Computer Graphics

Lecture 5

View Transformation and Clipping

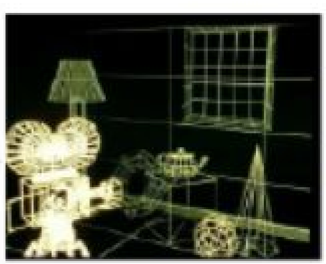
Taku Komura

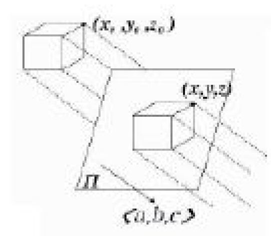
Overview

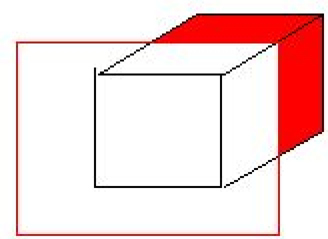
- View transformation
  - Recap of homogeneous transformation
  - Parallel projection
  - Perspective projection
  - Canonical view volume
- Clipping
  - Line / Polygon clipping

Procedure

1. Transform into camera coordinates. (done in Lecture 3)
2. Perform projection into *view volume* or *screen coordinates*.
3. Clip geometry outside the *view volume*.

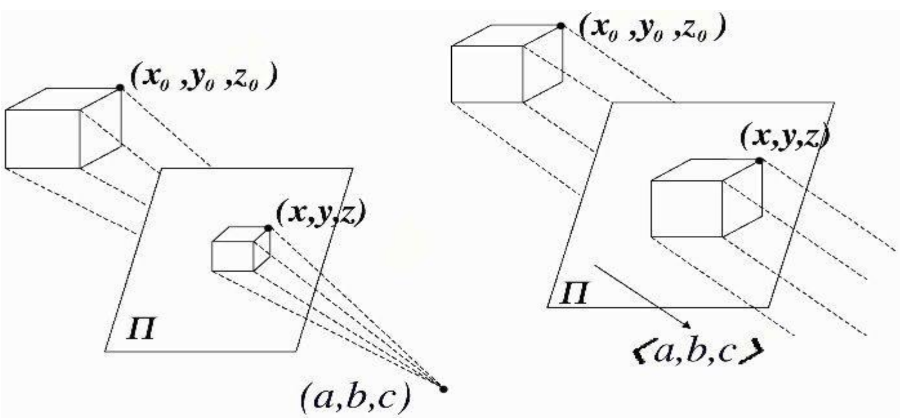


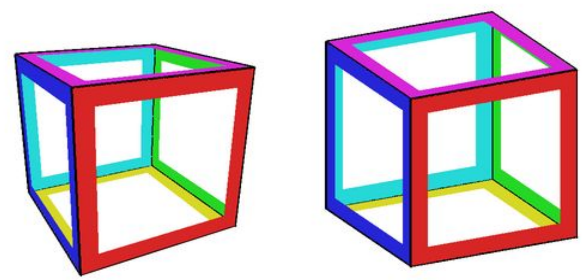




View Projection : Topics

- Homogenous transformation
- Parallel projection
- Perspective projection
- Canonical view volume

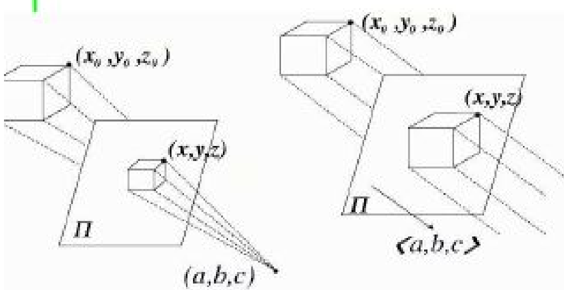


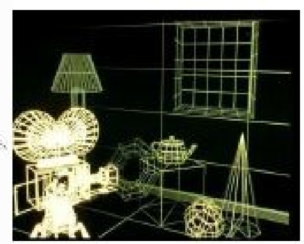
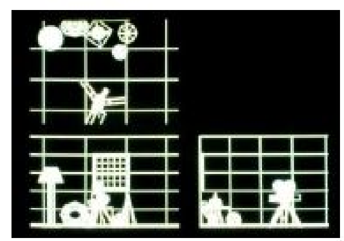
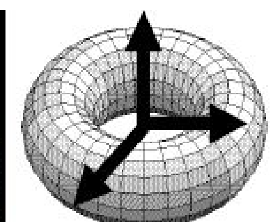


Homogeneous Transformations

$$v = M_{proj}\, M_{c\leftarrow w}\, M_{w\leftarrow l}\, v_l$$

Projection matrix	World to camera matrix	Local to world matrix	Local coordinates



The projection matrix should be 4x4 matrices

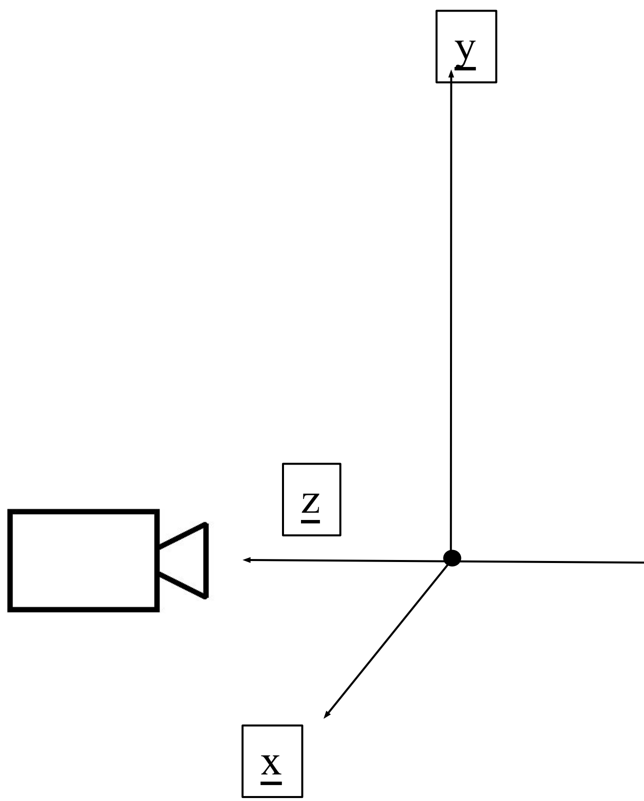
to allow general concatenation

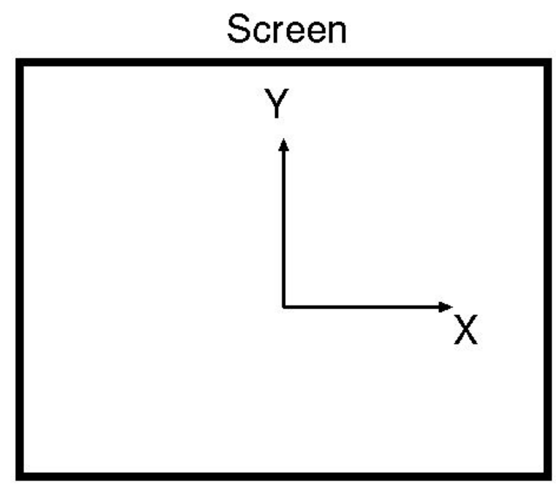
Homogeneous Coordinates

- Introduced to represent various transformations by multiplications (in Lecture 2)
- In homogenous coordinates, (x,y,z,w) represent the same point when all elements are multiplied by the same factor
  - (2,0,1,1) and (4,0,2,2) are the same points
  - To bring back to Cartesian space, need to divide the other elements by the fourth element *w*
    - $(x, y, z, w) \rightarrow (x/w, y/w, z/w, 1)$

Camera Coordinate System we use

(same as OpenGL)





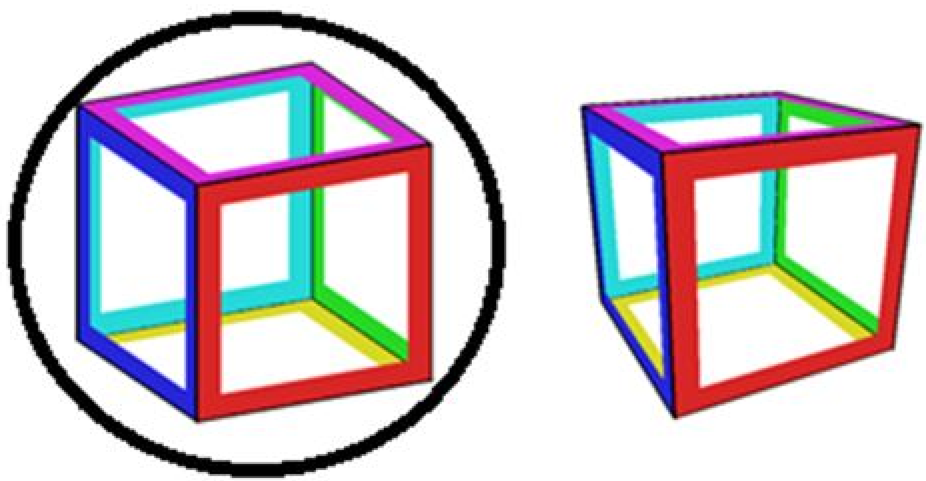
Facing the –z direction

X axis facing the right side

Y axis facing upwards

Parallel projections (Orthographic projection)

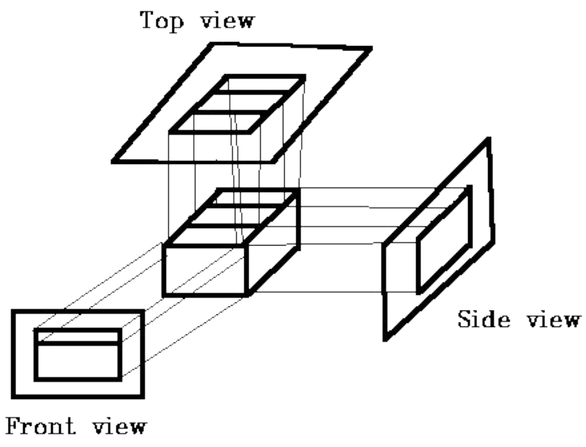
- Specified by a direction of projection, rather than a point.
- Objects of same size appear at the same size after the projection



Parallel projection.

Orthographic Projection onto a plane at z = 0.

$$x_p = x\ ,\ y_p = y\ ,\ z = 0.$$

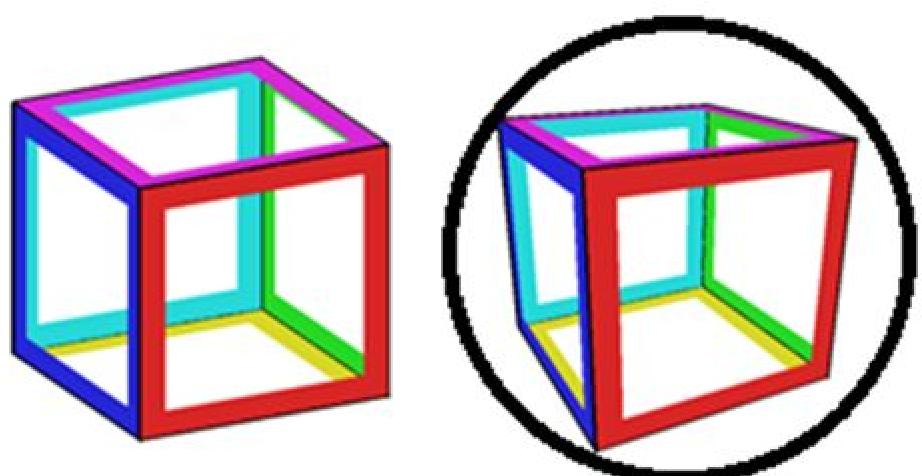


$$M_{orth} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

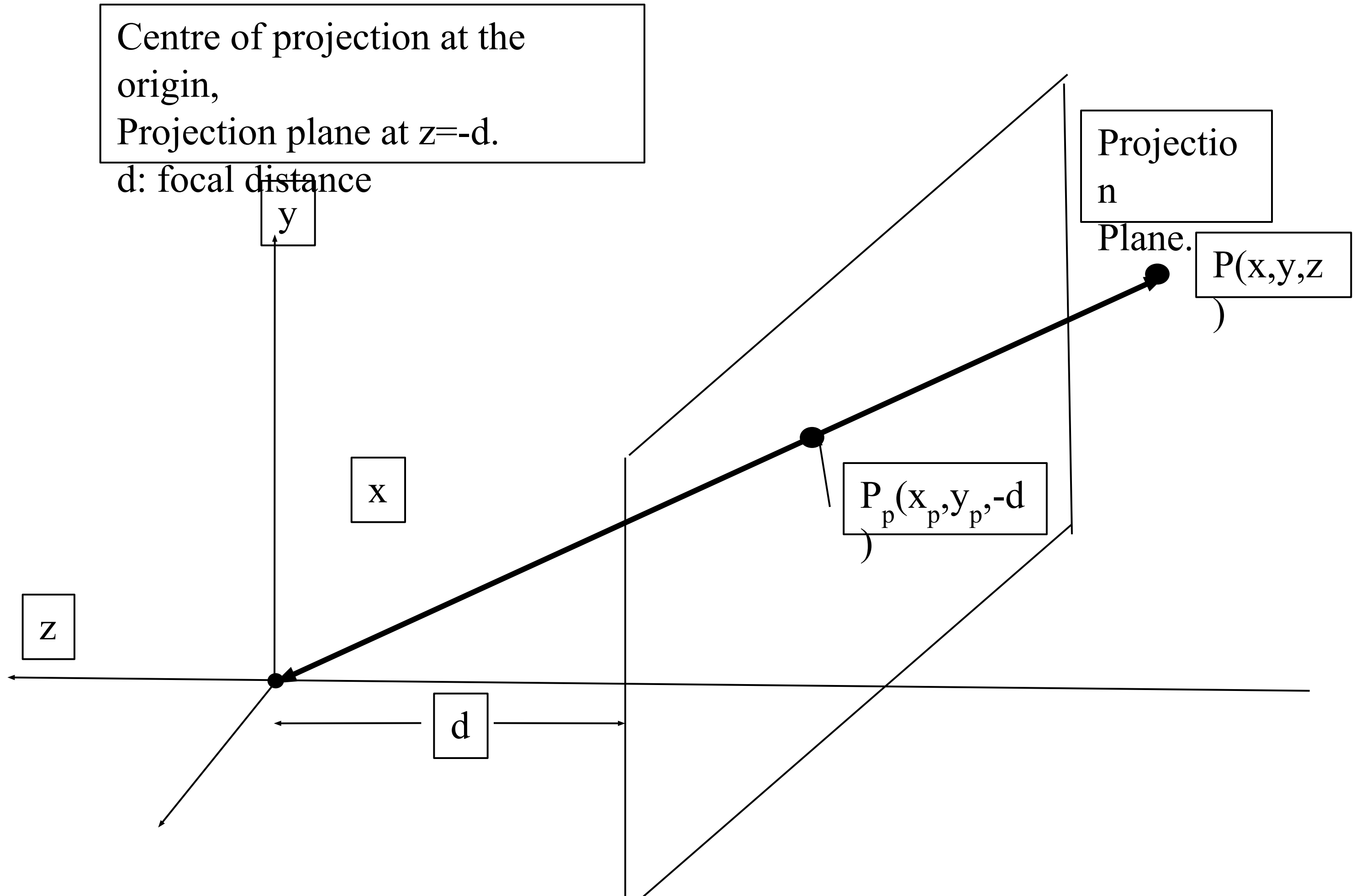
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \\ 1 \end{bmatrix}$$

Perspective Projection

- Objects far away appear smaller, closer objects appear bigger
- Specified by a center of projection and the focal distance (distance from the eye to the projection plane)



Perspective projection

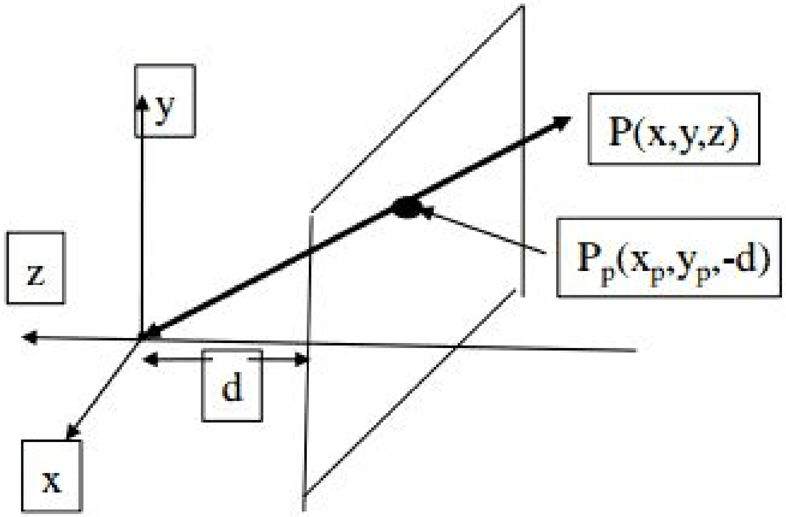


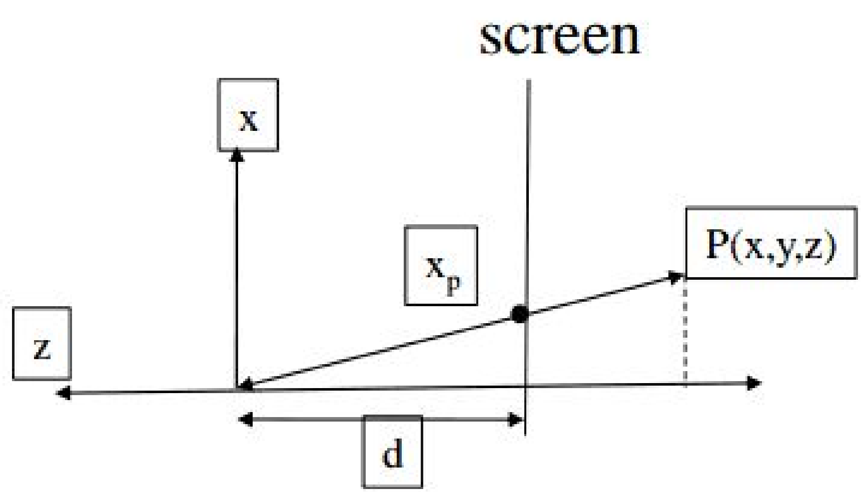
Perspective projection – simplest case.

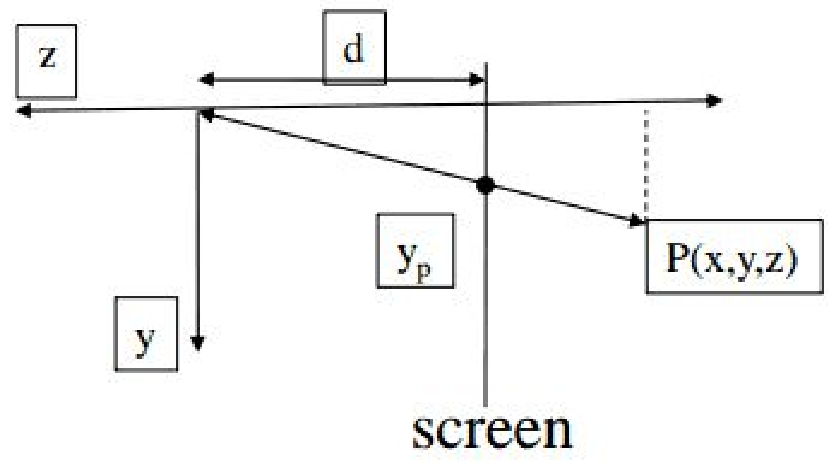
From similar triangles:

$$\frac{x_p}{d} = \frac{x}{-z}; \quad \frac{y_p}{d} = \frac{y}{-z}$$

$$x_p = \frac{d \cdot x}{-z} = \frac{x}{-z/d}; \quad y_p = \frac{d \cdot y}{-z} = \frac{y}{-z/d}$$







Perspective projection.

$$\begin{bmatrix} x_p & y_p & -d & 1 \end{bmatrix}^T = \begin{bmatrix} -d.x/z & -d.y/z & -d & 1 \end{bmatrix}^T = \begin{bmatrix} x & y & z & -z/d \end{bmatrix}^T$$

Using homogeneous transformation, perspective projection can be represented as a 4x4 matrix multiplication :

$$M_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix}$$

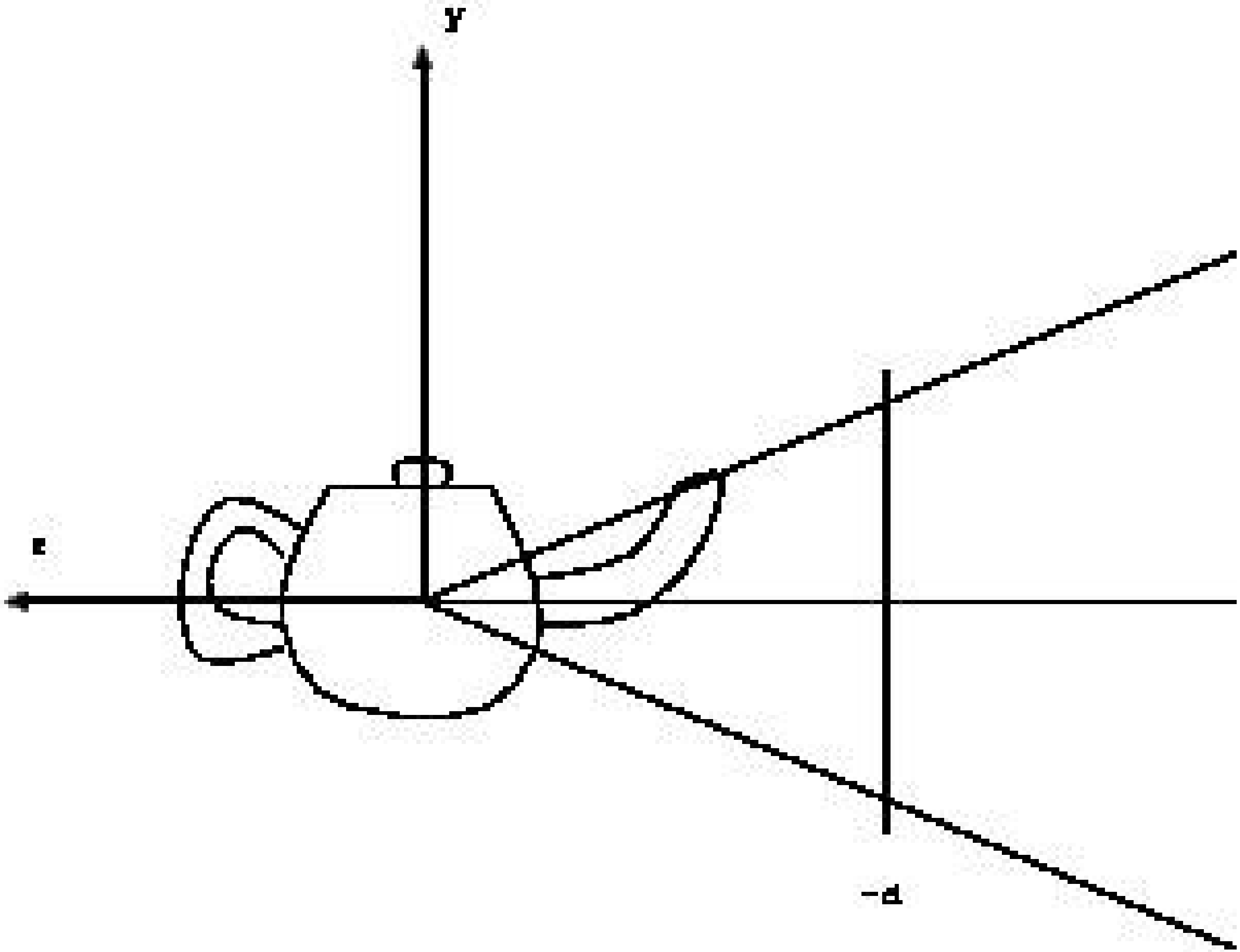
Perspective projection.

Projected point : $P_p = \begin{bmatrix} X & Y & Z & W \end{bmatrix}^T$

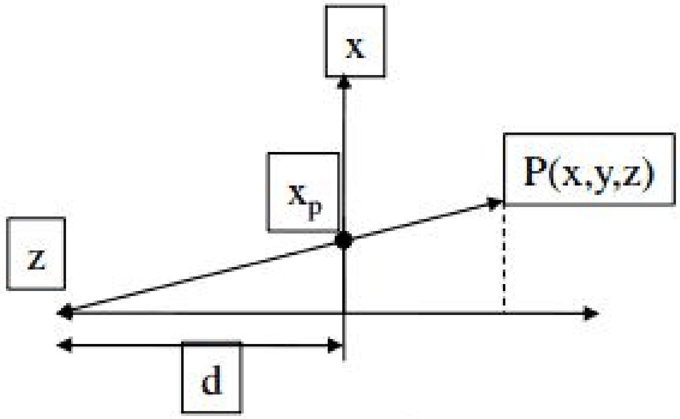
$$P_p = M_{per} \cdot P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

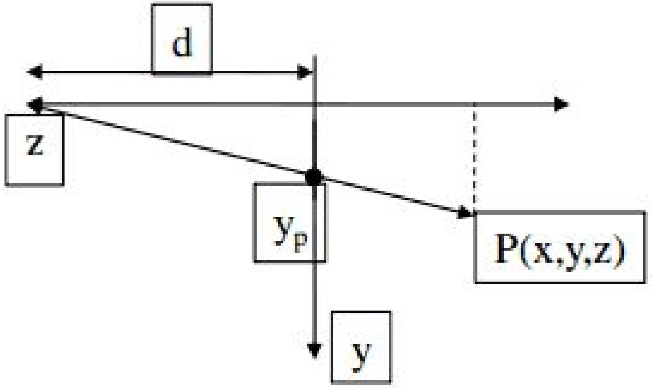
$$= \begin{bmatrix} X & Y & Z & W \end{bmatrix}^T = \begin{bmatrix} x & y & z & -z/d \end{bmatrix}^T$$

$$\left(\frac{X}{W}, \ \frac{Y}{W}, \ \frac{Z}{W} \right) = \left(\frac{x}{-z/d}, \ \frac{y}{-z/d}, \ -d \right)$$


y
z
-d

Alternative formulation.





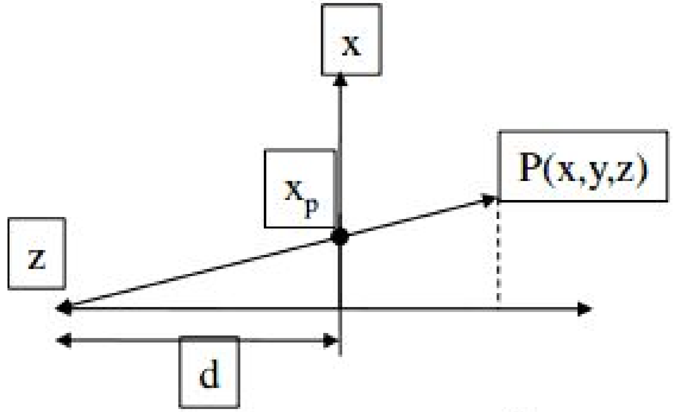
Projection plane at z = 0
Centre of projection at
z = d

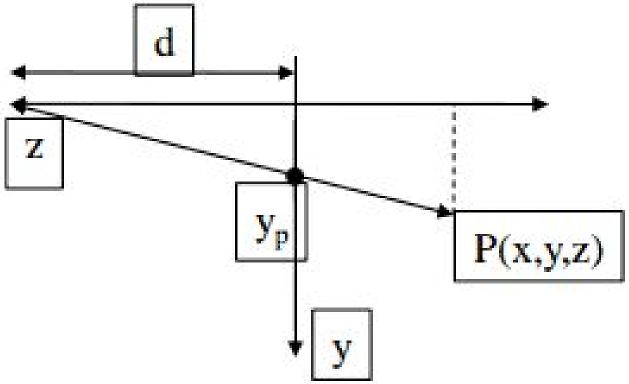
$$\frac{x_p}{d} = \frac{x}{-z+d}; \qquad \frac{y_p}{d} = \frac{y}{-z+d}$$

Multiply by d

$$x_p = \frac{d \cdot x}{-z+d} = \frac{x}{(-z/d)+1}; \qquad y_p = \frac{d \cdot y}{-z+d} = \frac{y}{(-z/d)+1}$$

Alternative formulation.



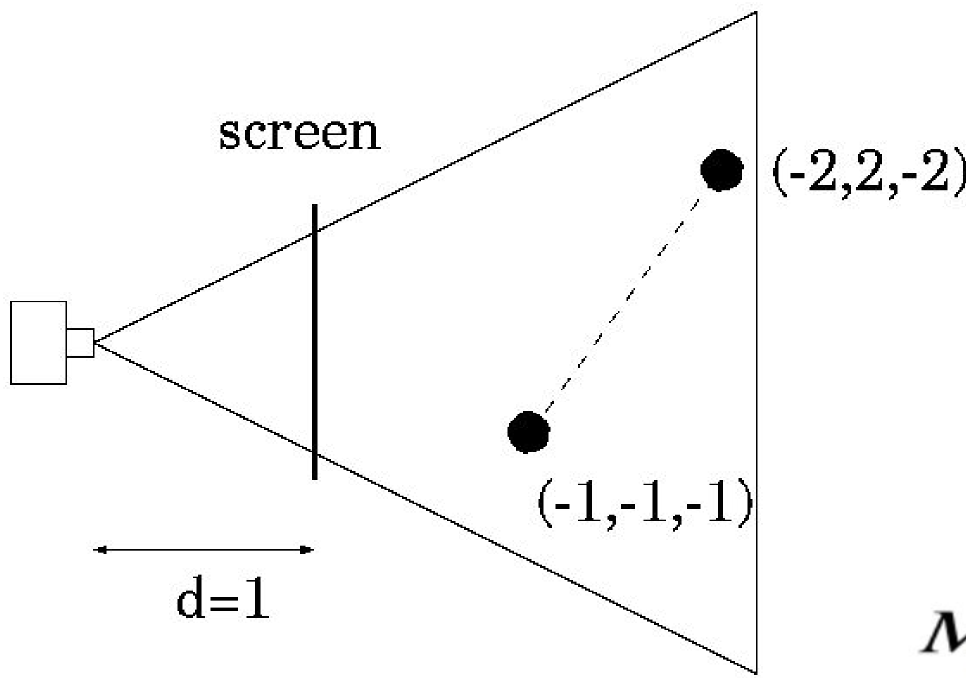


Projection plane at z = 0,
Centre of projection at
z = d

Now we can allow $d \rightarrow \infty$

$$M_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/d & 1 \end{bmatrix}$$

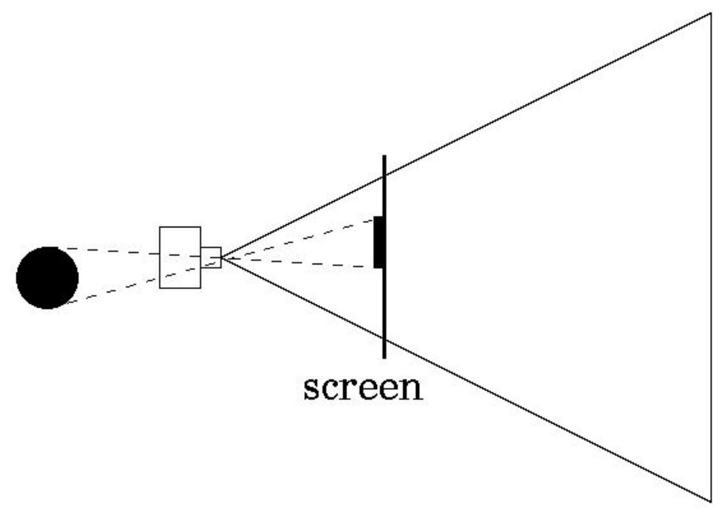
Exercise: where will the two points be projected onto?



$$M_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/d & 1 \end{bmatrix}$$

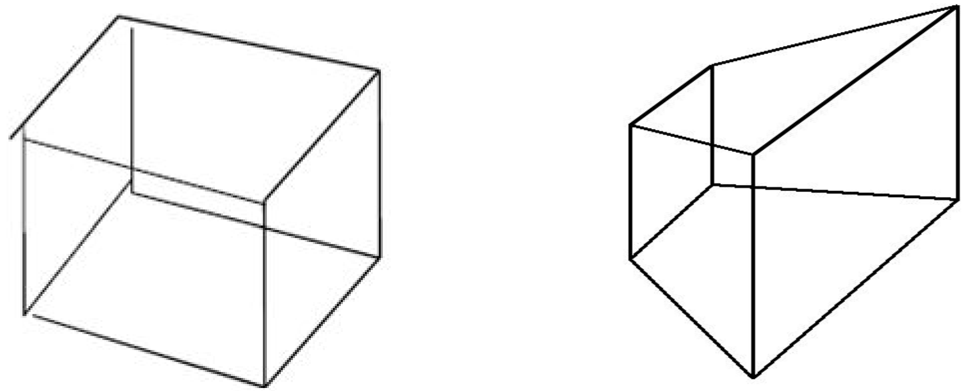
Problems

- After projection, the depth information is lost
  - We need to preserve the depth information for hidden surface removal
- Objects behind the camera are projected to the front of the camera



3D View Volume

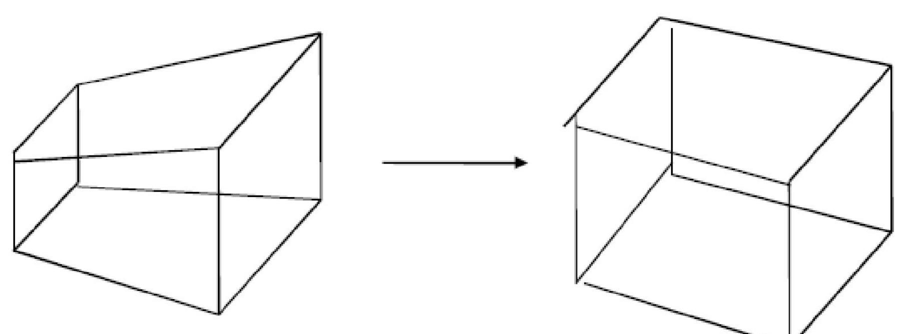
- The volume in which the visible objects exist
  - For parallel projection, view volume is a box.
  - For perspective projection, view volume is a *frustum*.
- The surfaces outside the view volume must be clipped



Canonical View Volume

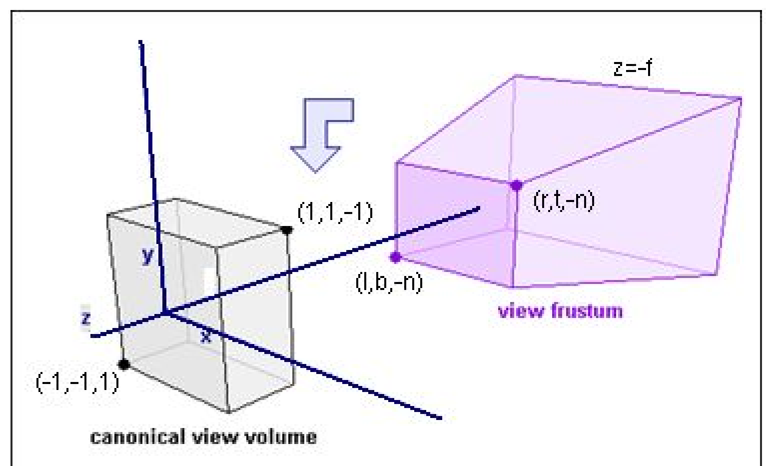
- Checking if a point is within a frustum is costly
- We can transform the frustum view volume into a normalized canonical view volume
- By using the idea of perspective transformation

- Much easier to clip surfaces and calculate hidden surfaces



Transforming the View Frustum

- Let us define parameters (l,r,b,t,n,f) that determines the shape of the frustum
- The view frustum starts at z=-n and ends at z=-f, with 0<n<f
- The rectangle at z=-n has the minimum corner at (l,b,-n) and the maximum corner at (r,t,-n)



Transforming View Frustum into a Canonical view-volume

The perspective canonical view-volume can be transformed to the parallel canonical view-volume with the following matrix:

$$If\ \ z \in [-n, -f](0 < n < f)\ then$$

$$P_p = \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Final step.

- Divide by w to get the 3-D Cartesian coordinates
- 3D Clipping
  - The Canonical view volume is defined by:

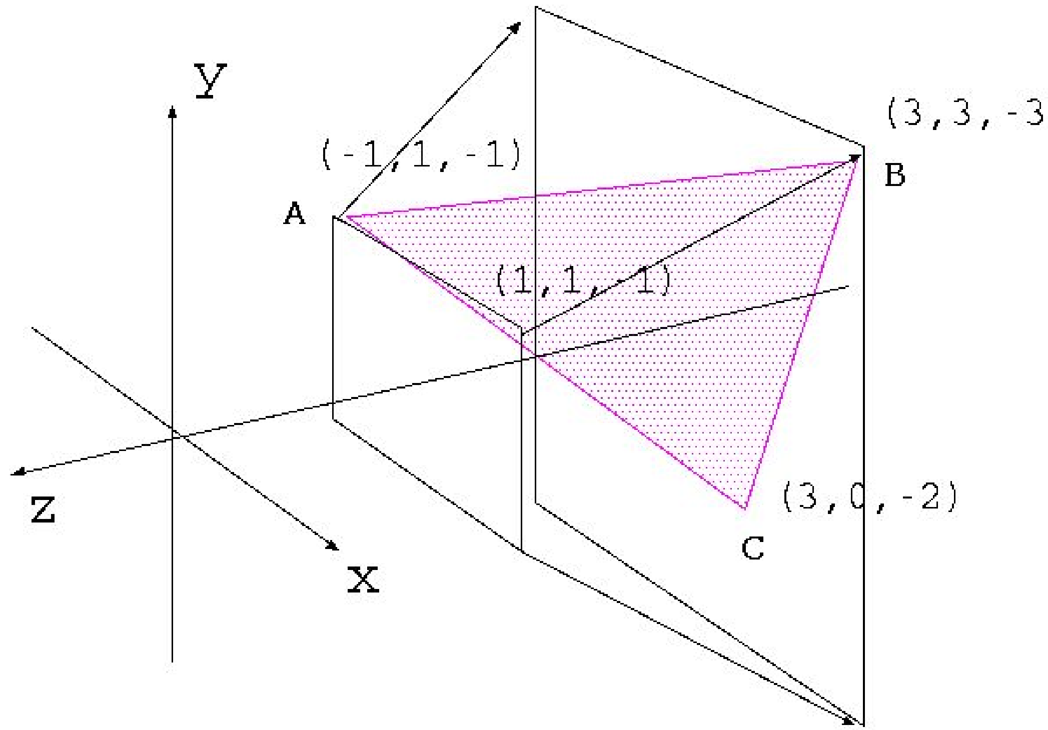
$$-1 \leq x \leq 1,\ -1 \leq y \leq 1\ ,\ -1 \leq z \leq 1$$

  - Simply need to check the (x,y,z) coordinates and see if they are within the canonical view volume

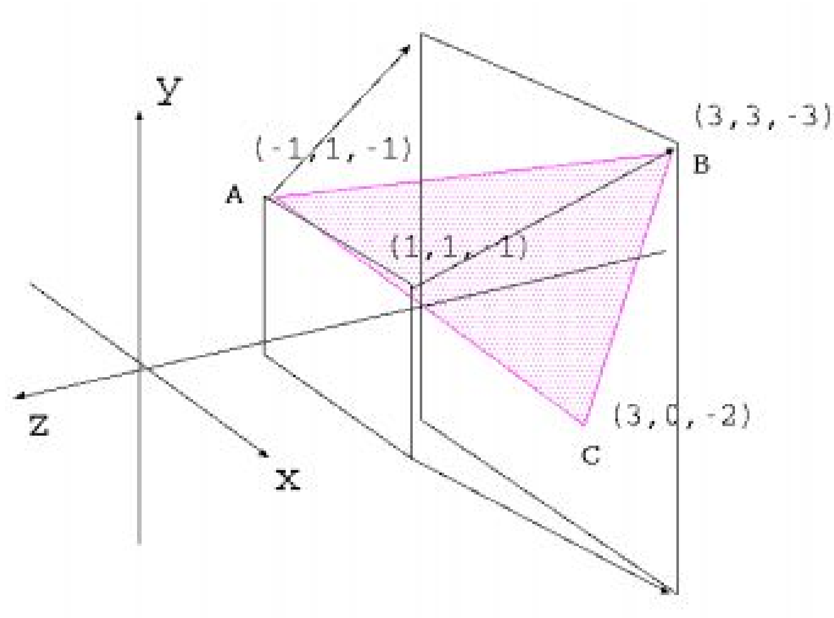
$$If\ z \in [-n, -f](0 < n < f)\ then$$

$$P_p = \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Exercise



- How does ABC look like after the projection?



$$\text{If } z \in [-n,-f](0<n<f) \text{ then}$$

$$P_p = \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

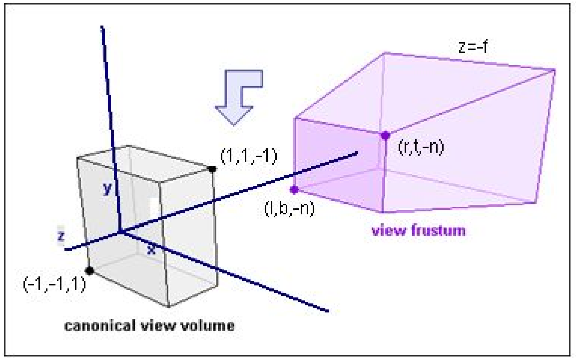
$$n=1, f=3, r=1, l=-1, t=1, b=-1$$

$$P_p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & -3 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} -1 & 3 & 3 \\ 1 & 3 & 0 \\ -1 & -3 & -2 \\ 1 & 1 & 1 \end{bmatrix} \end{array} = \begin{bmatrix} -1 & 3 & 3 \\ 1 & 3 & 0 \\ -1 & 3 & 1 \\ 1 & 3 & 2 \end{bmatrix} = \begin{array}{c} \text{projected} \\ \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} -1 & 1 & \frac{3}{2} \\ 1 & 1 & 0 \\ -1 & 1 & \frac{1}{2} \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

Summary of Projection

- Two kind of projections:
  - parallel and perspective
- We can project points onto the screen by using projection matrices
- Canonical view volume is useful for telling if the point is within the view volume
  - parts outside must be clipped



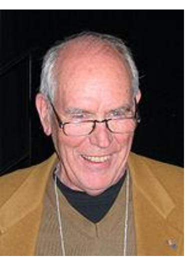
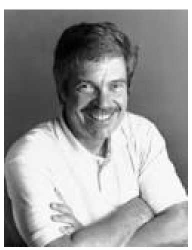
Some side story

SCHOOL OF COMPUTING
THE UNIVERSITY OF UTAH

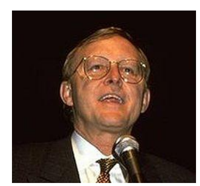
http://www.cs.utah.edu/about/history/

University of Utah

Like the holy place of computer graphics.





Adobe PIXAR Animation Studios E&S Evans & Sutherland Myricom CIRRUS LOGIC SiliconGraphics WordPerfect

Graduates/Faculties include

Ivan Sutherland

Ed Catmull

Alan Kay

Henri Gouraud

Bui Tuong Phong

Jim Clark

Jim Blinn

Jim Kajiya

Ed Catmull

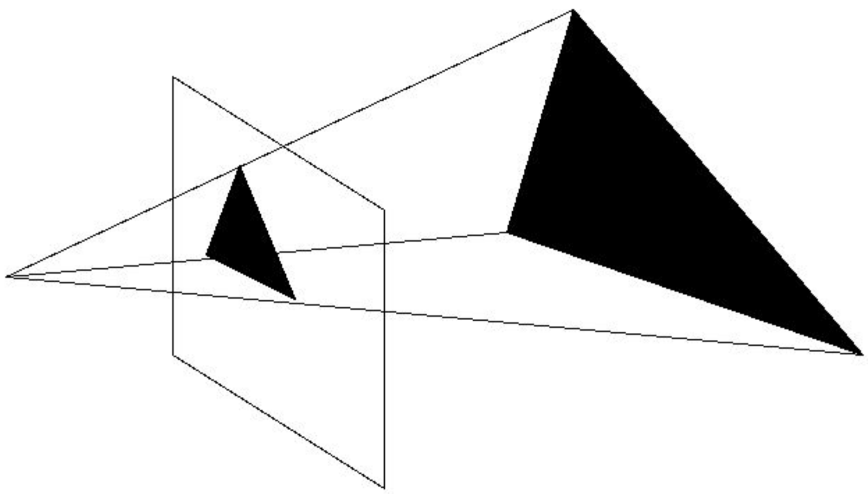
and many others

Overview

- View transformation
  - Parallel projection
  - Perspective projection
  - Canonical view volume
- **Clipping**
  - **Line / Polygon clipping**

Projecting polygons and lines

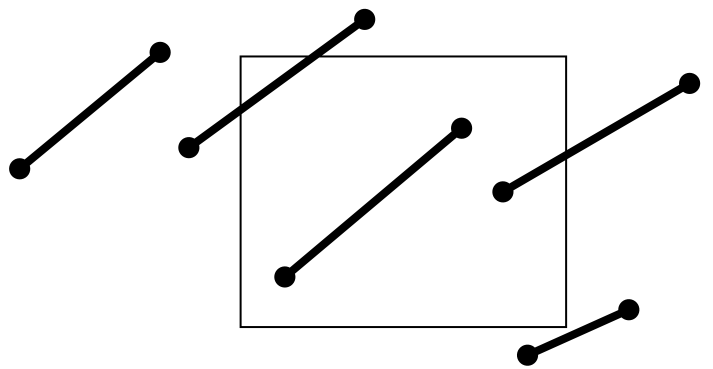
- After projection, a line in 3D space becomes a line in 2D space
- A polygon in 3D space becomes a polygon in 2D space

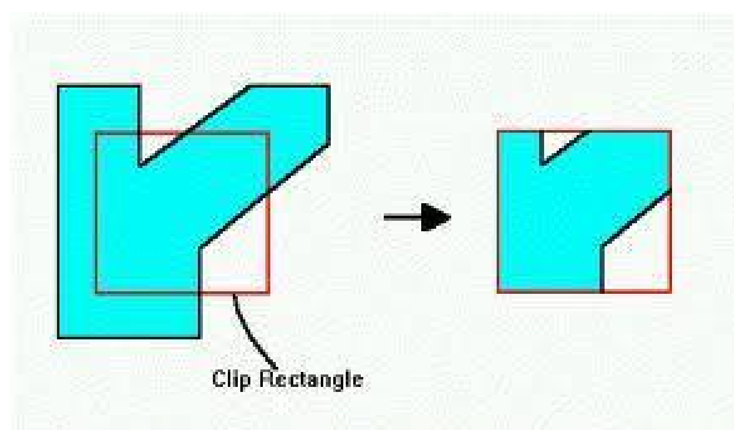


Clipping

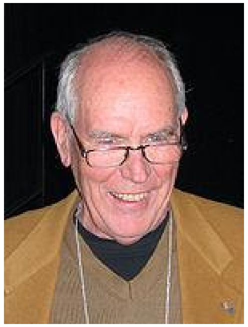
- We need to clip objects outside the canonical view volume

- Clipping lines (Cohen-Sutherland algorithm)
- Clipping polygons (Sutherland-Hodgman algorithm)





Cohen-Sutherland algorithm

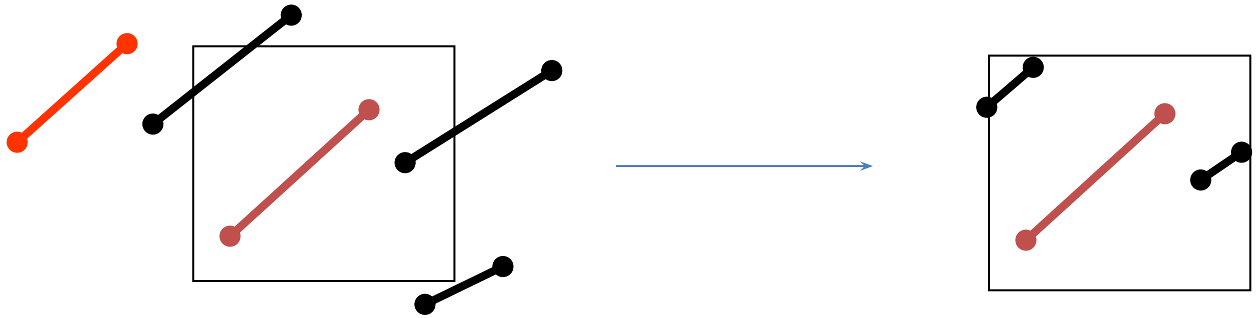


A systematic approach to clip lines

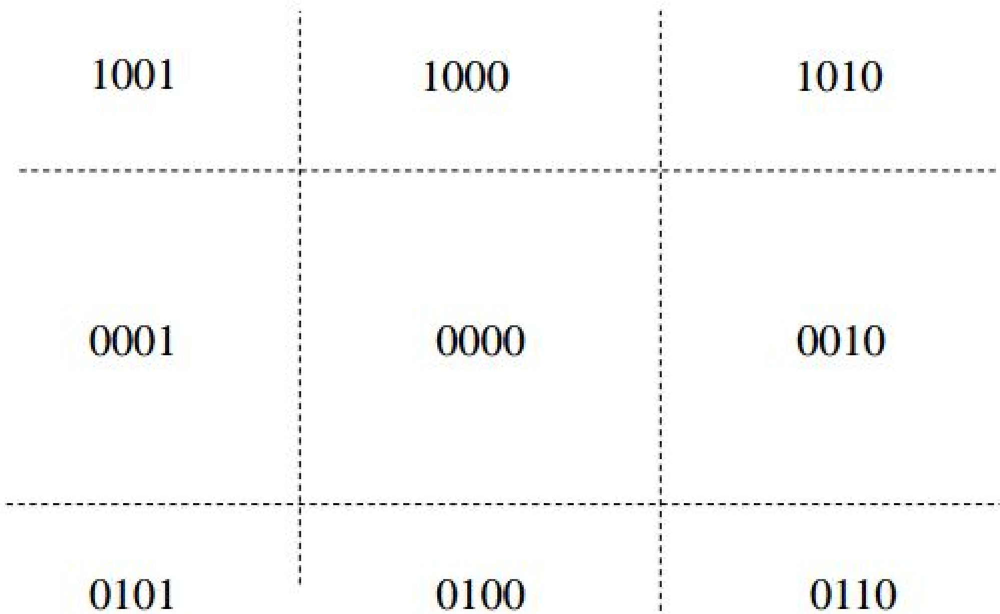
Input: The screen and a 2D line segment

(let’s start with 2D first)

Output: Clipped line segment

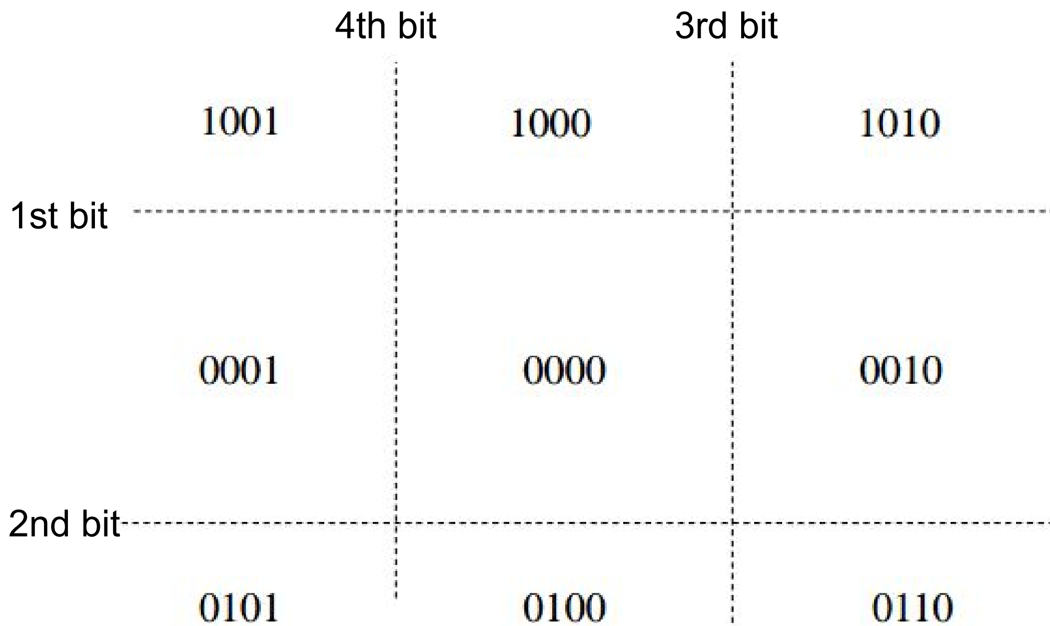


Cohen-Sutherland 2D outcodes



- The whole space is split into 9 regions
- Only the center region is visible
- Each region is encoded by four bits

Cohen-Sutherland 2D outcodes



- 4-bit code called: *Outcode*
- First bit : above top of window, $y > ymax$
- Second bit : below bottom, $y < ymin$
- Third bit : to right of right edge, $x > xmax$
- Fourth bit : to left of left edge, $x < xmin$

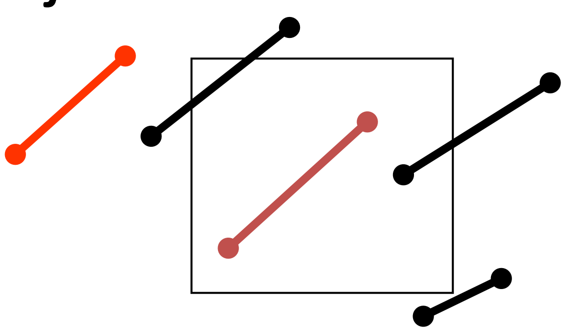
Cohen-Sutherland algorithm

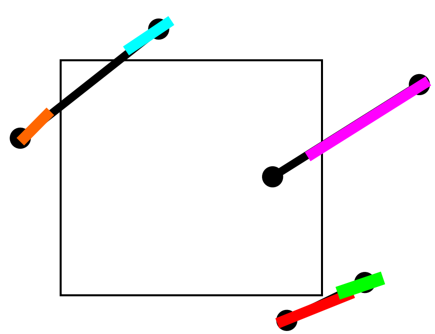
While (true) {

1. Check if the line segment is trivial accept/reject

2. Otherwise clip the edge and shorten

}





Recap of AND/OR operators

0 OR 0 = 0

0 OR 1 = 1 (if either is true, true)

1 OR 1 = 1

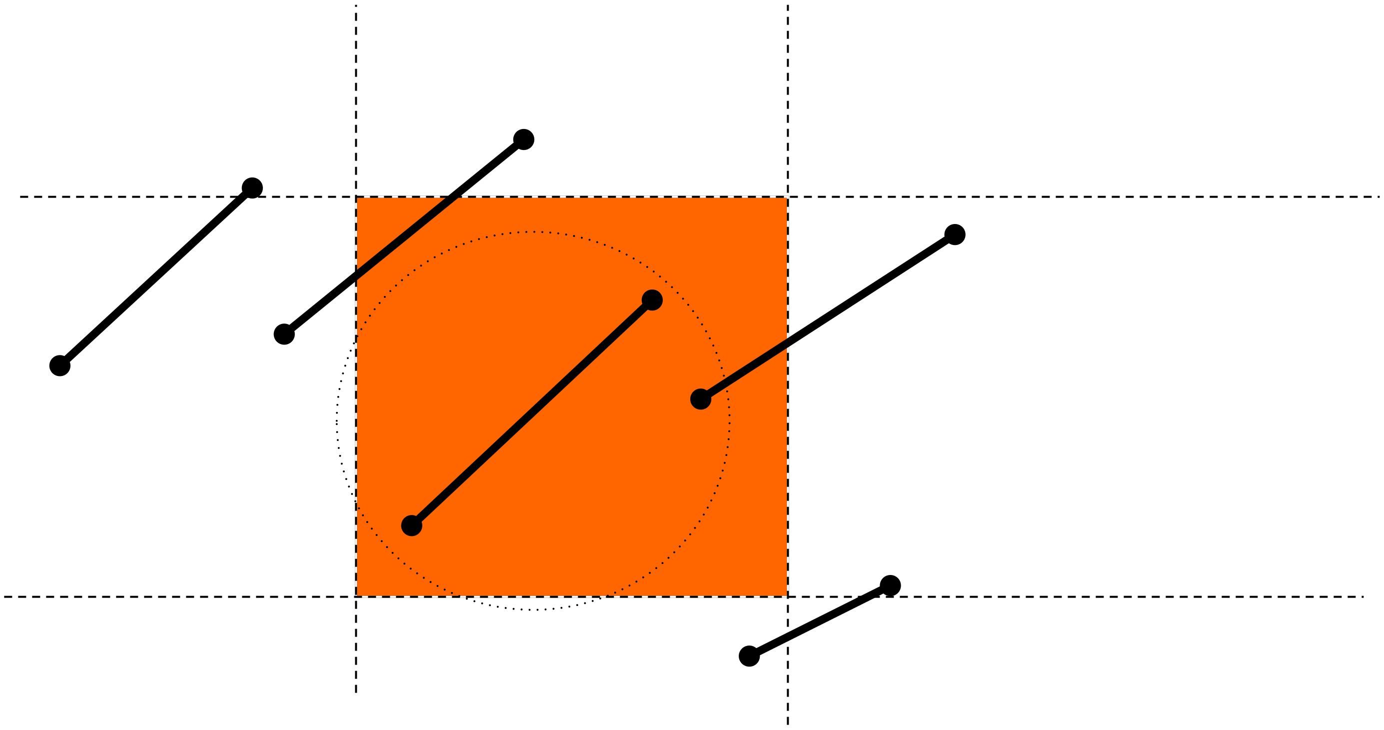
0 AND 0 = 0

1 AND 0 = 0 (if both are true, true)

1 AND 1 = 1

What is a trivial accept?

- All line vertices lie inside box → accept.
  - Apply an ‘OR’ operation to the outcodes of two endpoints



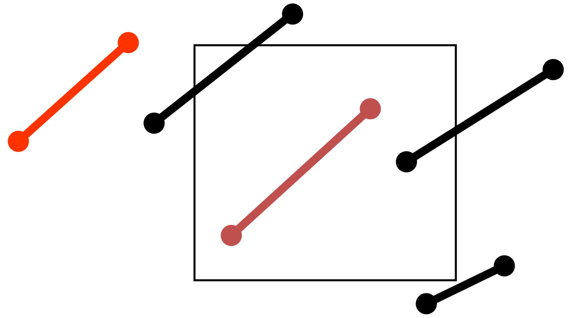
Cohen-Sutherland algorithm

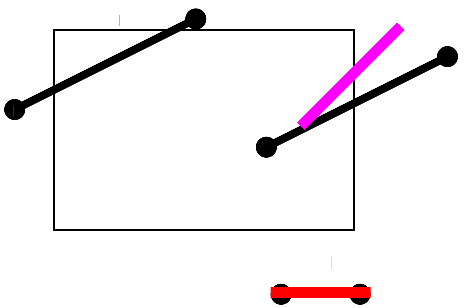
While (true) {

1. Check if the line segment is trivial accept/**reject**

2. Otherwise clip the edge and shorten

}

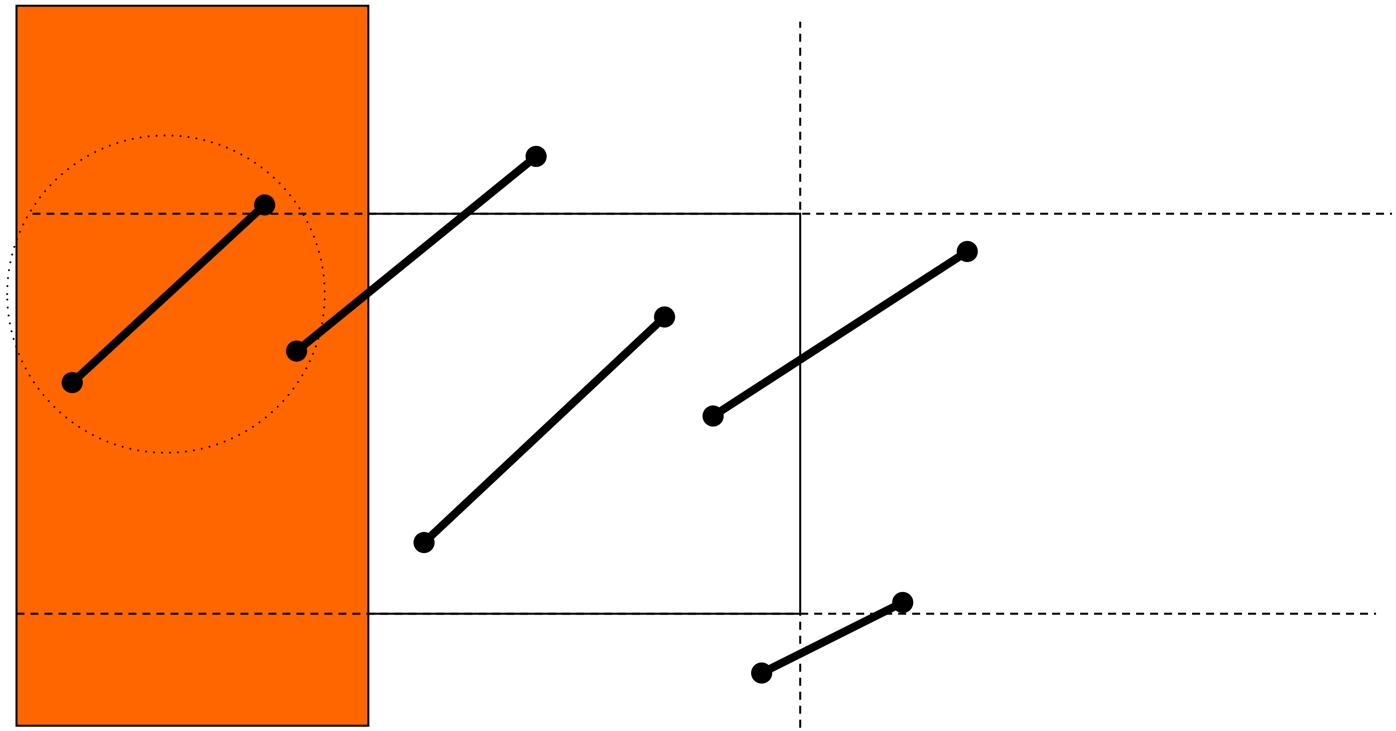




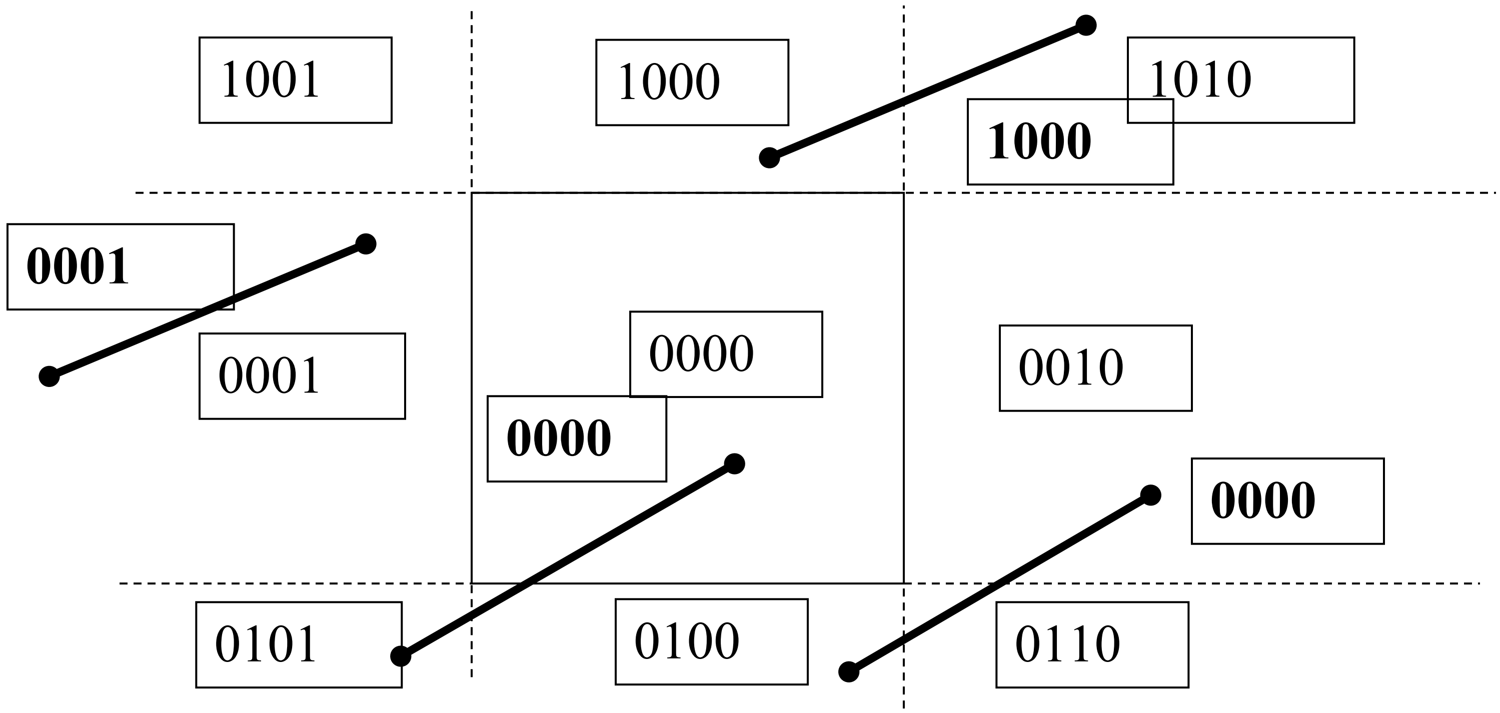
What is a trivial reject?

All line vertices lie outside and on same side → reject.

- Apply an 'AND' operation to the two endpoints
- If not '0000', then reject

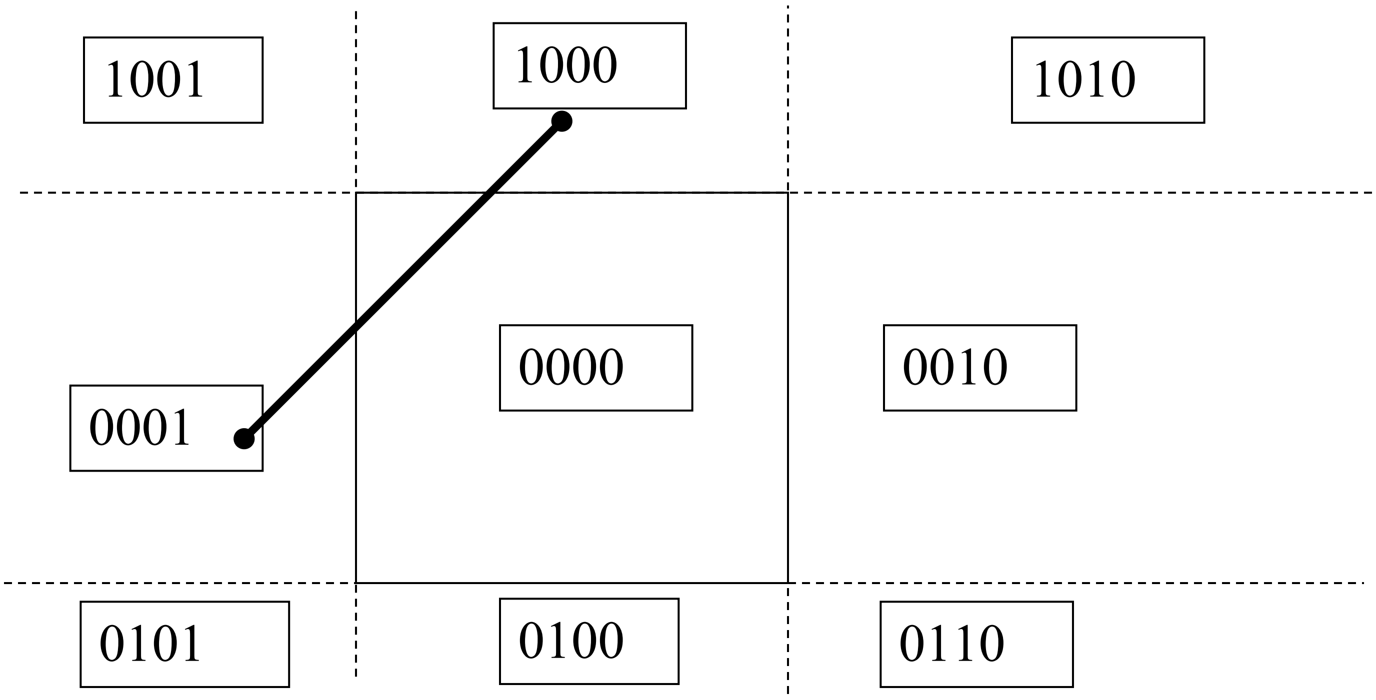


Cohen-Sutherland 2D outcodes



Logical AND between codes for 2 endpoints,
Reject line if non-zero – trivial rejection.

What about this one?



Logical AND between codes for 2 endpoints,
Reject line if non-zero – trivial rejection.

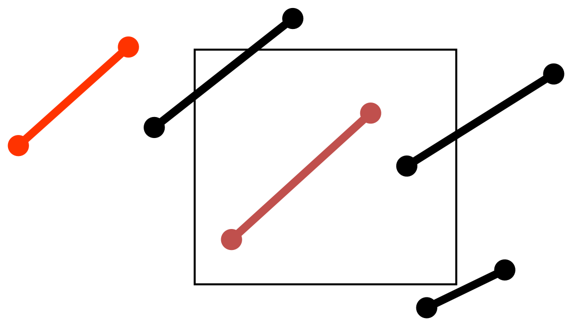
Cohen-Sutherland algorithm

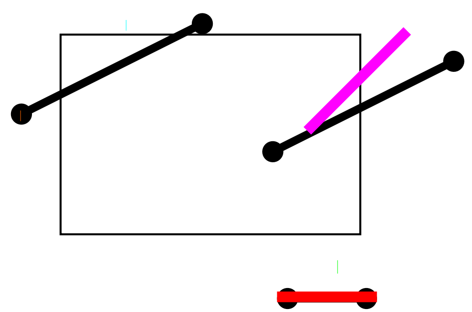
While (true) {

1. Check if the line segment is trivial accept/**reject**

2. **Otherwise clip the edge and shorten**

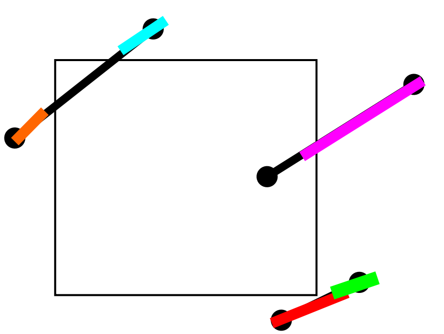
}





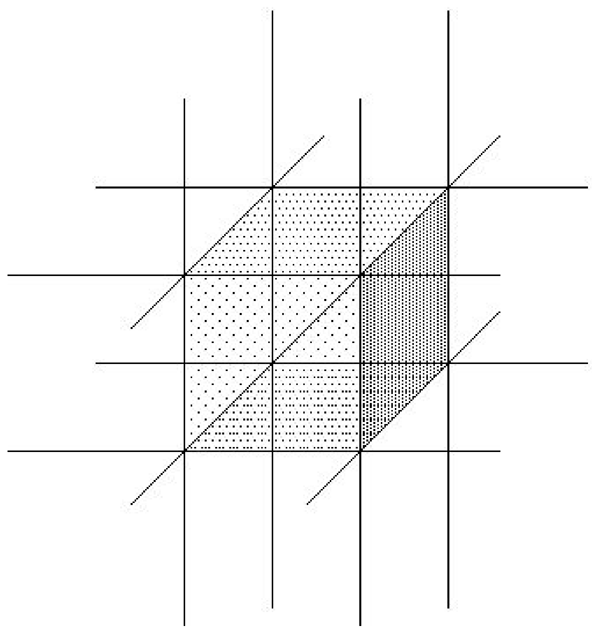
Line Intersection.

- Clip the line by edges of the rectangle
- Select a clip edge based on the outcode, split and feed the new segment on the side of the rectangle back into algorithm



Cohen-Sutherland algorithm

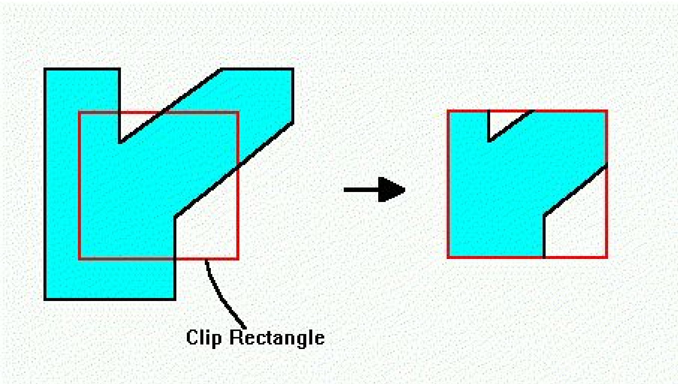
- How to extend to 3D?
  - Also clipping the lines using front / back planes
- How many bits needed for the outcode?



Polygon Clipping: Sutherland-Hodgman's algorithm

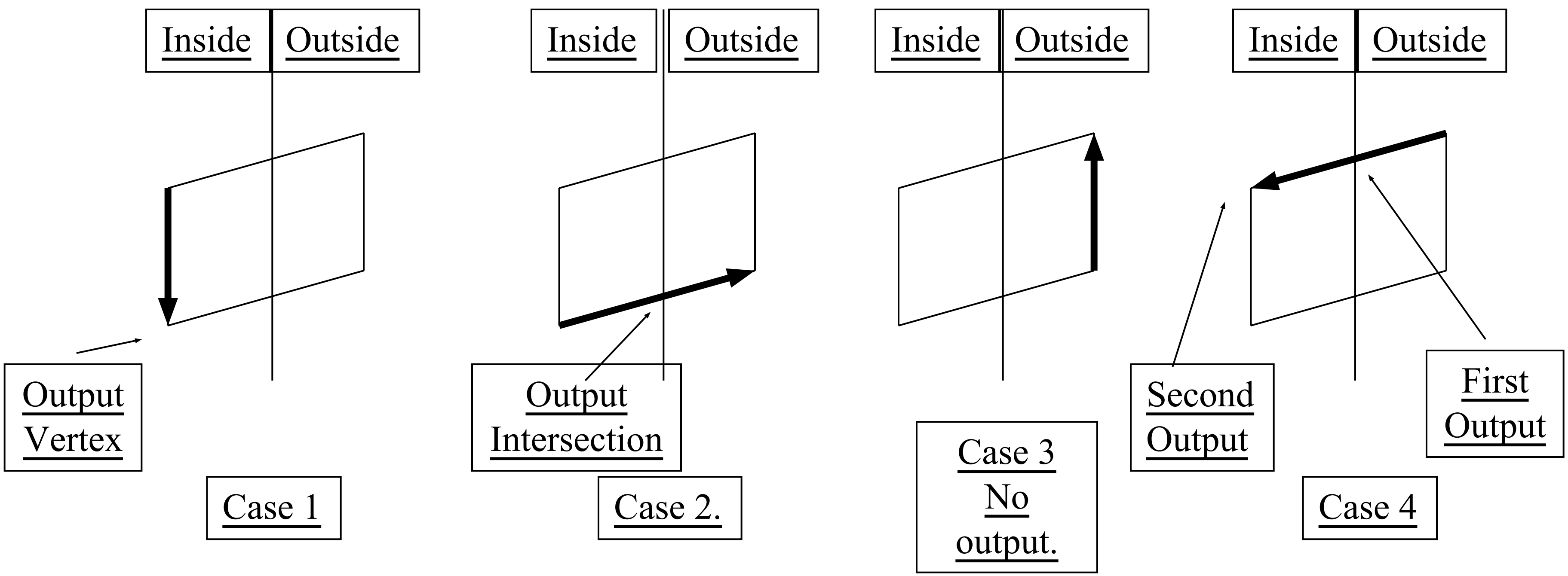
- A systematic approach to clip polygons
- Input : A 2D polygon
- Output : a list of vertices of the clipped polygon

Polygons are clipped at each edge of the window while traversing the polygon



Sutherland-Hodgman's algorithm

- The edges of the polygon are traversed
- The edges can be divided into four types

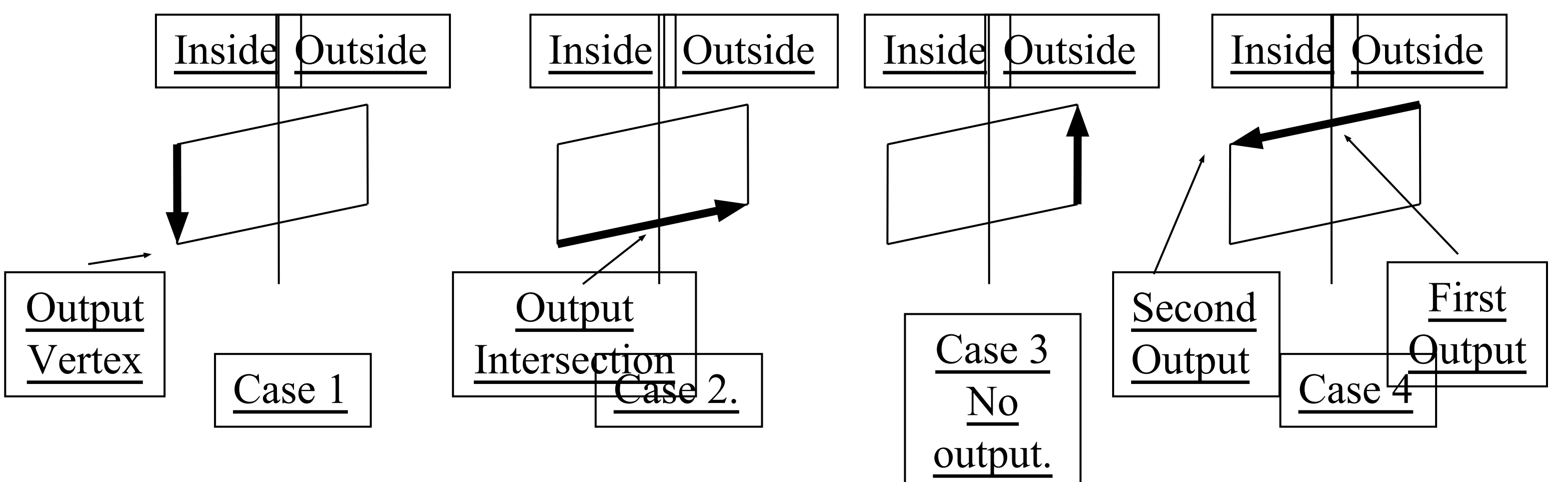


Sutherland-Hodgman's algorithm

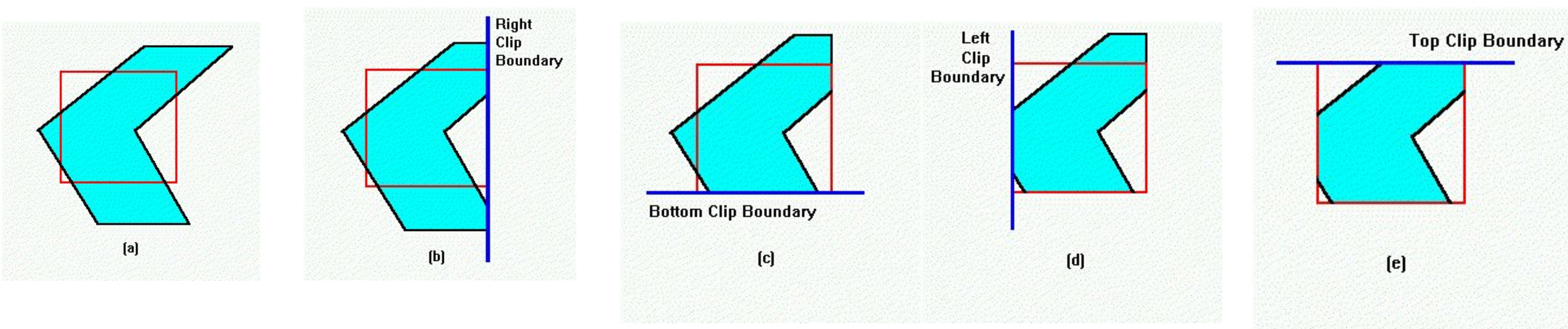
For each edge of the clipping rectangle

For each edge of the polygon (connecting pi, pi+1)

- If case 1 add p+1 to the output
- If case 2 add interaction to output
- If case 4 add intersection and p+1 to output



Example


(a)
Right
Clip
Boundary
(b)
Bottom Clip Boundary
(c)
Left
Clip
Boundary
(d)
Top Clip Boundary
(e)

Sutherland-Hodgman algorithm

- How to extend to 3D?

Summary

- Projection
  - Perspective, parallel (orthographic) projection
  - Canonical view volume
- Clipping
  - Cohen-Sutherland's algorithm
  - Sutherland-Hodgmans's algorithm

Readings

- Foley et al. Chapter 6 – all of it,
  - Particularly section 6.5
- Introductory text, Chapter 6 – all of it,
  - Particularly section 6.6
- Akenine-Moller, Real-time Rendering Chapter 3.5
- Clipping lines, polygons
  - Foley et al. Chapter 3.12, 3.14
  - http://www.cc.gatech.edu/grads/h/Hao-wei.Hsieh/Haowei.Hsieh/mm.html