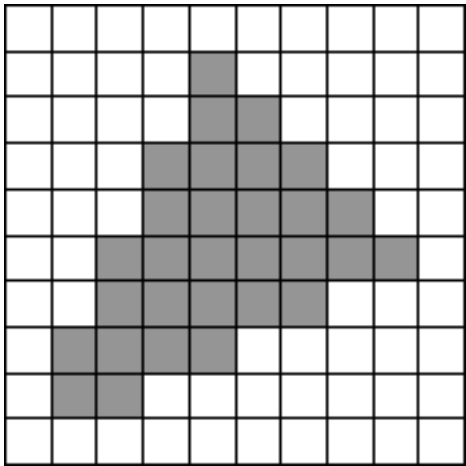
Computer Graphics

Lecture 7
Rasterization
Taku Komura

Rasterization

- After projection, the polygons are still in the continuous screen space
- We need to decide which pixels to lit how much
- This is called rasterization (or scan conversion)





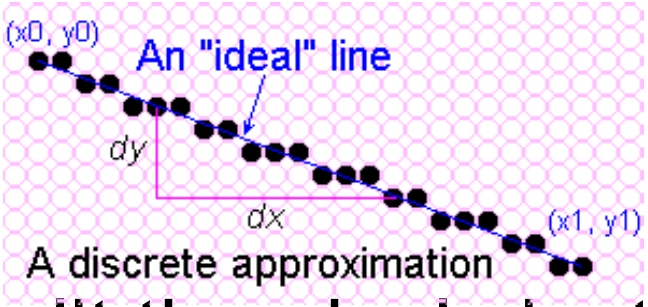
Overview

Rasterization

- Line Rasterization
- Polygon Rasterization
  - Scanline Algorithm
  - Rasterizing triangles
    - Edge walking
    - Interpolation by barycentric coordinates
- Mean value coordinates
- Dividing polygons into triangles

Rasterizing Lines

Converting a continuous object in the world into a discrete object in the computer



We need to lit the pixels instead of drawing a continuous line

Simple Line

From linear algebra

$$y = m x + n$$

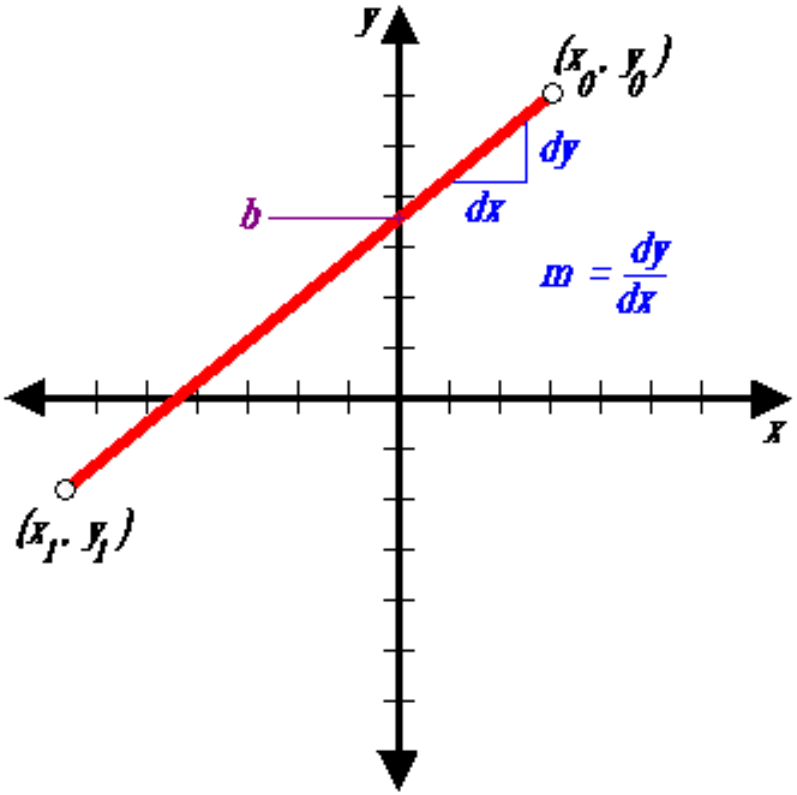
Simple approach:

increment x, calculate y

Then cast y to an integer

(x, (int)y)

Will this work?



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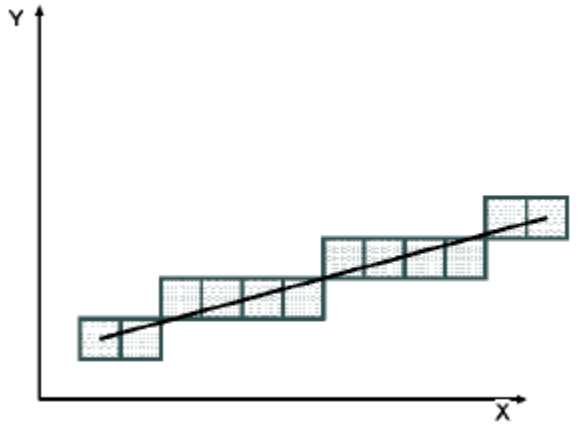
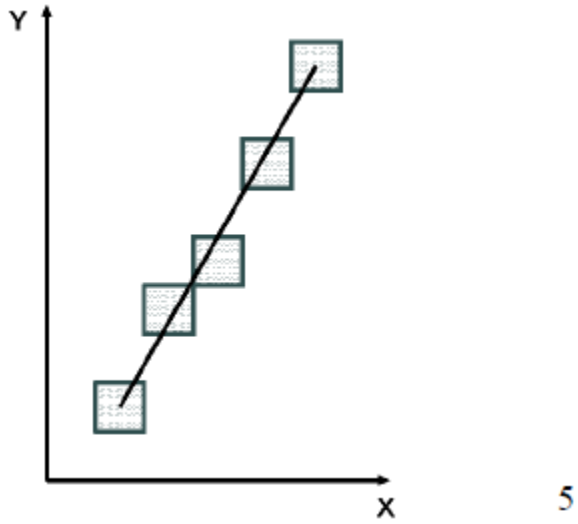
Does it Work?

It seems to work okay for lines with a slope of 1 or less,

But doesn't work well for lines with slope greater than 1

- Lines become more discontinuous in appearance
- We must add more than 1 pixel per column to make it work.





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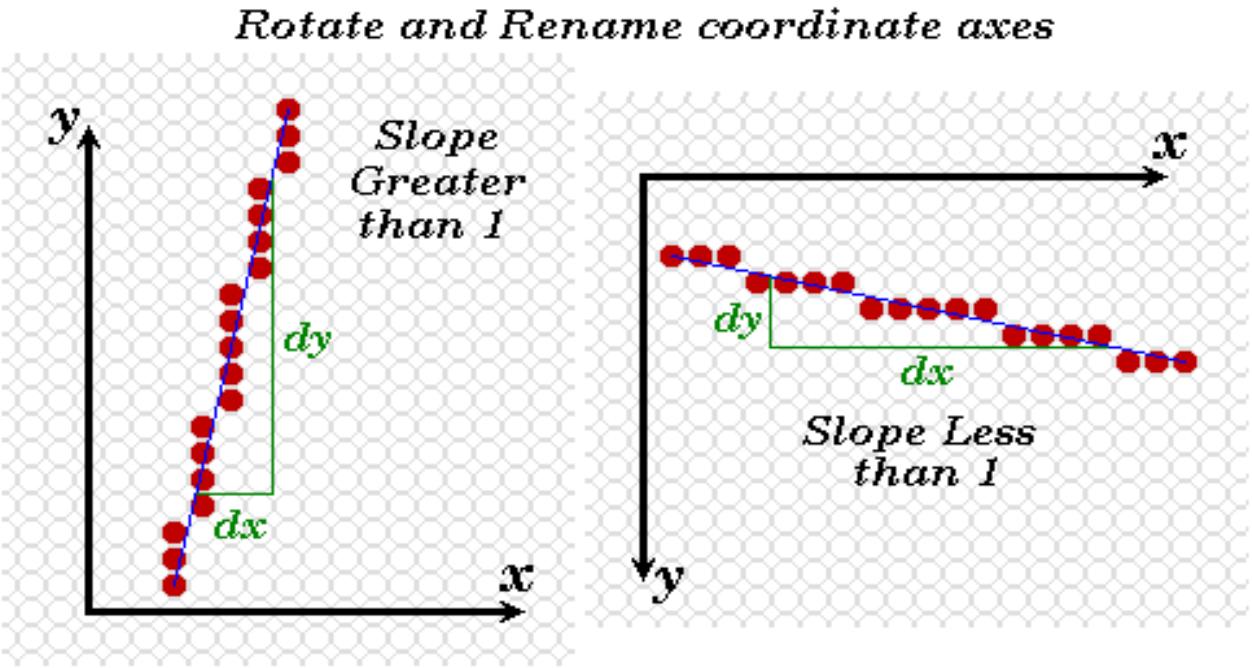
Use Symmetry



Increment along x-axis if dy<dx else increment along y-axis

.**But still we need to do a lot of floating point arithmetic!**

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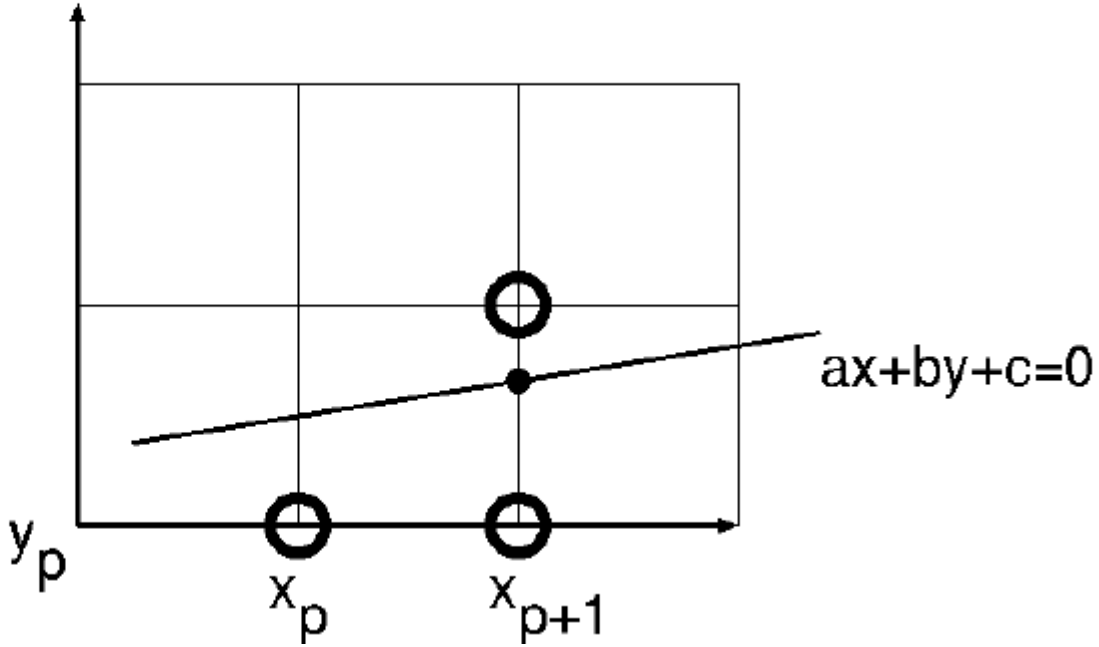
Midpoint Algorithm: Overview

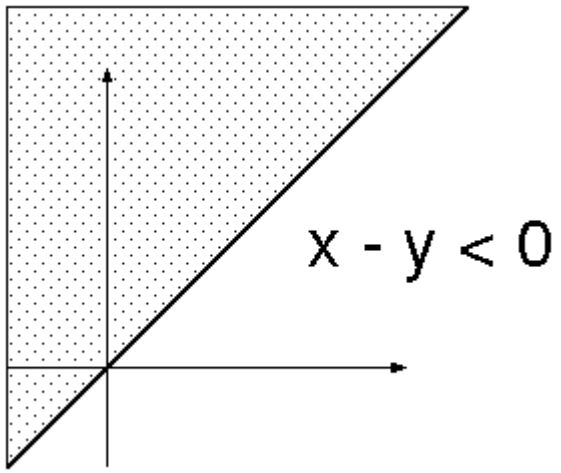
- At step p, assume pixel (x_p, y_p) was lit
- Check where the line intersects with $x = x_{p+1}$
- Lit (x_{p+1}, y_p) or $(x_{p+1}, y_p + 1)$, whichever is closer to the intersection



Testing above or below a line

$F(x,y) = ax + by + c = 0.$

If $b < 0$, $F > 0$ if the line is above the point $F < 0$ if the line is below the point



Testing for the side of a line.

- Assume the line is connecting $(x_l, y_l), (x_r, y_r)$
- The slope will be $\frac{dy}{dx}$ where $dx = x_r - x_l$, $dy = y_r - y_l$

$$y = mx + n \text{ and so } y = \frac{dy}{dx}x + n$$

$$F(x, y) = ax + by + c = 0$$

$$F(x, y) = dy.x - dx.y + c = 0$$

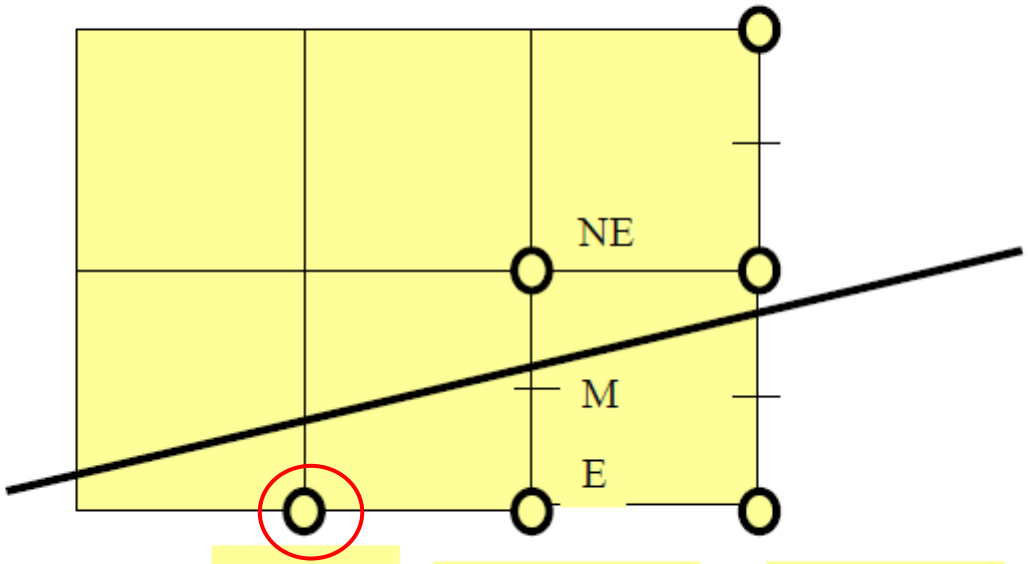
Decision variable.

Let's assume dy/dx < 1 (we can use symmetry)

Evaluate F at point M $\quad d = F(x_p + 1, y_p + \frac{1}{2}) = a(x_p + 1) + b(y_p + \frac{1}{2}) + c$

Referred to as *decision variable*

If $d \leq 0$, select E as the next pixel
otherwise select NE as the next pixel



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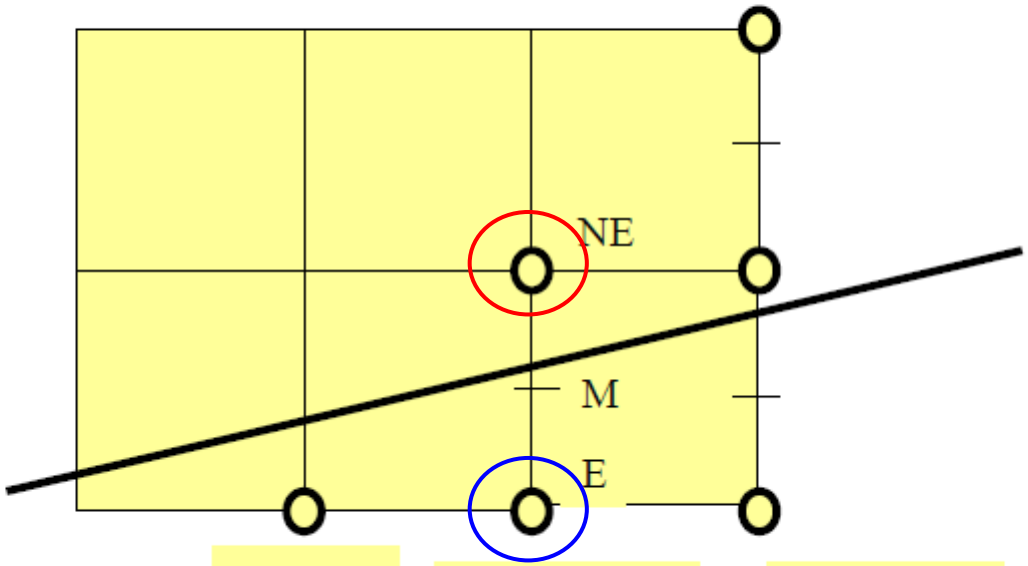

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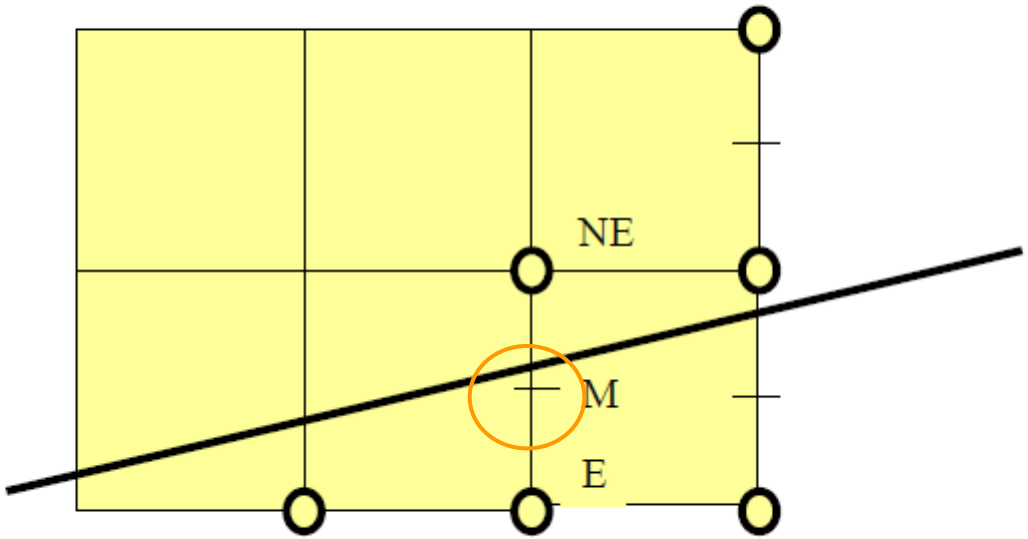
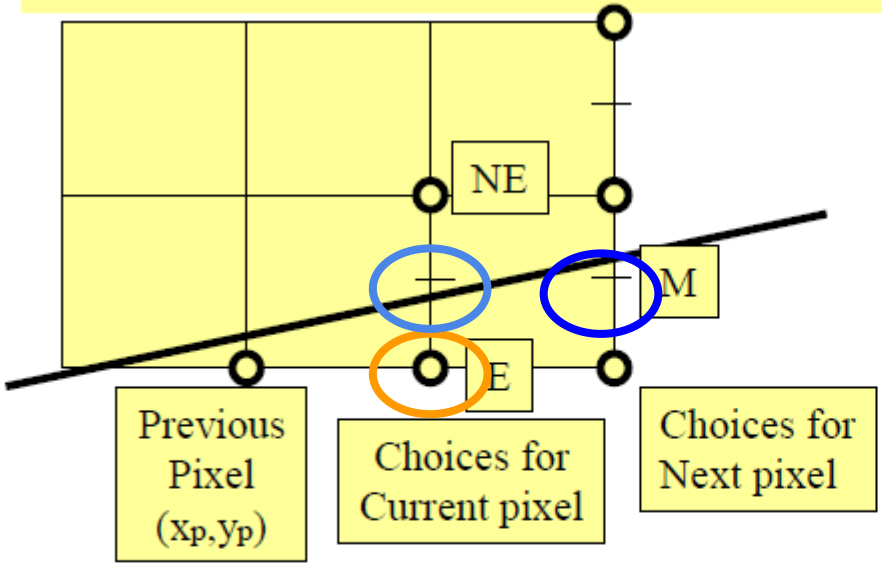


Updating the decision variable

Evaluate d for next pixel, Depends on whether E or NE Is chosen :

If E chosen :

$$d_{new} = F(x_p + 2, y_p + \frac{1}{2}) = a(x_p + 2) + b(y_p + \frac{1}{2}) + c$$



But recall :

$$d_{old} = F(x_p + 1, y_p + \frac{1}{2})$$

$$= a(x_p + 1) + b(y_p + \frac{1}{2}) + c$$

So :

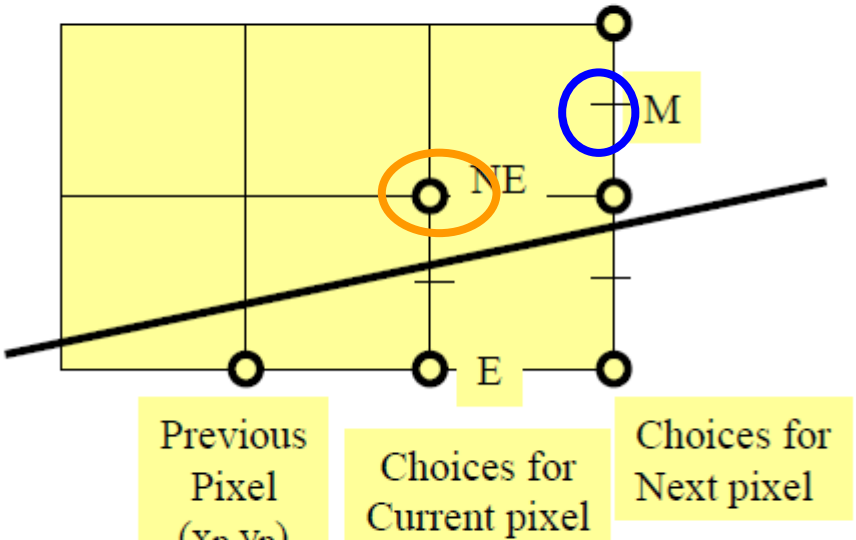
$$d_{new} = d_{old} + a$$

$$= d_{old} + dy$$

Decision variable.

If NE was chosen :

$$d_{new} = F(x_p + 2, y_p + \frac{3}{2}) = a(x_p + 2) + b(y_p + \frac{3}{2}) + c$$



But recall :

$$d_{old} = F(x_p + 1, y_p + \frac{1}{2})$$

$$= a(x_p + 1) + b(y_p + \frac{1}{2}) + c$$

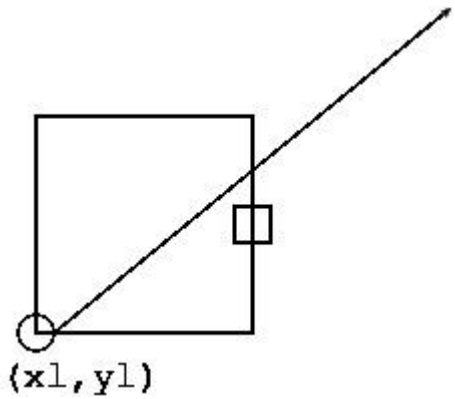
So :

$$d_{new} = d_{old} + a + b$$

$$= d_{old} + dy - dx$$

Initial value of d.

Start point is (x_1, y_1)



$$d_{start} = F(x_1 + 1, y_1 + \frac{1}{2}) = a(x_1 + 1) + b(y_1 + \frac{1}{2}) + c$$

$$= ax_1 + by_1 + c + a + \frac{b}{2}$$

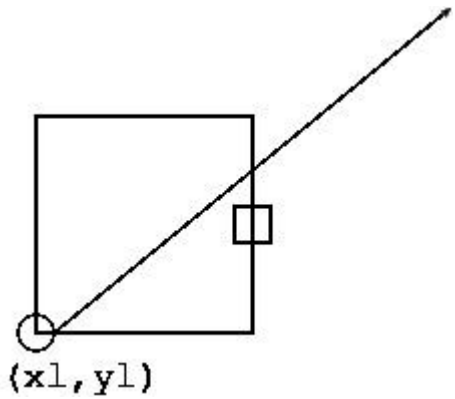
$$= F(x_1, y_1) + a + \frac{b}{2}$$

But (x_1, y_1) is a point on the line, so $F(x_1, y_1) = 0$

$$d_{start} = dy - dx/2$$

Initial value of d.

Start point is (x_1, y_1)



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But (x_1, y_1) is a point on the line, so $F(x_1, y_1) = 0$

$$d_{start} = dy - dx/2$$

Multiply by 2 to avoid floating point operations
(We are only using the sign of the decision variable)

Decision variable : summary

- New equation for decision variable
- Initialization
- If E point is selected
- If NE point is selected

$$d = F(x, y) = 2(ax + by + c)$$

$$d_{start} = 2dy - dx$$

$$d_{new} = d_{old} + 2dy$$

$$d_{new} = d_{old} + 2dy - 2dx$$

Only requires integer operations

Summary of mid-point algorithm

- Start point is simply first endpoint (x_l, y_l).
- Calculate the initial value for d
- Choose between 2 pixels at each step based upon the sign of a decision variable.
- Update the decision variable based upon which pixel is chosen.

- **No floating point arithmetic needed**

Midpoint algorithm

```
  void MidpointLine(int
x1,y1,x2,y2)
  {
  int dx=x2-x1;
  int dy=y2-y1;
  int d=2*dy-dx;
  int increE=2*dy;
  int incrNE=2*(dy-dx);
  x=x1;
  y=y1;
  WritePixel(x,y);
```

```
while (x < x2) {
    if (d<= 0) {
        d+=incrE;
        x++
    } else {
        d+=incrNE;
        x++;
        y++;
    }
    WritePixel(x,y);
}
}
```

Overview

Rasterization

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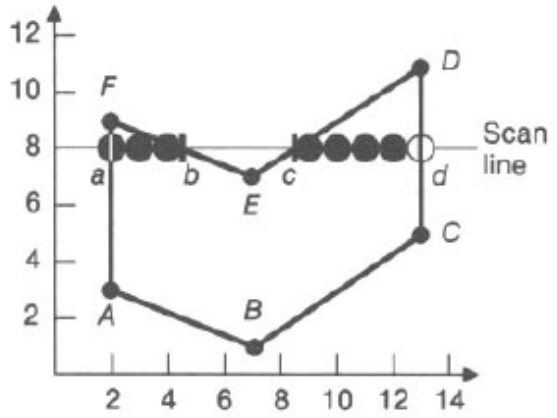
- **Polygon Rasterization**
  - Scanline Algorithm
  - Rasterizing triangles
    - Edge walking
    - Interpolation by barycentric coordinates
- Mean value coordinates
- Dividing polygons into triangles

Scanline algorithm

A traditional approach of rasterization
Filling in the pixels within the polygon along the scan line

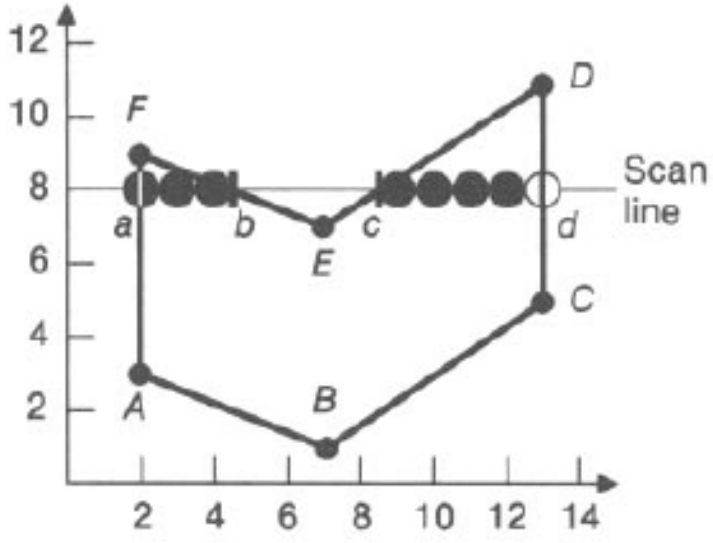


Scanline algorithm

For each scan line:

1.Find the intersections of the scan line with all edges of the polygon.
2.Sort the intersections in the increasing order of the x coordinate.
3.Fill in all pixels between pairs of intersections.

Can also deal with concave polygons



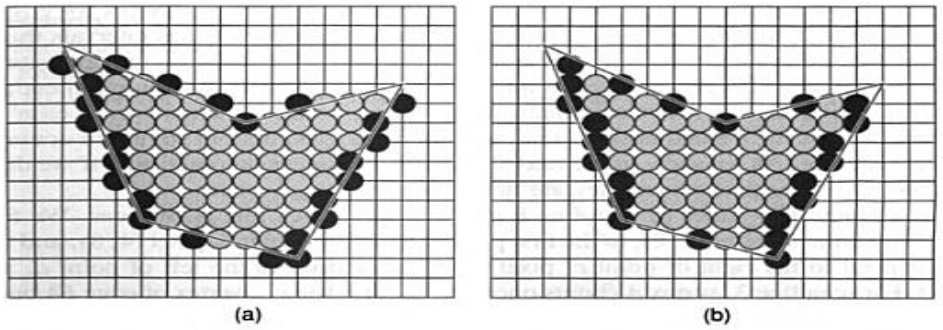
Span extrema

Only turn on pixels whose centers are *interior* to the polygon:

Otherwise will intrude other adjacent polygons

round up values on the left edge of a span,

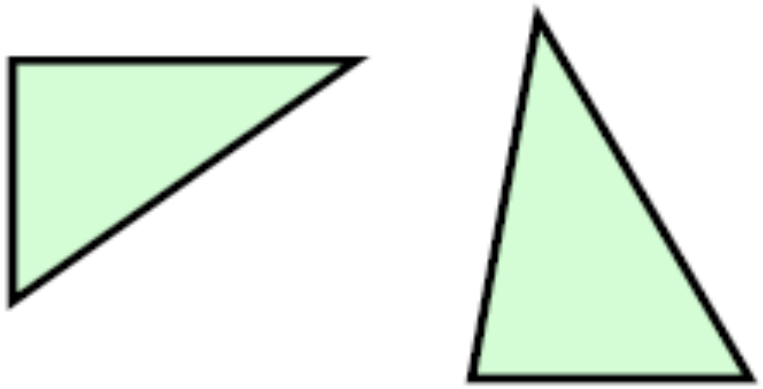
round down on the right edge



Scanline Algorithm
Cons and Pros

- Simple

- Very serial (cannot be efficiently parallelized)
- Special cases require exception handling



• sliver: not even a single pixel wide

Triangle Rasterization by Barycentric Coordinates

Barycentric coordinates

- Can check whether a pixel is inside / outside the triangle
- Can interpolate the attributes at the vertices
- Often used in modern graphics cards
- Can be easily parallelized

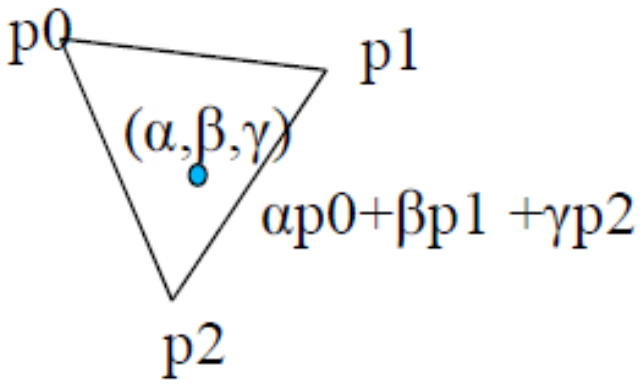
Triangle Rasterization

Consider a 2D triangle with vertices $\boldsymbol{p_0}, \boldsymbol{p_1}, \boldsymbol{p_2}$.

Let $\boldsymbol{p}$ be any point in the plane, it can be expressed by

$$\begin{aligned}\boldsymbol{p} &= \boldsymbol{p_0} + \beta(\boldsymbol{p_1} - \boldsymbol{p_0}) + \gamma(\boldsymbol{p_2} - \boldsymbol{p_0}) \\ &= (1 - \beta - \gamma)\boldsymbol{p_0} + \beta\, \boldsymbol{p_1} + \gamma \boldsymbol{p_2} \\ &= \alpha \boldsymbol{p_0} + \beta\, \boldsymbol{p_1} + \gamma \boldsymbol{p_2}\end{aligned}$$

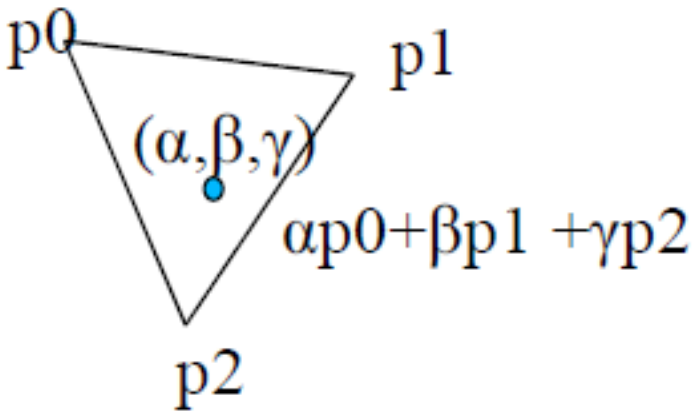
$$\alpha + \beta + \gamma = 1, \alpha, \beta, \gamma \in \mathcal{R}$$



Barycentric Coordinates

We will have $\alpha, \beta, \gamma \in [0,1]$ if and only if $\boldsymbol{p}$ is inside the triangle.

We call the α, β, γ the **barycentric coordinates** of $\boldsymbol{p}$.



Computing Barycentric Coordinates

The triangle is composed of 3 points

p0 *(x0,y0),* ***p1*** *(x1, y1),* ***p2****(x2,y2).*

For point (*x,y*), its barycentric coordinates can be computed by

where $\alpha = \frac{f_{12}(x,y)}{f_{12}(x_0,y_0)} \quad \beta = \frac{f_{20}(x,y)}{f_{20}(x_1,y_1)} \quad \gamma = \frac{f_{01}(x,y)}{f_{01}(x_2,y_2)}$

$$f_{ab}(x,y) \stackrel{\text{def}}{=} (y_a - y_b)\,x + (x_b - x_a)\,y + x_a y_b - x_b y_a$$

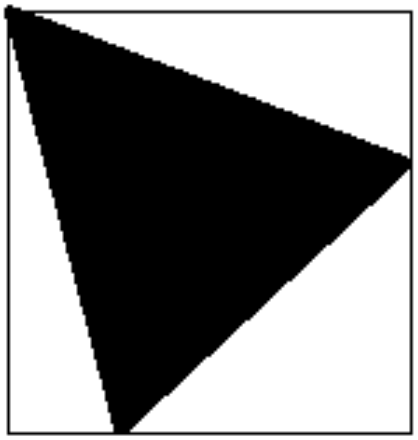
for $a, b \in \{0,1,2\}$.

Bounding Box of a Triangle

Calculate a tight bounding box for a triangle: simply calculate pixel coordinates for each vertex, and find the minimum/maximum for each axis

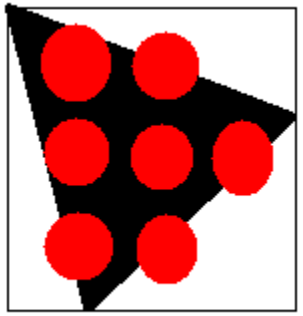
min (x_0,x_1,x_2), max (x_0,x_1,x_2)

min (y_0,y_1,y_2), max (y_0,y_1,y_2)



Scanning inside the triangle

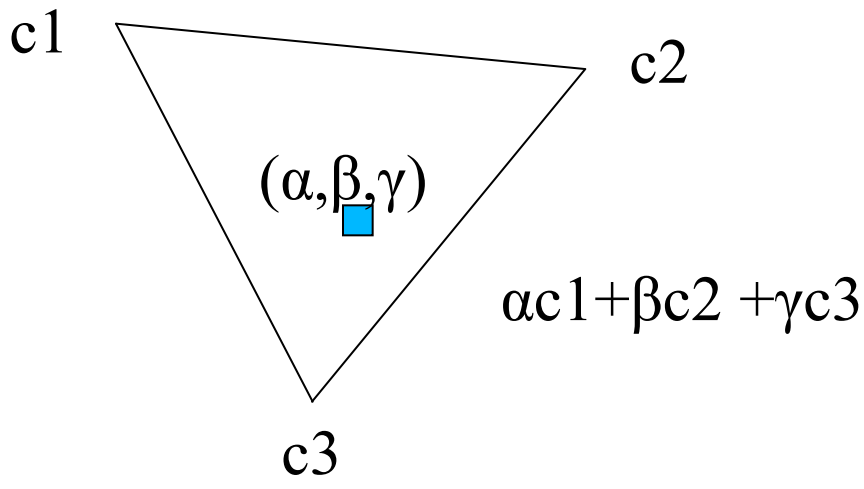
- For each pixel, compute the barycentric coordinates
- Color it if all the three values are in the range of [0,1]



Interpolation by Barycentric Coordinates

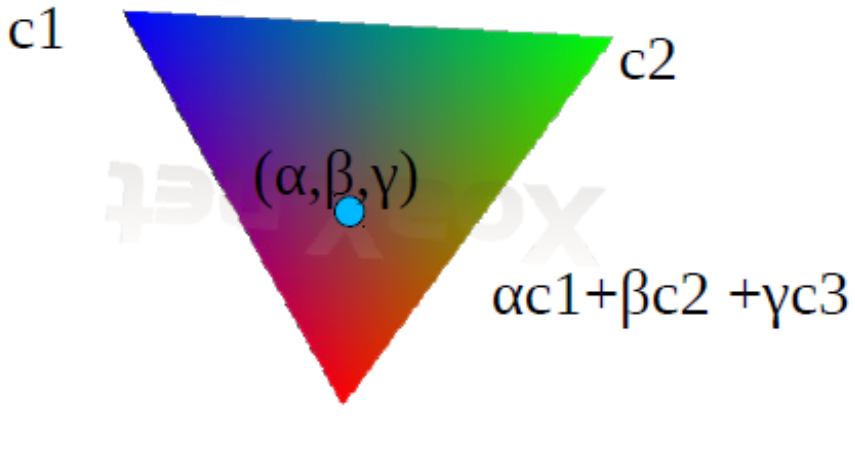
We can use the barycentric coordinates to interpolate attributes of the triangle vertices

- color, depth, normal vectors, texture coordinates



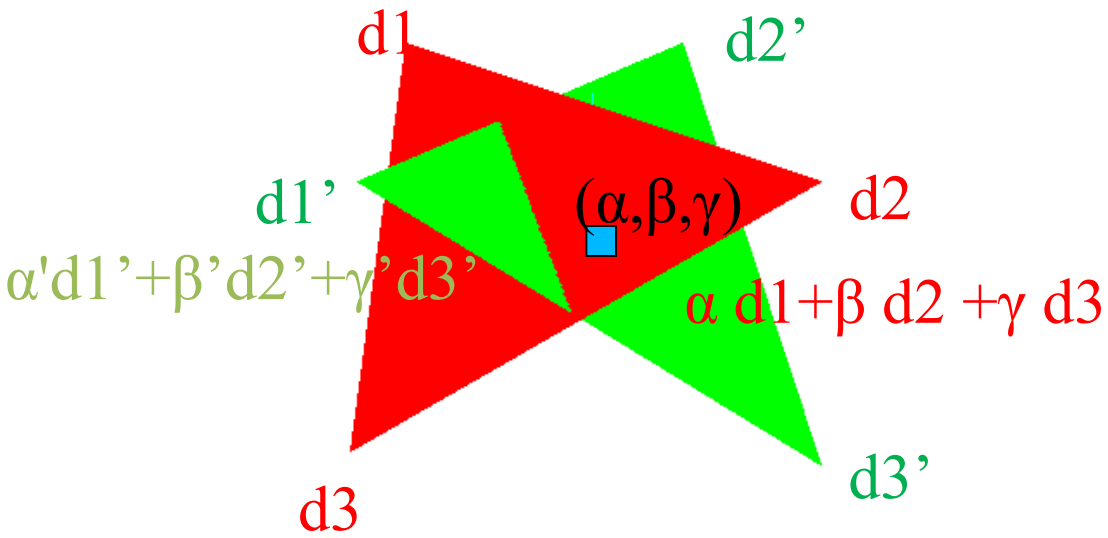
Interpolation of Color

- We can compute the color at the vertices (computed using the lighting equation) and interpolate the color on the surface of the triangle
- This is called Gouraud shading

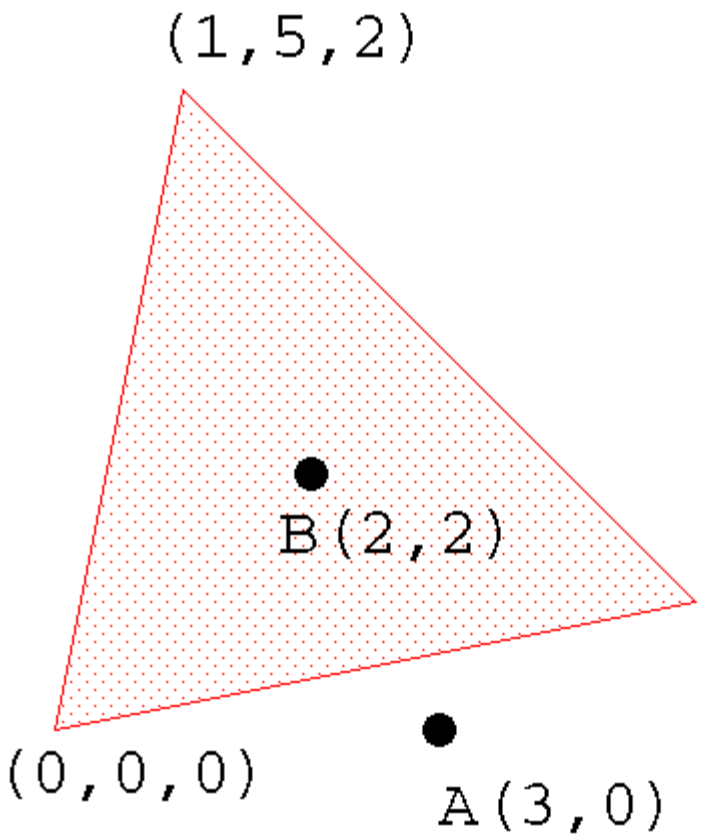


Interpolation of Depth

- When triangles are overlapped, need to compute the depth at each pixel
- Can be computed by barycentric coordinates
- Compare the depth of the pixel at different triangles and only show the closest one
- This is called Z-buffering



Exercise



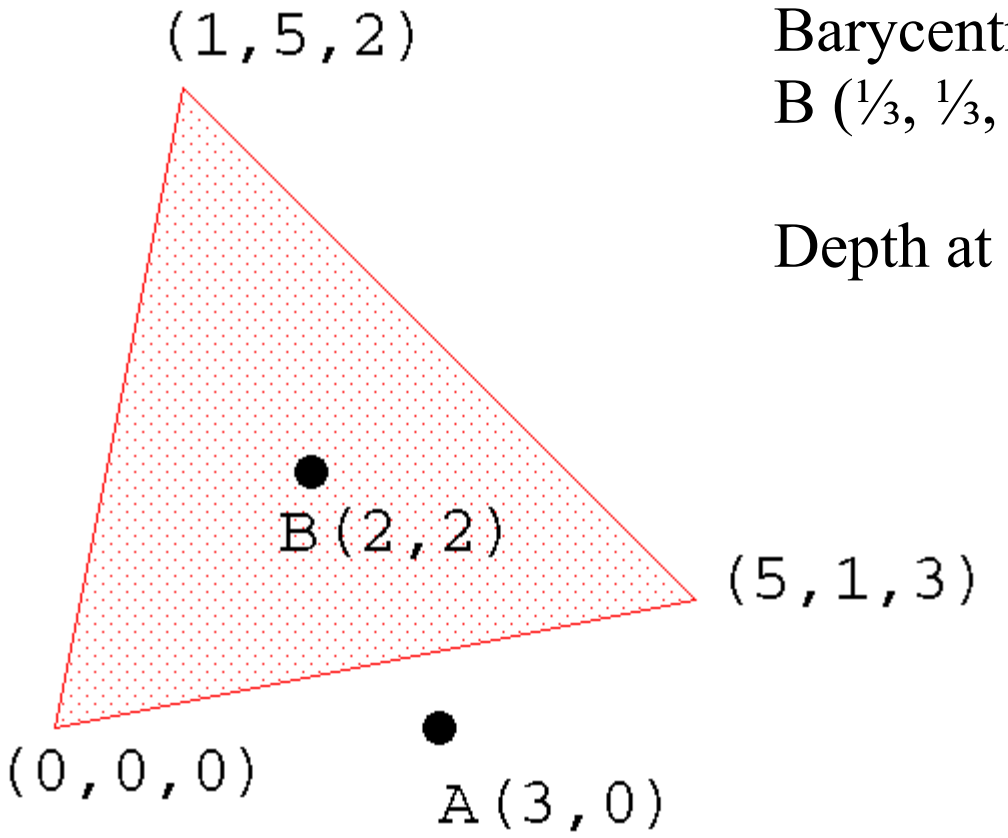
1.What are the barycentric coordinates at point A and B?

2. What is the depth of the triangle surface at point B?

$$\gamma = \frac{(y_0 - y_1)x + (x_1 - x_0)y + x_0y_1 - x_1y_0}{(y_0 - y_1)x_2 + (x_1 - x_0)y_2 + x_0y_1 - x_1y_0}$$

$$\beta = \frac{(y_0 - y_2)x + (x_2 - x_0)y + x_0y_2 - x_2y_0}{(y_0 - y_2)x_2 + (x_2 - x_0)y_2 + x_0y_2 - x_2y_0}$$

Exercise

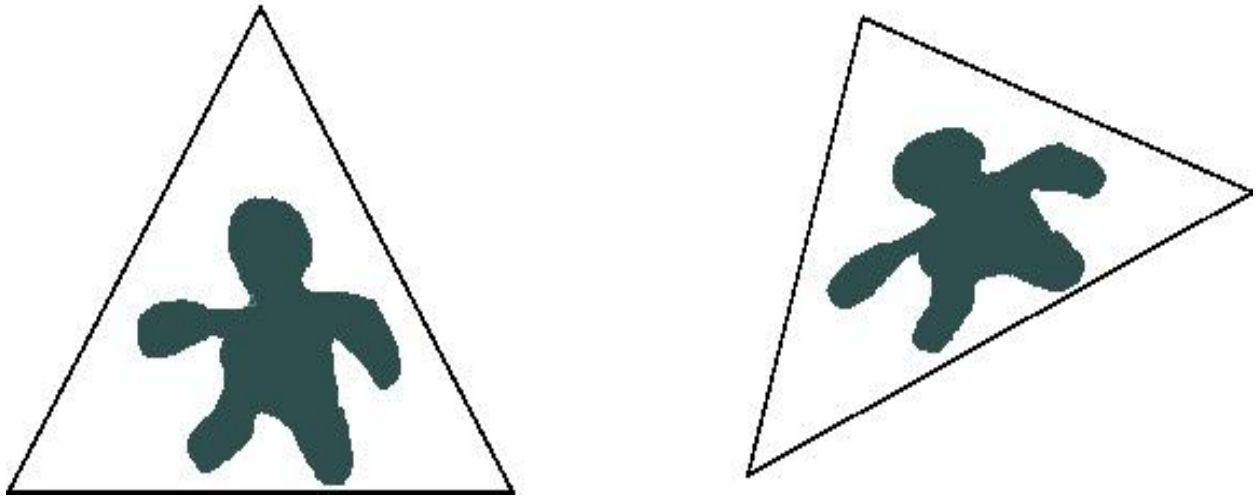


Barycentric coordinates
B (⅓, ⅓, ⅓), A(½, ⅝, -⅛)

Depth at B = 5/3

Another usage : Shape Editing

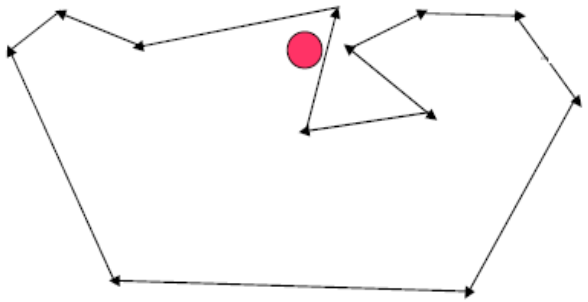
We can re-apply the same barycentric coordinates within a triangle when its shape is edited



What about polygons with many vertices?

- Can we compute barycentric coordinates for polygons with more vertices?

$$v = \frac{\sum_i w_i p_i}{\sum_i w_i}$$

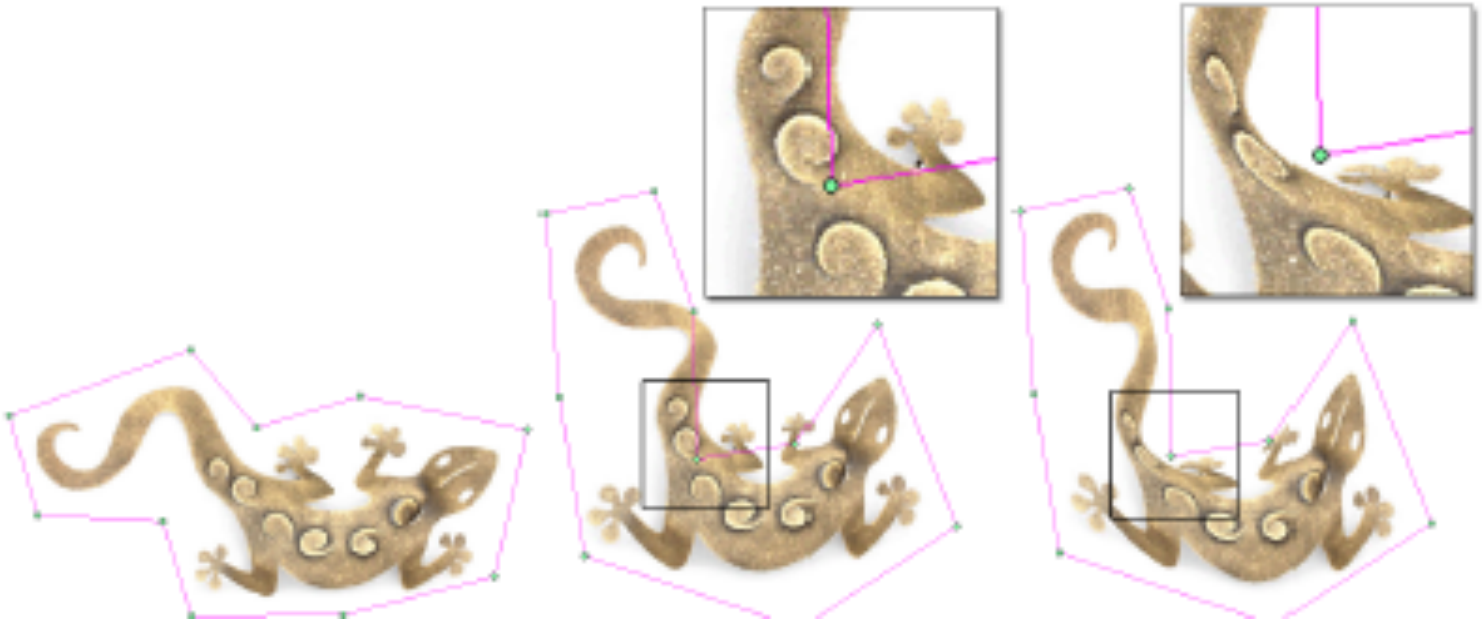


- Can we compute barycentric coordinates for 3D meshes?

-> Mean value coordinates

Harmonic coordinates

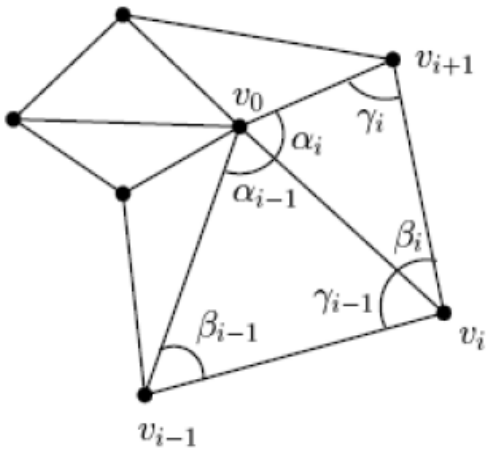
(generalized barycentric coordinates)



Mean Value Coordinates

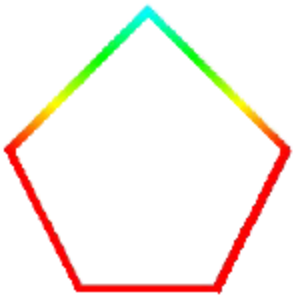
- A good and smooth barycentric coordinates that can
  - smoothly interpolate the boundary values
  - Also works with concave polygons
  - There is also a 3D version

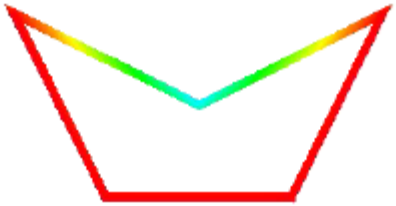
$$w_i = \frac{\tan(\alpha_{i-1}/2) + \tan(\alpha_i/2)}{||v_i - v_0||}$$



Mean Value Coordinates

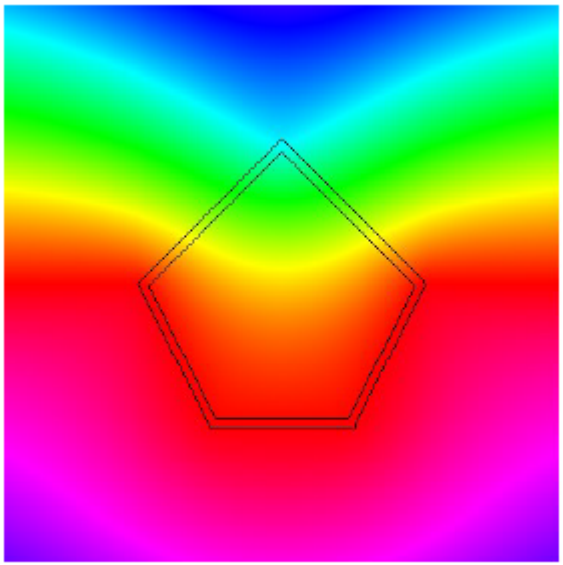
- Can interpolate convex and concave polygons
- Smoothly interpolate the interior as well as exterior

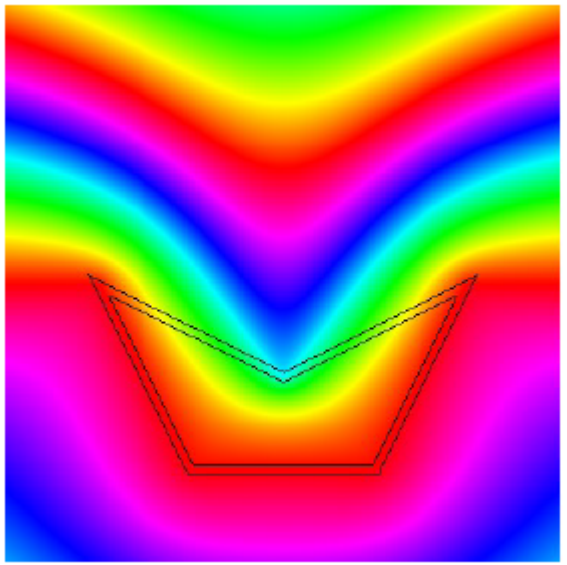




Mean Value Coordinates

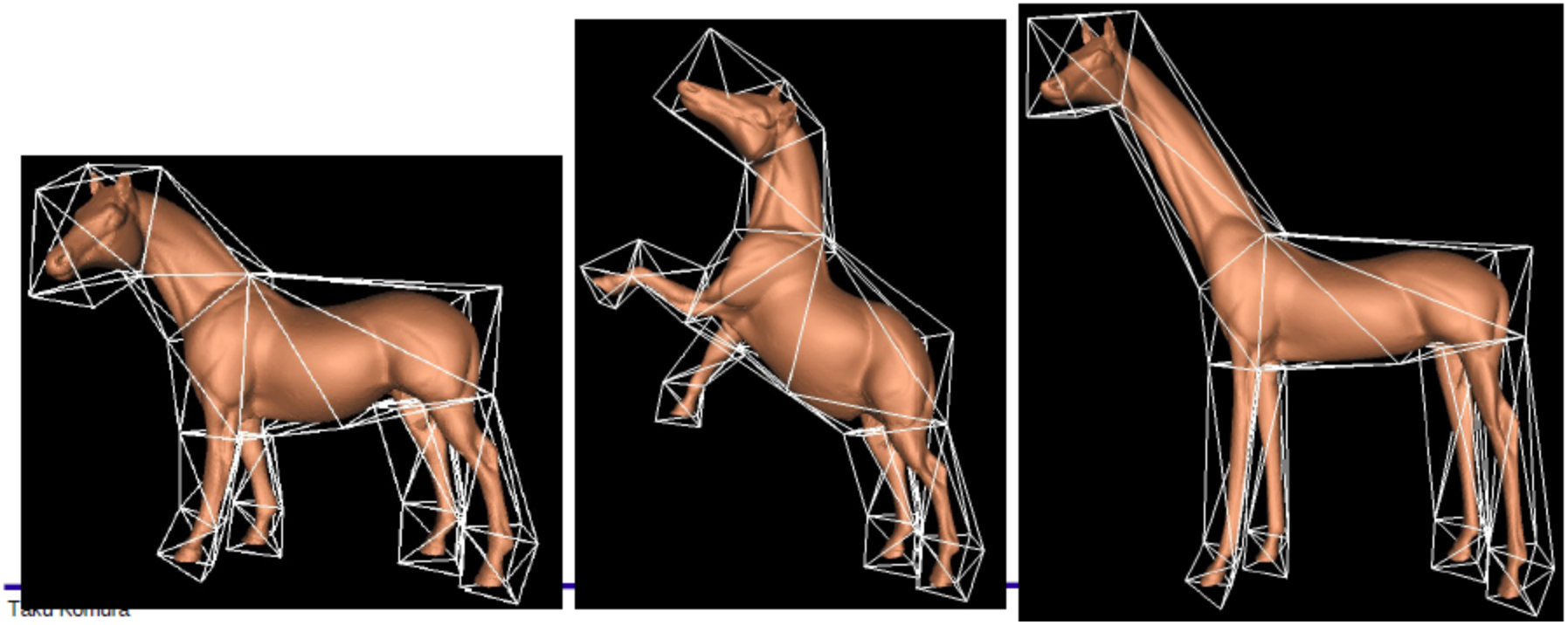
- Can interpolate convex and concave polygons
- Smoothly interpolate the interior as well as exterior





Mean Value Coordinates

- Can be computed in 3D
- Applicable for mesh editing



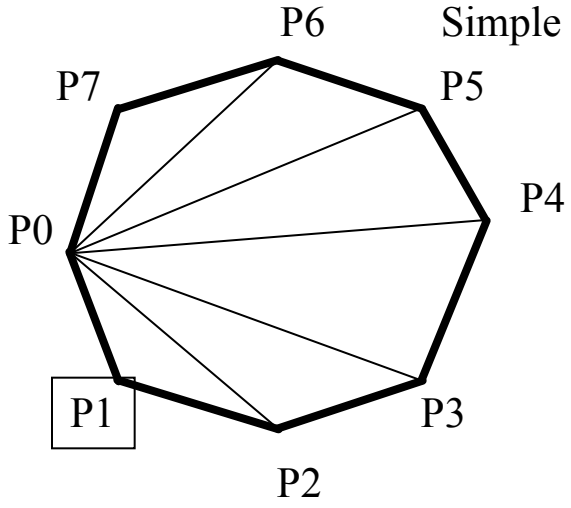
Overview

Rasterization

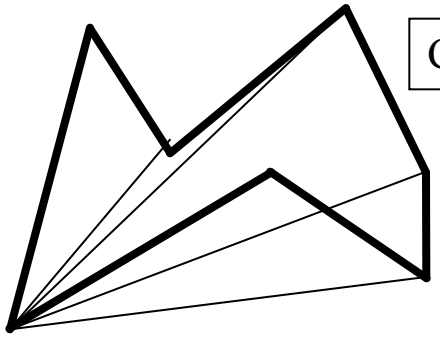
- Line Rasterization
- Polygon Rasterization
  - Scanline Algorithm
  - Rasterizing triangles
    - Edge walking
    - Interpolation by barycentric coordinates
- Mean value coordinates
- **Dividing polygons into triangles**

Polygon Decomposition

For polygons with more than three vertices, we usually decompose them into triangles



Simple for convex polygons.



Concave more difficult.

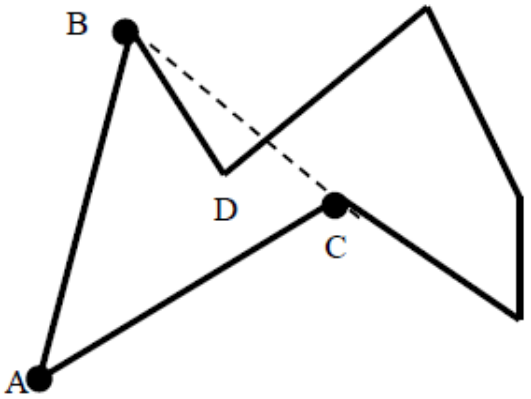
Polygon Decomposition: Algorithm

Start from the left and form the leftmost triangle:

- Find leftmost vertex (smallest x) – A
- Compose possible triangle out of A and the two adjacent vertices B and C
- Check to ensure that no other polygon point P is inside of triangle ABC
- If all other polygon points are outside of ABC then cut it off from polygon and proceed with next leftmost triangle

Polygon decomposition (2)

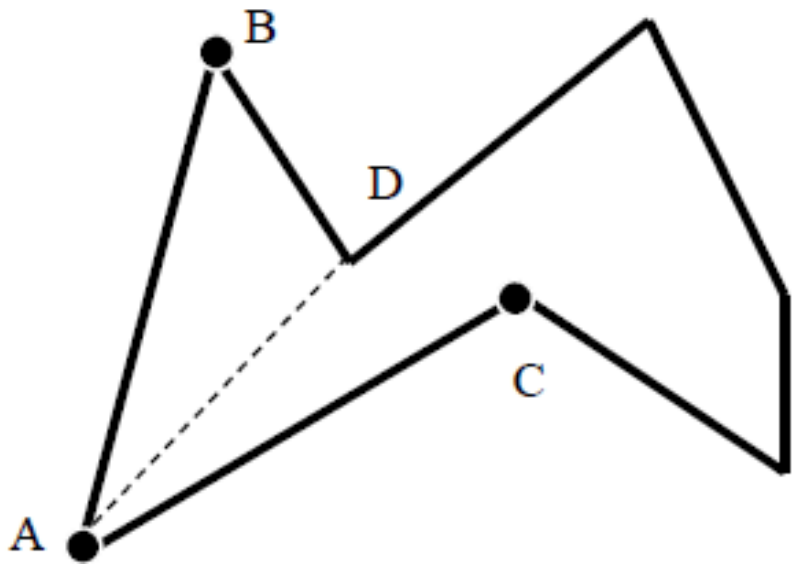
- The left most vertex A
- A triangle is formed by A and the two adjacent B and C
- Check if all the other vertices are outside the triangle



Vertex ‘D’ fails test.

Polygon decomposition (3)

If a vertex is inside, split the polygon by the inside vertex and point A, proceed as before.

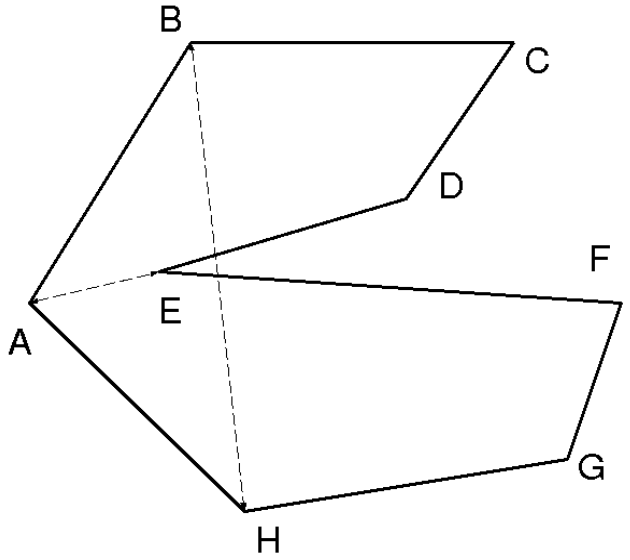


Test ABD in same manner as before,

Polygon decomposition (4)

This edge may split the polygon into two.
Then, recurse the method with each polygon

(ABCDE and AEFGH in the example below)



Summary

Rasterization

- Line Rasterization - Midpoint algorithm
- Triangle Rasterization - barycentric coordinates
- Mean value coordinates
- Dividing polygons into triangles

Reading for rasterization

Scanline algorithm

Foley et al., Chapter 3.5, 3.6

Baricentric coordinates

www.cs.caltech.edu/courses/cs171/barycentric.pdf

Mean value coordinates for closed triangular meshes, SIGGRAPH 2005

Polygon decomposition

http://www.siggraph.org/education/materials/HyperGraph/scanline/outprims/polygon1.htm