

Computer Graphics

Review

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Overview

- Review
- Some additional things

Review

- Graphics pipeline
- Transformation of 3D objects, View transformation
- Projection and Rasterization
- Illumination
- Hidden surface removal
- Texture mapping, bump mapping, environment mapping
- Anti-aliasing
- Shadows
- Global Illumination
- Curves and surfaces

Graphics Pipeline

Three stages

- **Application stage**
 - Entirely done in the CPU
 - Loading data, getting user input
- **Geometric stage**
 - Applying transformation to vertices
 - Computing the attributes for the vertices
- **Rasterization stage**
 - Per pixel computation
 - Converting the continuous representation to the discrete representations

Which stage do the following processes belong to?

- Getting the input from the user and deciding the view point
- Anti-aliasing
- Transforming the vertex location into that of the canonical view volume
- Computing the color of the pixel by Phong shading

Object Representation

Parameteric surfaces

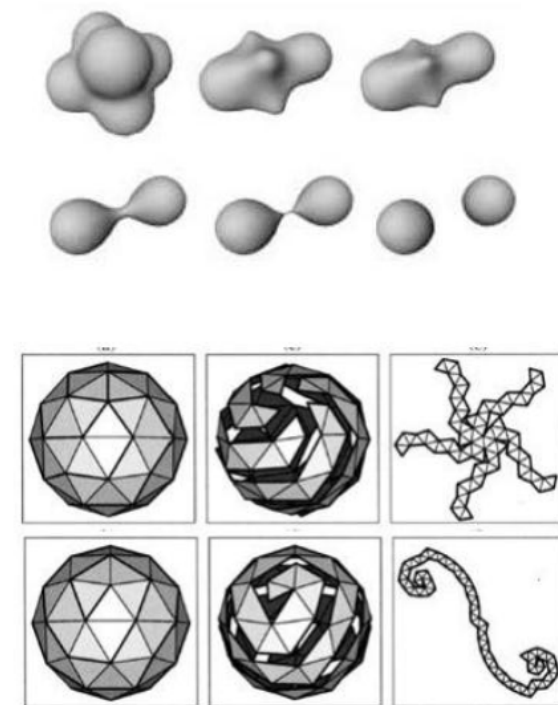
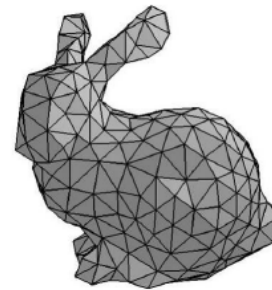
- Bezier, B-spline, NURBS

Implicit surfaces

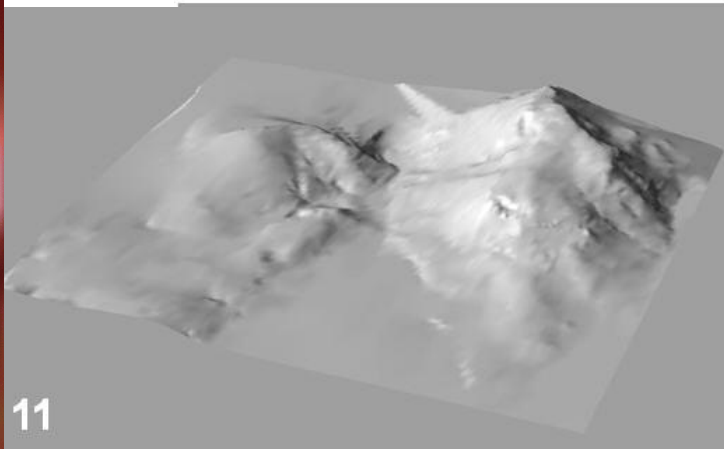
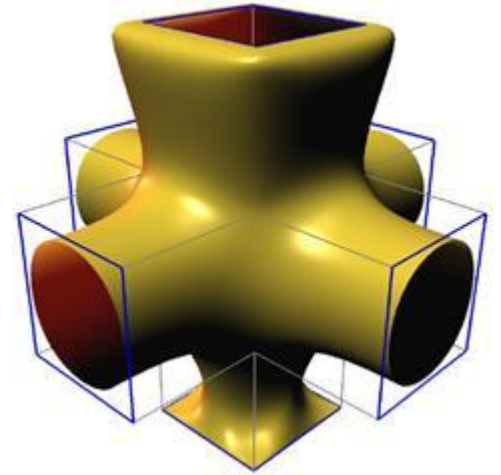
- Metaballs

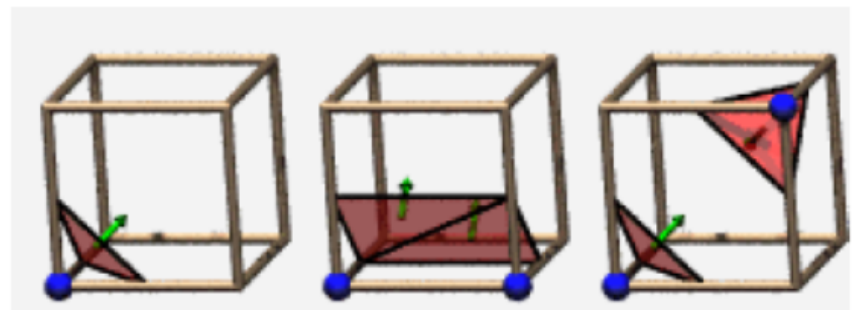
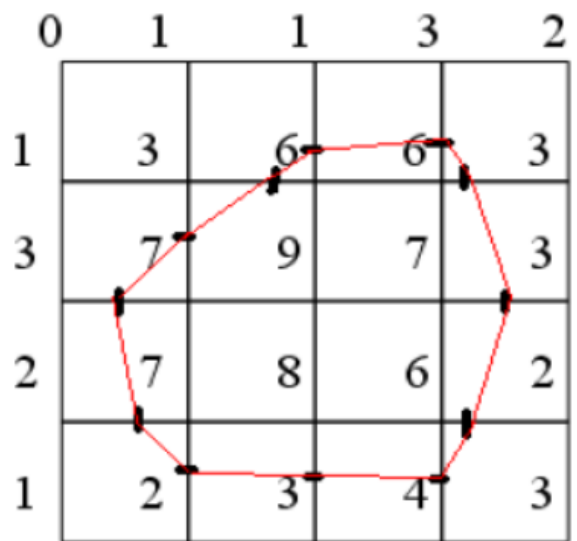
Polygon mesh

- Polygon format
- Triangle strips
- Subdivision surface



What are the good ways to model the following objects?





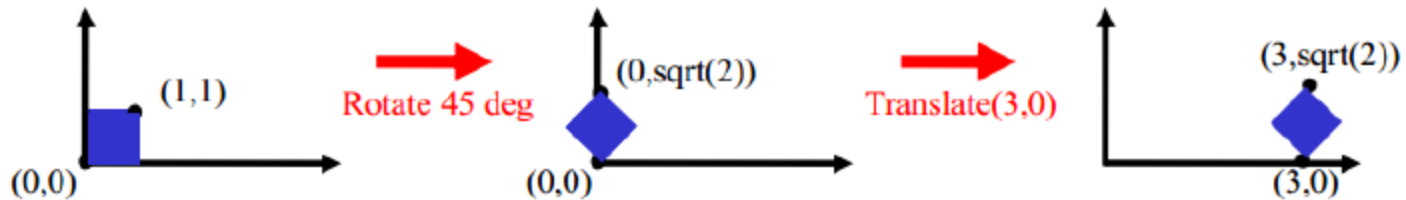
Transformation Matrix

Translation, Rotation and Scaling

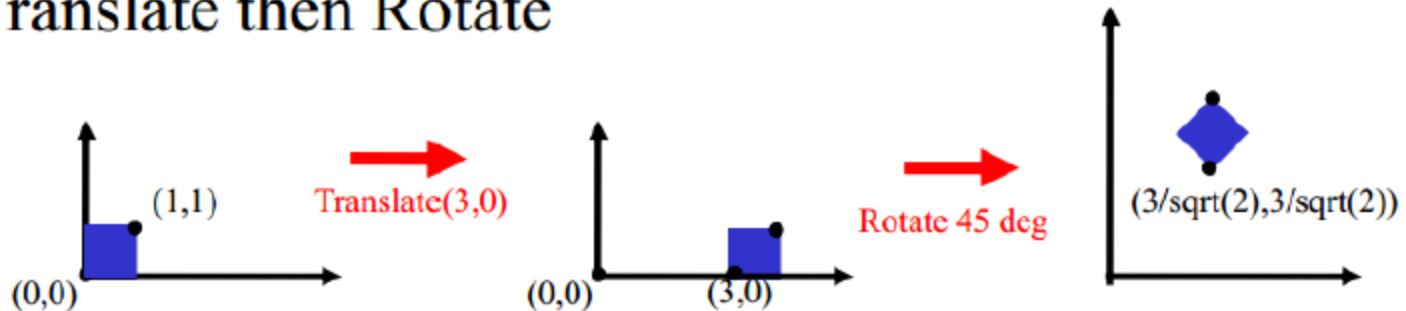
$$T(d_x, d_y, d_z) = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_z(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad S(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix concatenation

Rotate then Translate



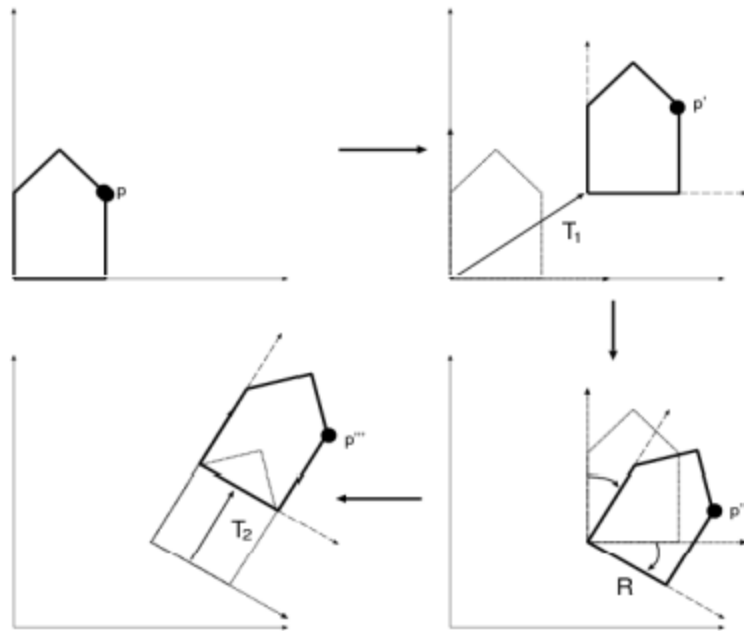
Translate then Rotate



$$x' = RT x$$

$$x'' = TR x$$

Transformations of coordinate systems - Example



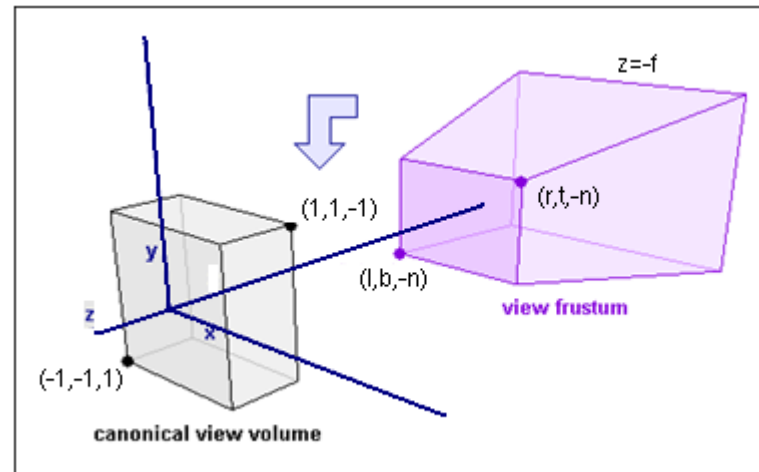
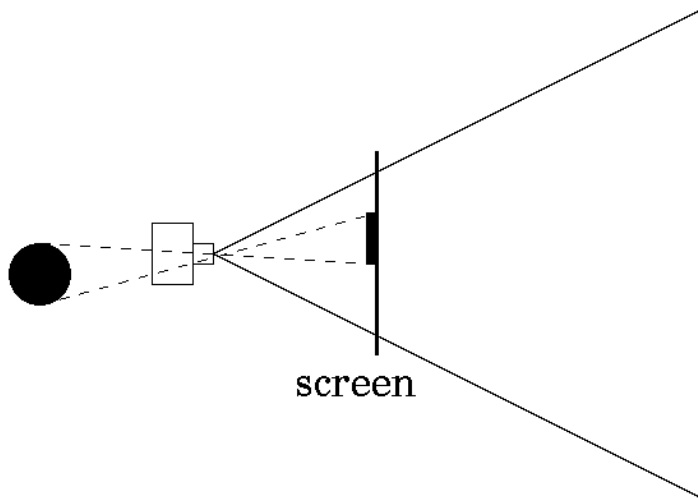
$$\begin{aligned} p' &= T_1 p \\ p'' &= T_1 R p \\ p''' &= T_1 R T_2 p \end{aligned}$$

Concatenate local transformation matrices from left to right

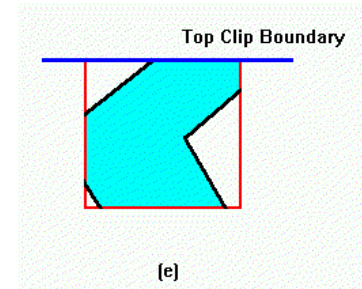
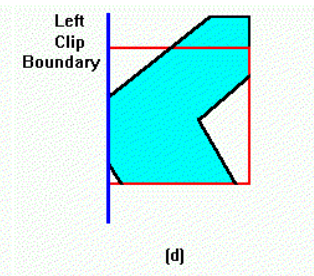
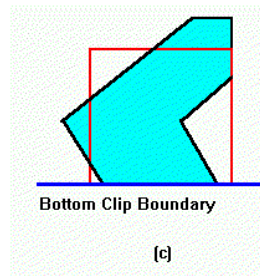
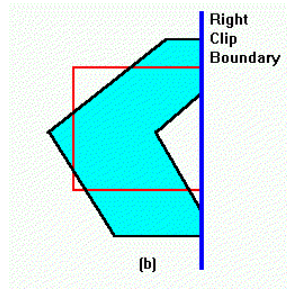
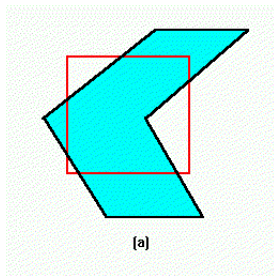
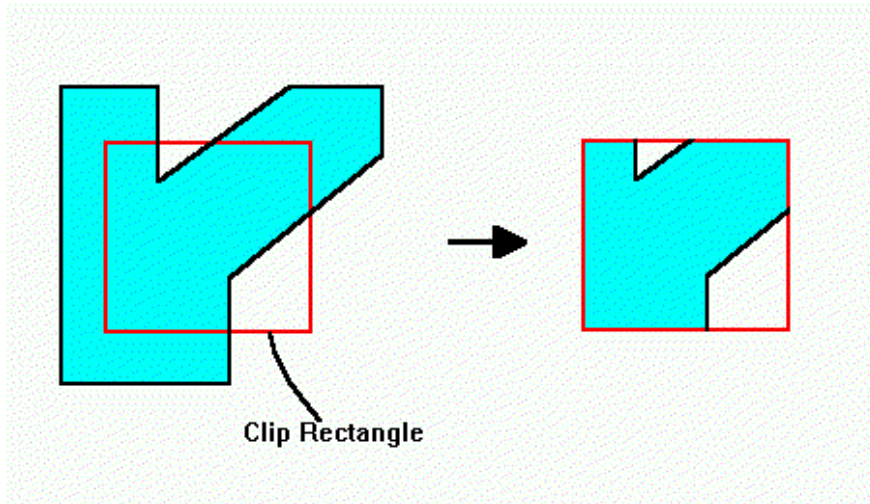
Can obtain the local – world transformation matrix

p', p'', p''' are the world coordinates of p after each transformation

Projection



Clipping



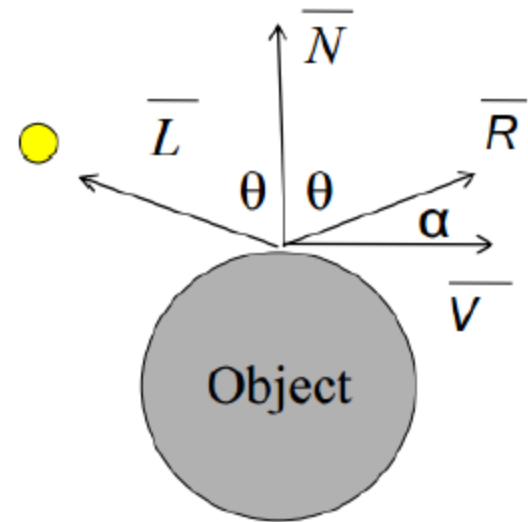
Illumination and Shading

Phong Illumination Model

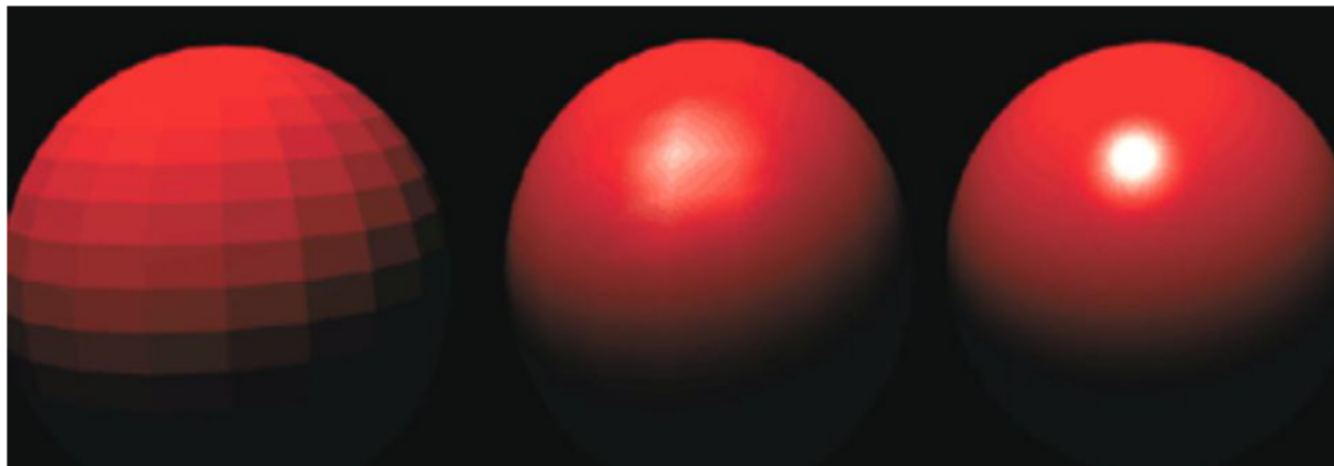
$$I_{\lambda} = I_a k_a + \sum_{p=1}^{\text{lights}} I_p [k_d (\overline{N} \cdot \overline{L}) + k_s (\overline{V} \cdot \overline{R})^n]$$

$\cos \theta$ $\cos \alpha$

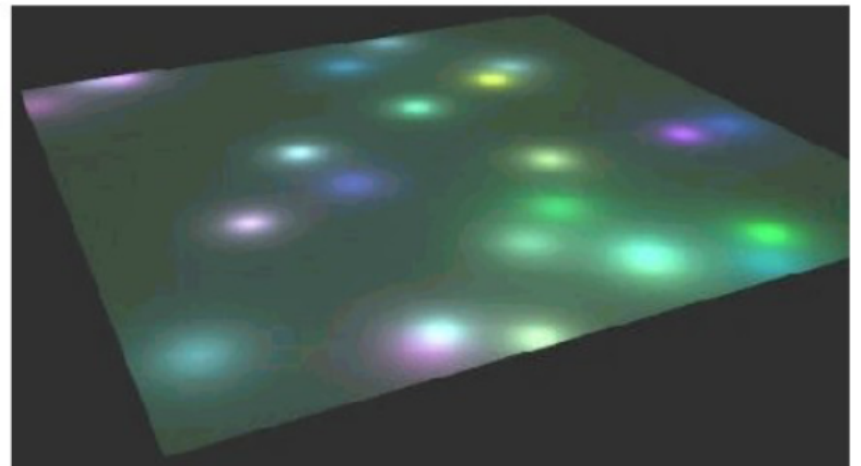
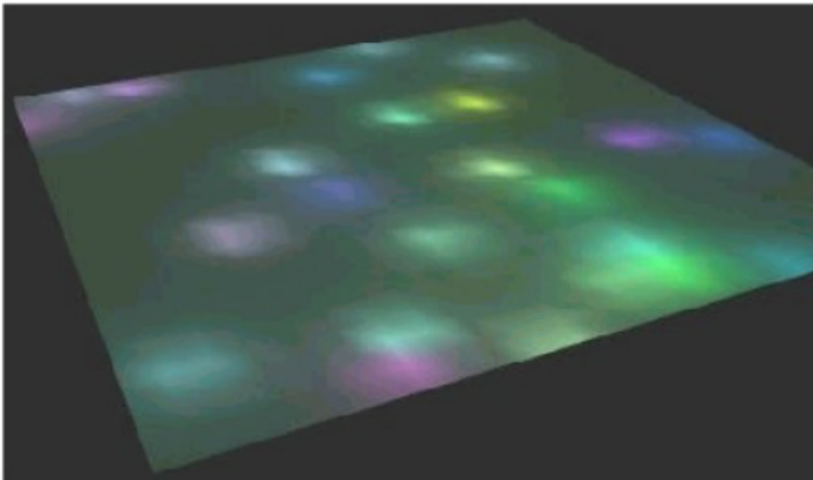
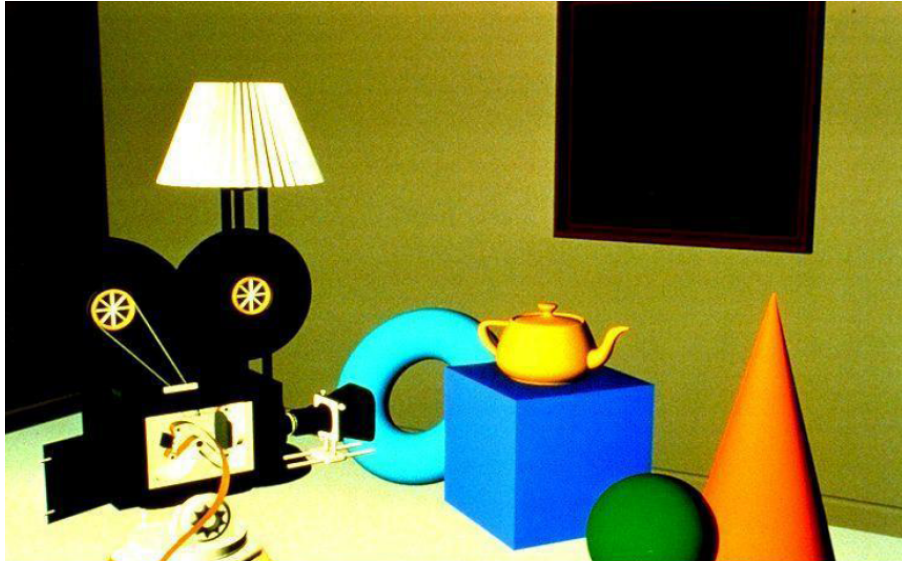
↓ ↓



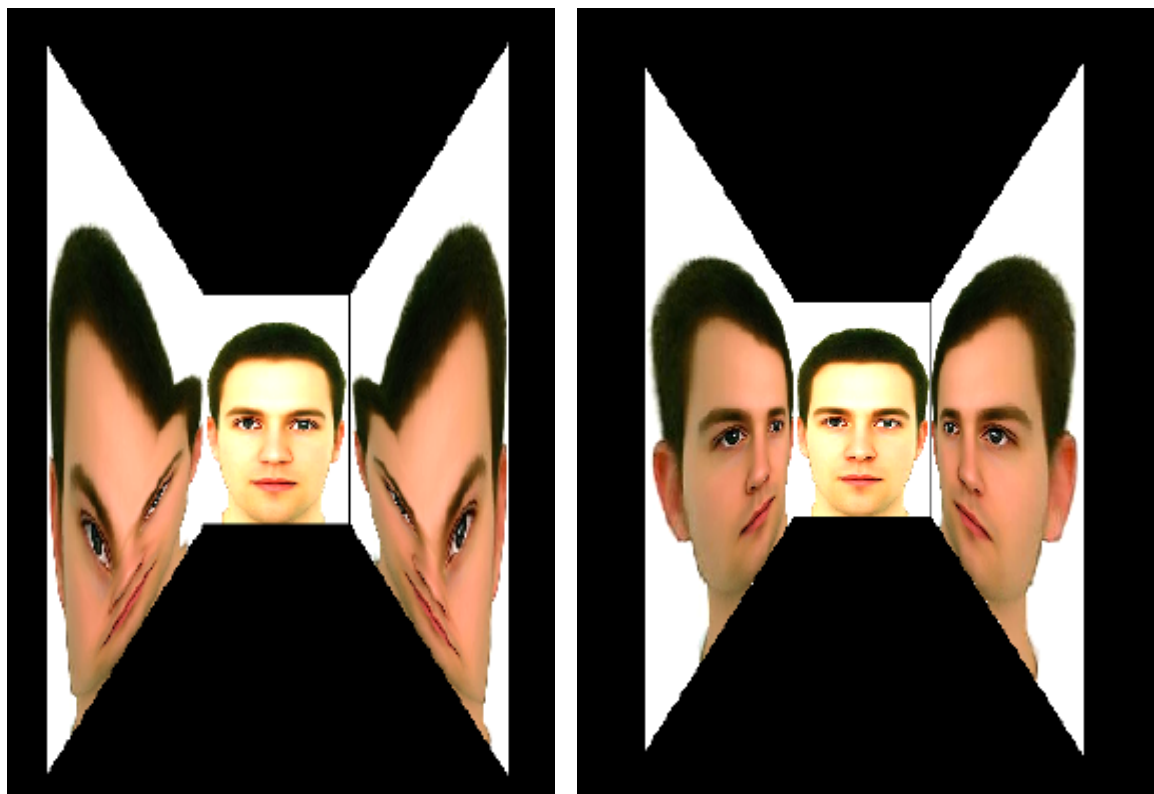
Flat shading, Gouraud shading, Phong shading



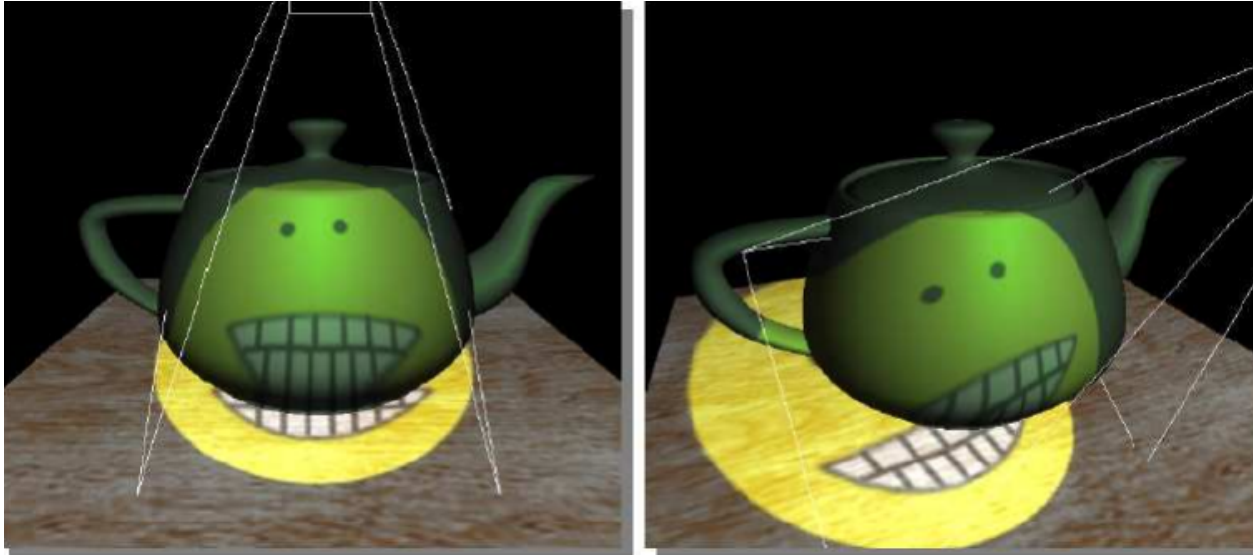
Illumination and Shading



Texture Mapping

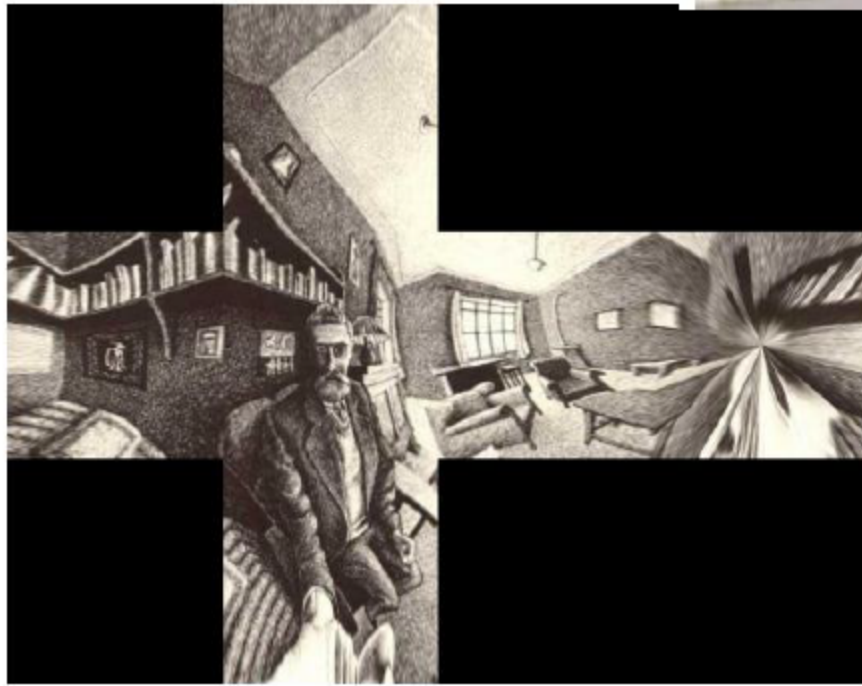
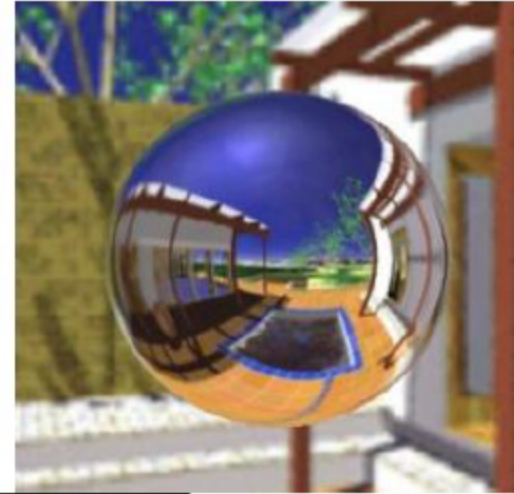


Texture Mapping

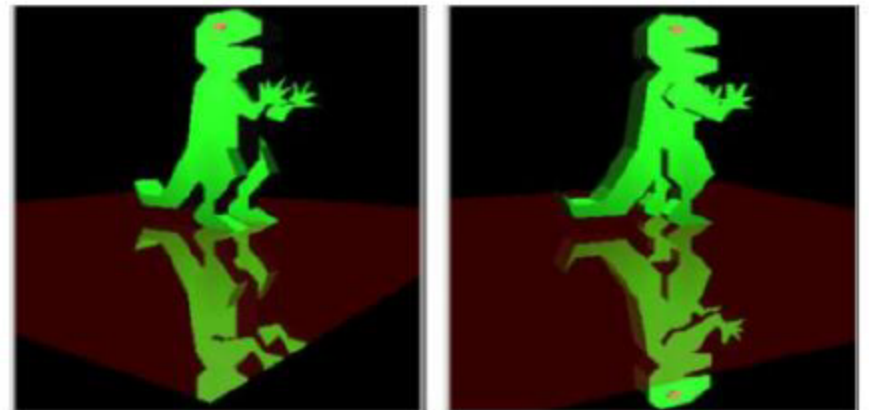
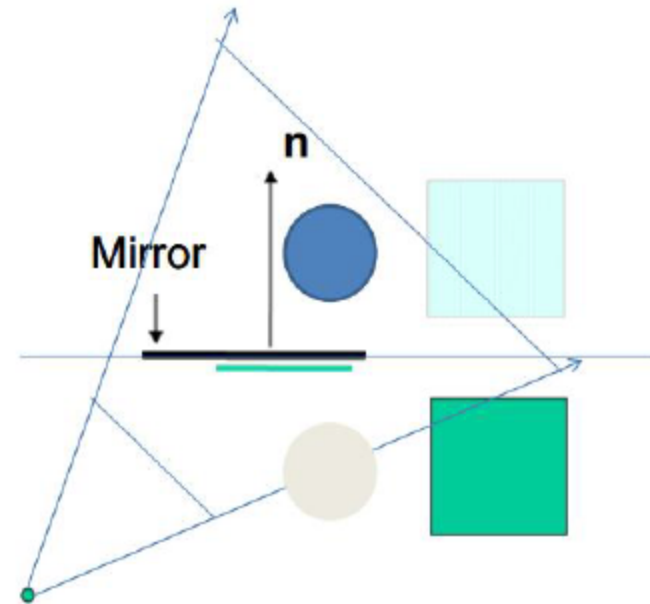
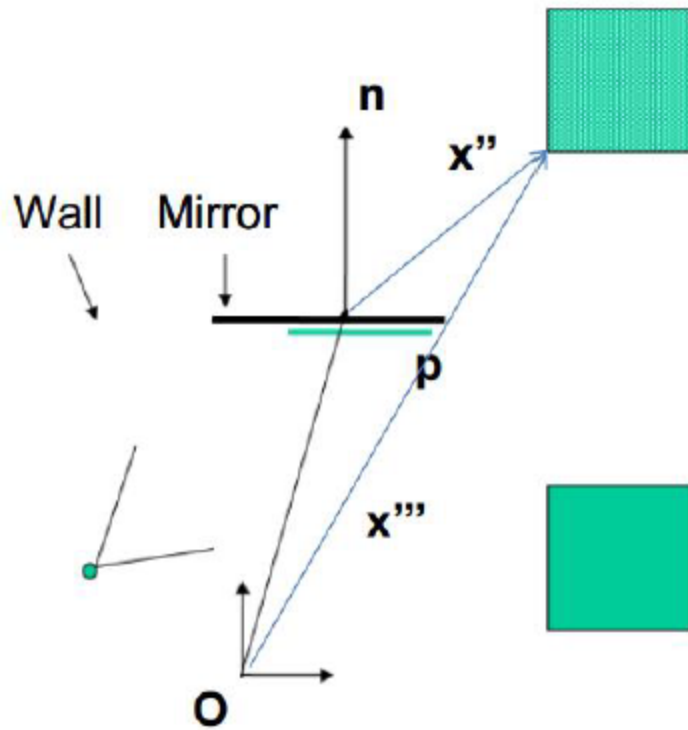


What is the mapping function?

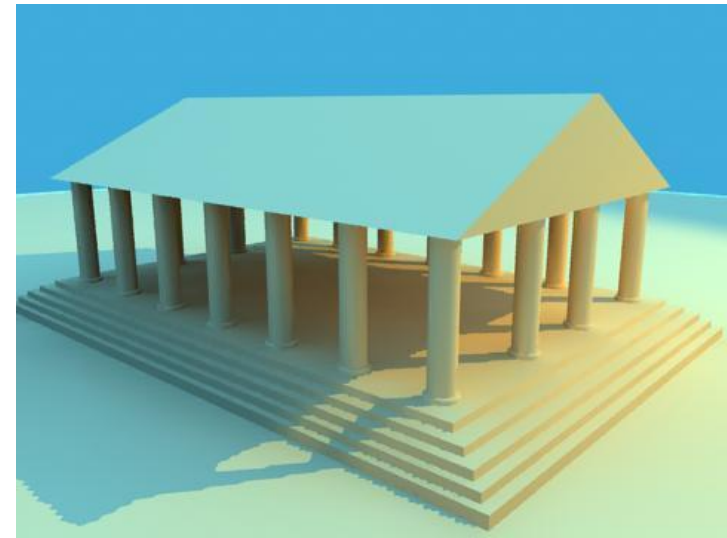
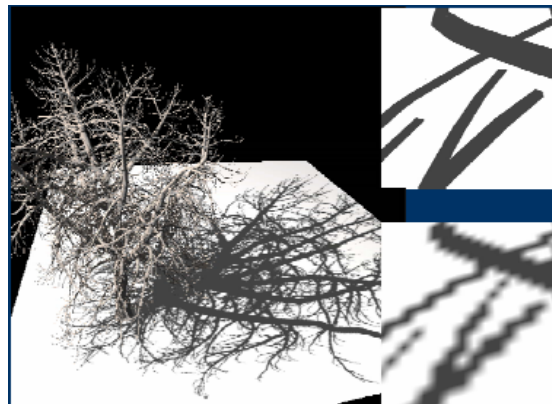
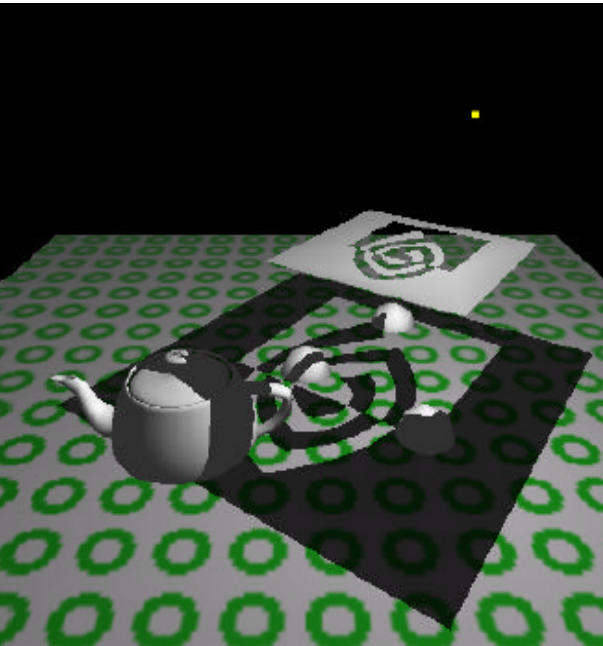
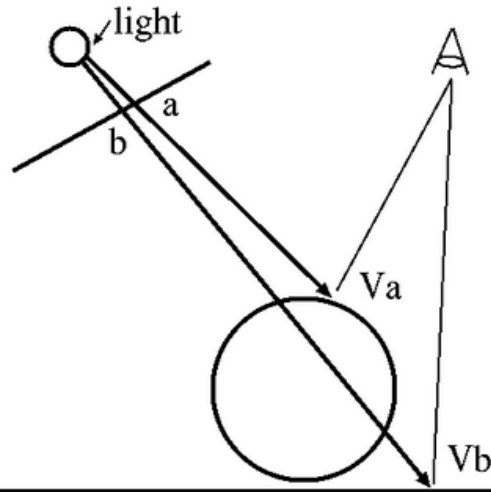
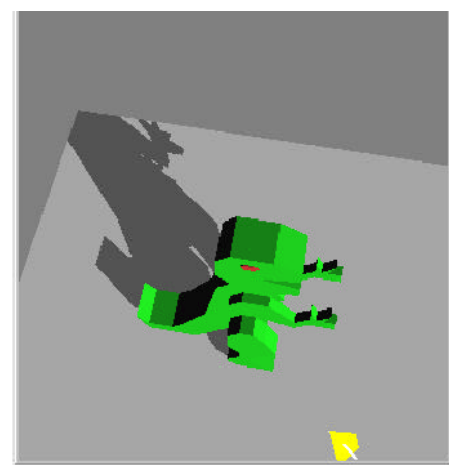
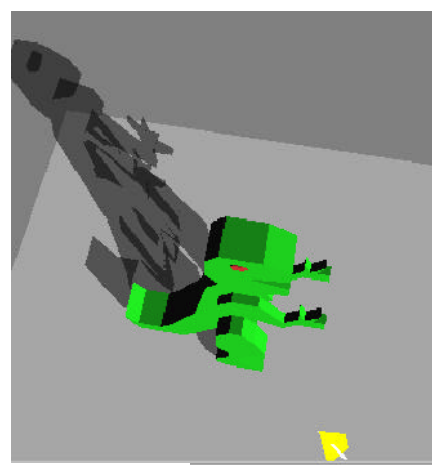
Environment Mapping



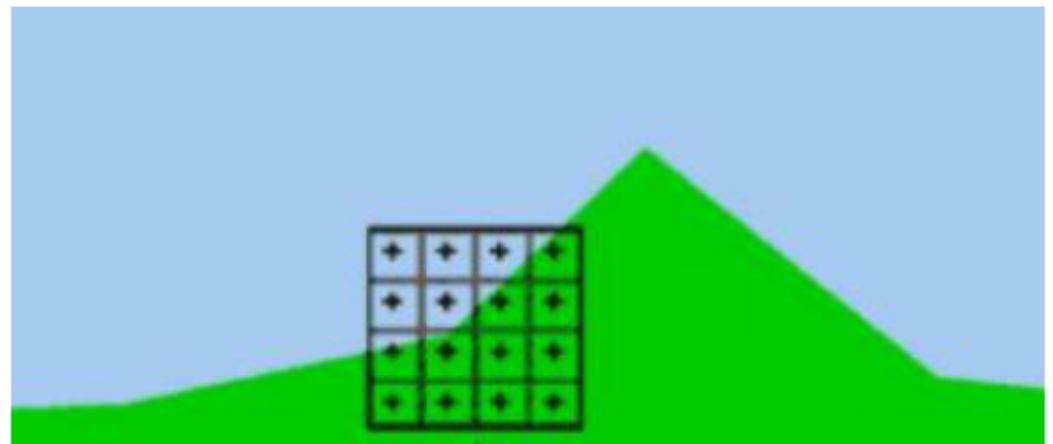
Mirrored world

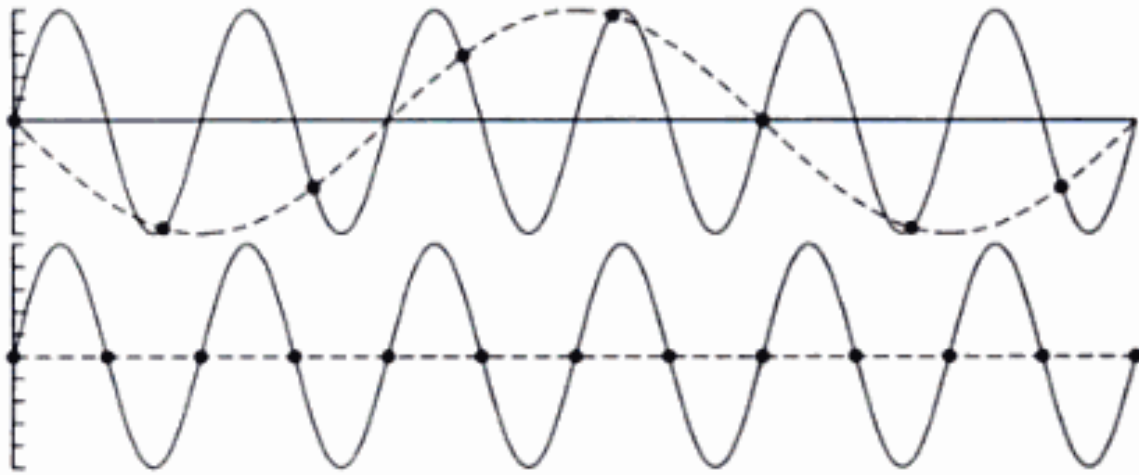


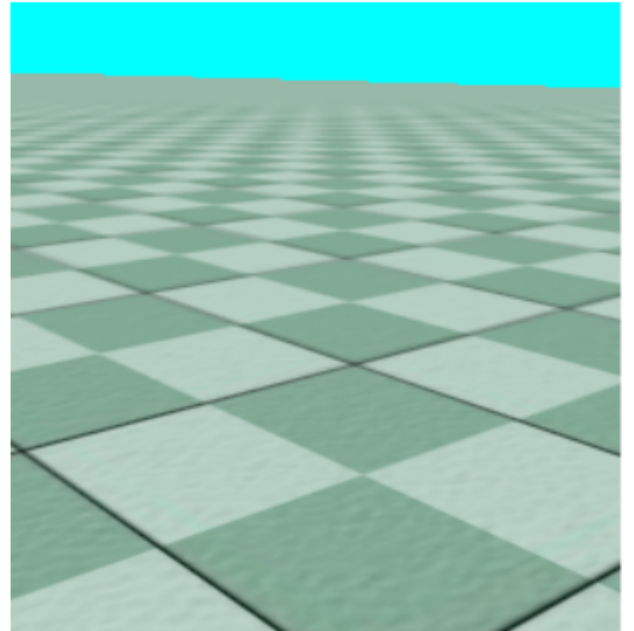
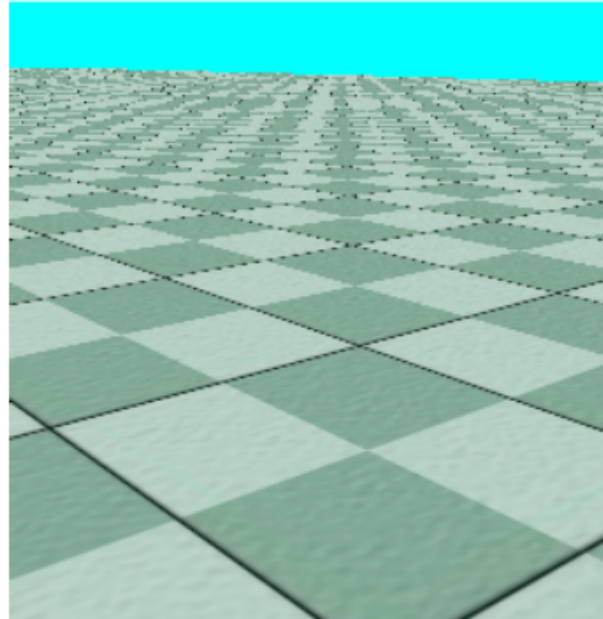
Shadows



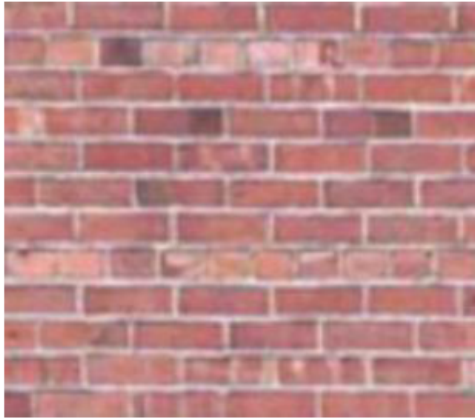
Anti-aliasing



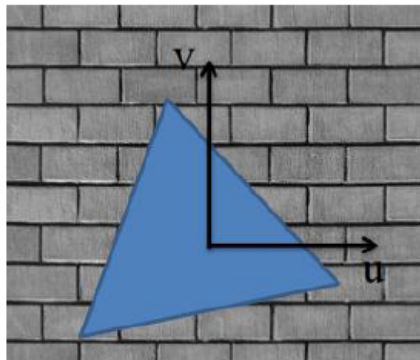
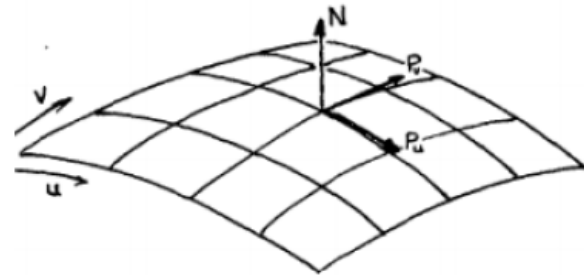




Bump mapping



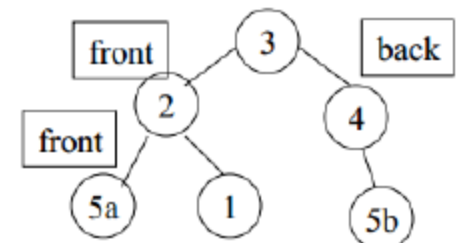
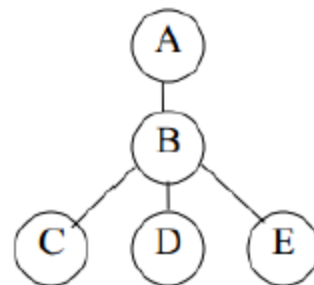
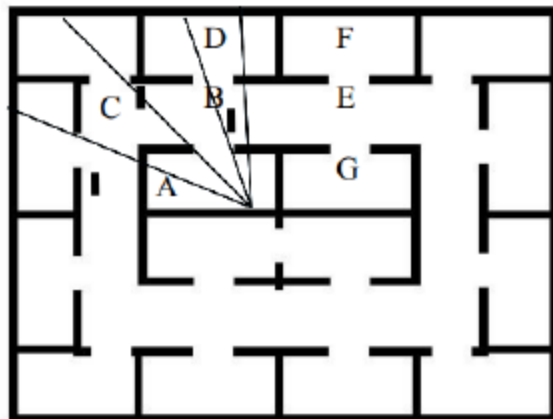
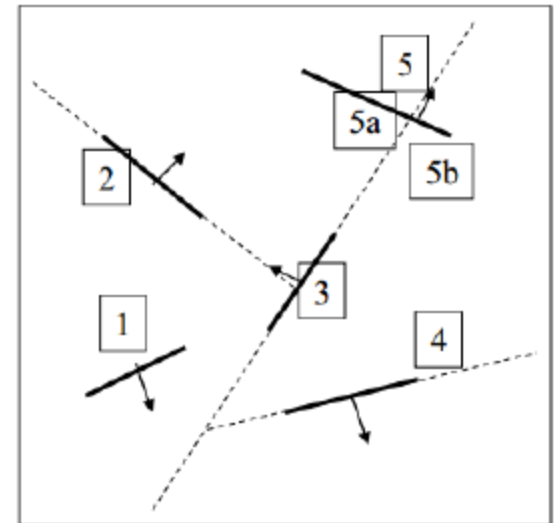
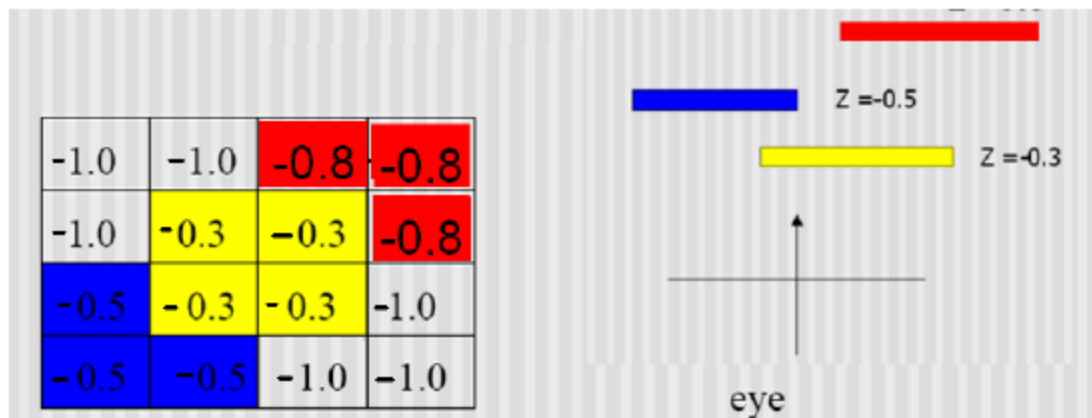
$$\mathbf{n}' = \mathbf{n} + \frac{F_u(\mathbf{n} \times \mathbf{P}_v) - F_v(\mathbf{n} \times \mathbf{P}_u)}{\|\mathbf{n}\|}$$

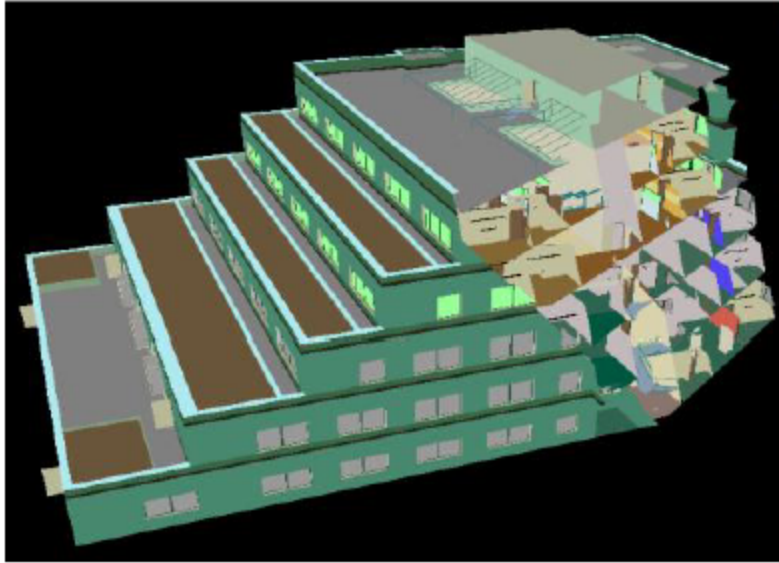


$$\begin{aligned} \frac{\partial P}{\partial x} &= \frac{\partial P}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial P}{\partial v} \frac{\partial v}{\partial x} \\ \frac{\partial P}{\partial y} &= \frac{\partial P}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial P}{\partial v} \frac{\partial v}{\partial y} \end{aligned}$$

Hidden Surface Removal

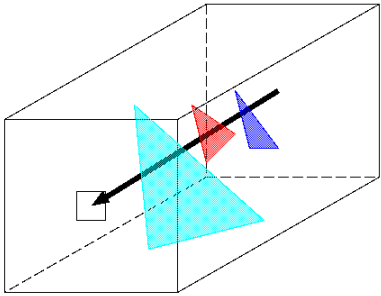
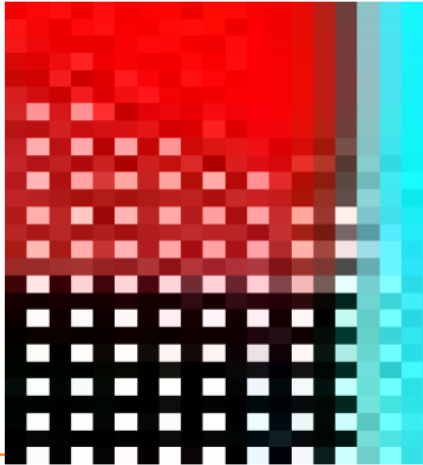
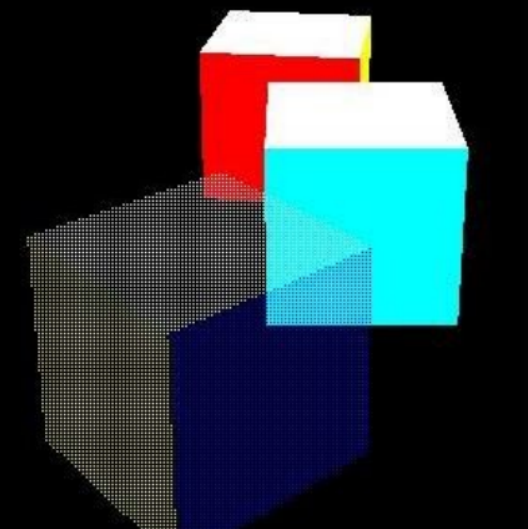
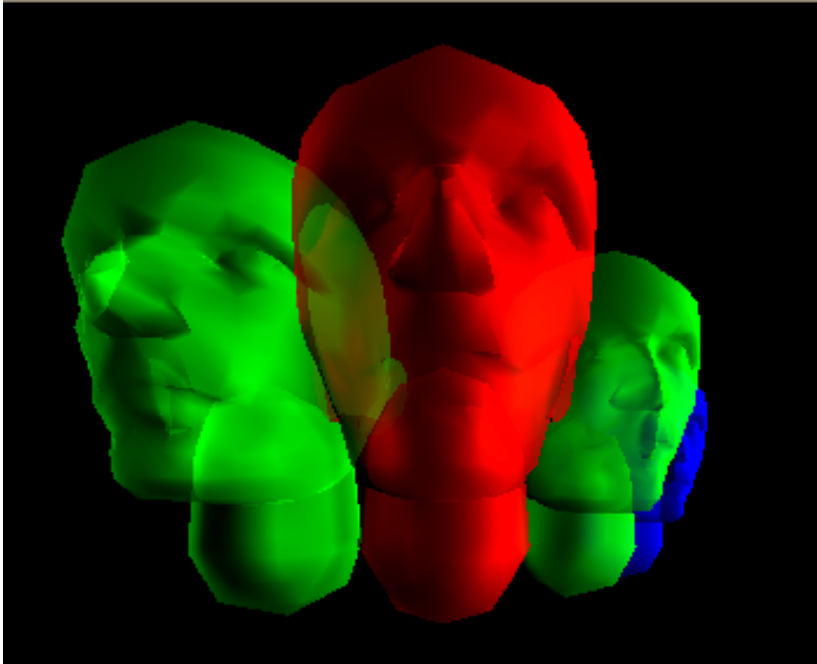
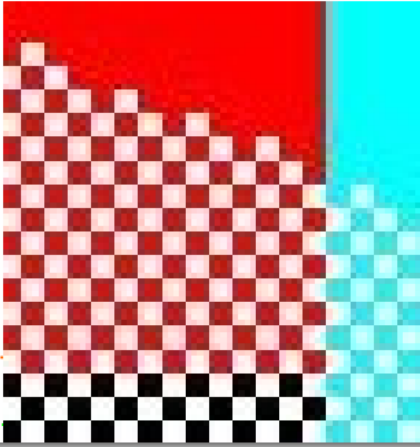
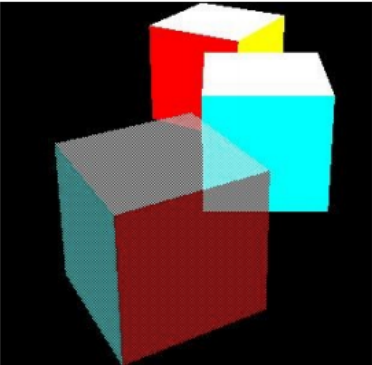
Z-buffer, BSP trees, Portal culling



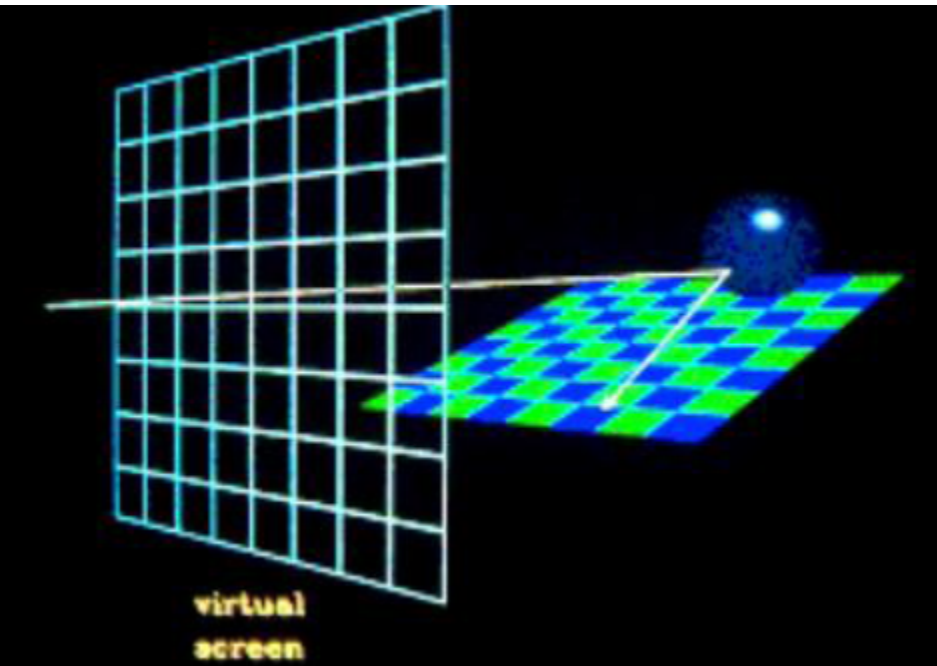
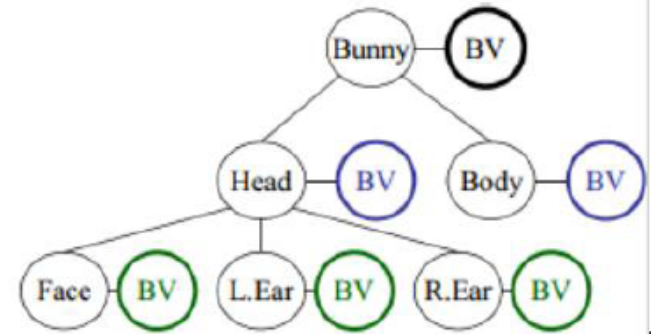
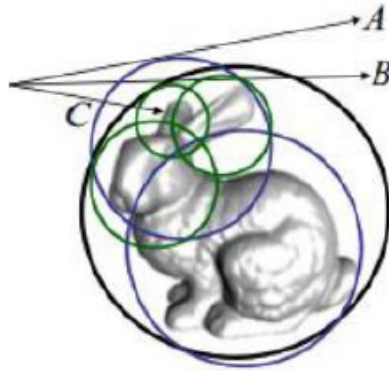


Transparency

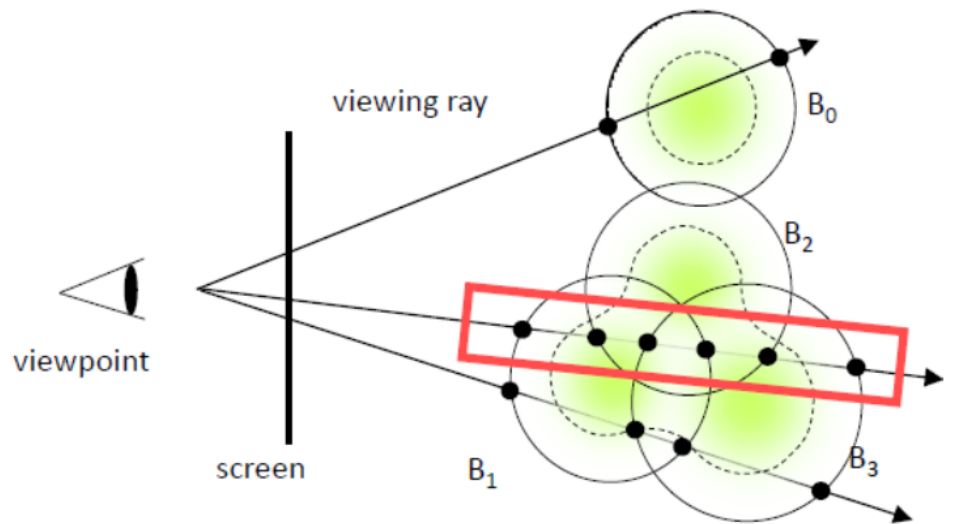
alpha = 0.5



Ray Tracing



- How to make a bounding sphere hierarchy?



Light Transport Notations

L a light source

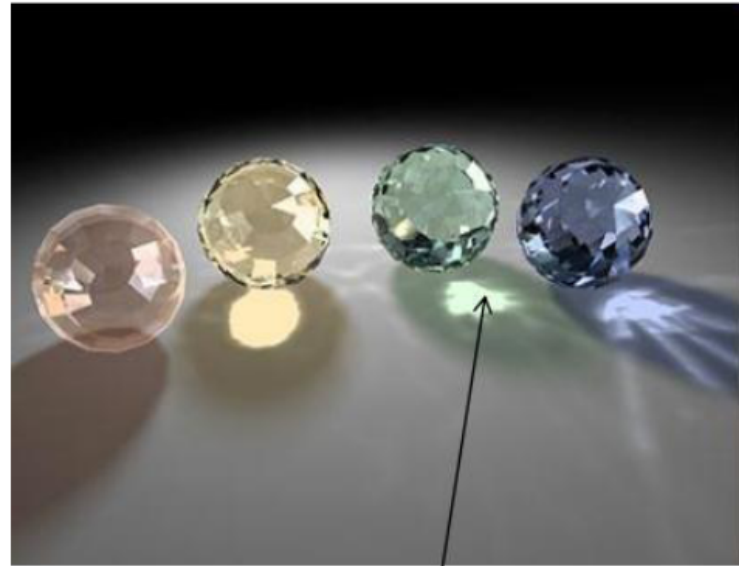
E the eye

S a specular reflection

D a diffuse reflection

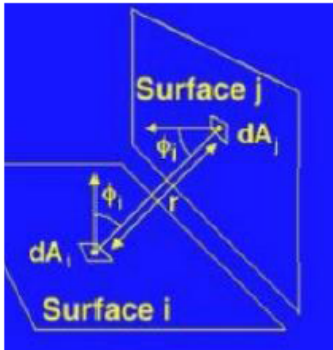


LDDE



LSDE

Radiosity



$$B_j = E_j + \rho_j \sum_{i=1}^N B_i F_{i,j}$$

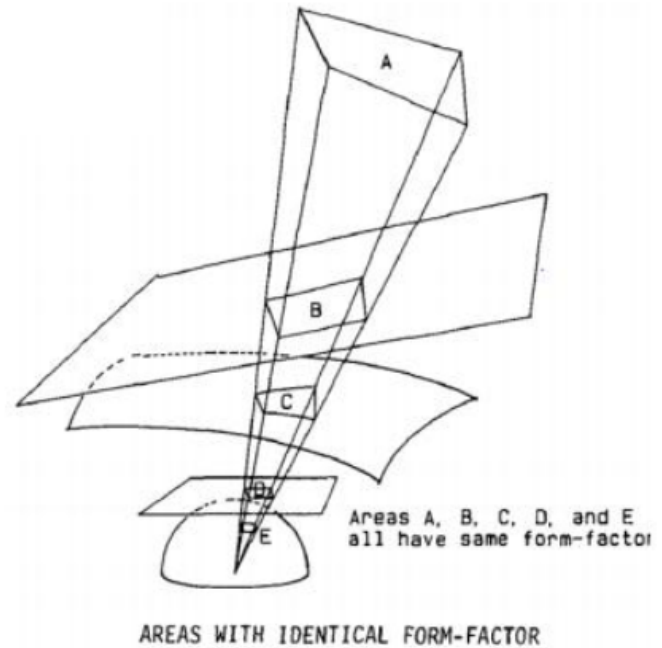
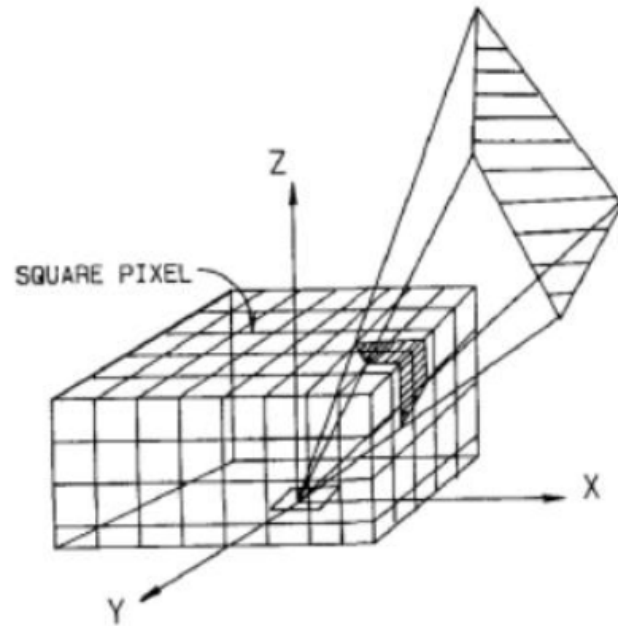
$$\begin{pmatrix} 1 - \rho_1 F_{11} & -\rho_1 F_{12} & \dots & -\rho_1 F_{1N} \\ -\rho_2 F_{21} & 1 - \rho_2 F_{22} & \dots & -\rho_2 F_{2N} \\ \vdots & \vdots & \dots & \vdots \\ -\rho_N F_{N1} & -\rho_N F_{N2} & \dots & 1 - \rho_N F_{NN} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{pmatrix} = \begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{pmatrix}$$



(a)



Hemicube



Projection, Z-buffer, symmetry

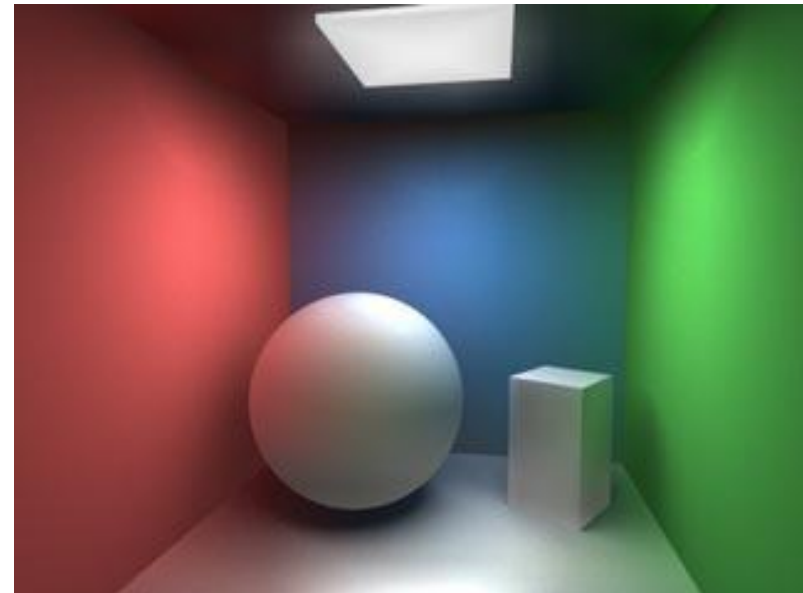
For computing the radiosity, you use an iterative approach

$$B_j = E_j + \rho_j \sum_{i=1}^N B_i F_{i,j}$$

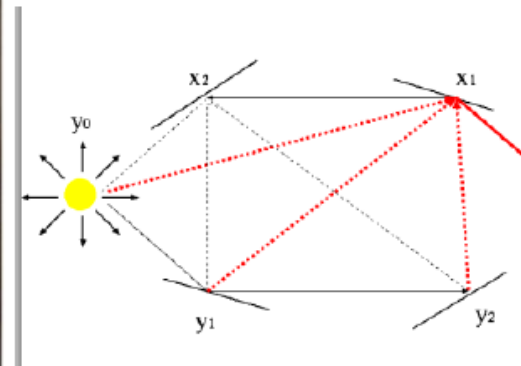
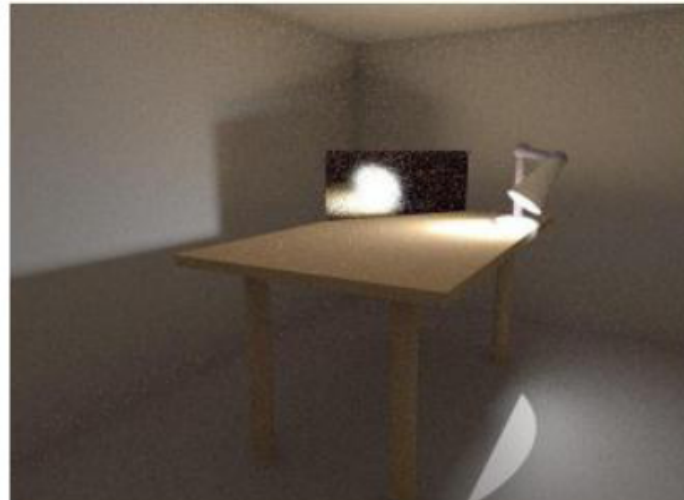
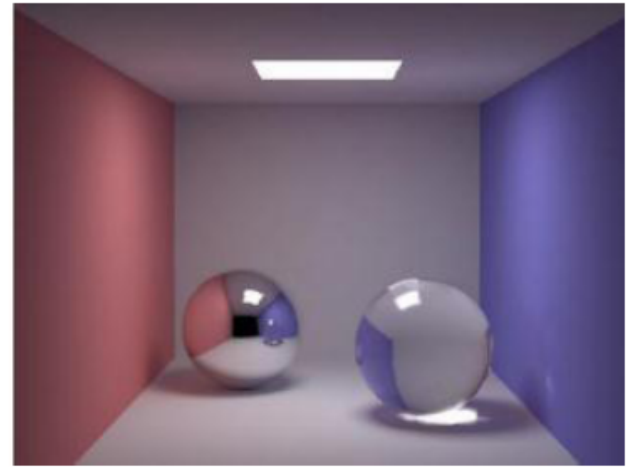
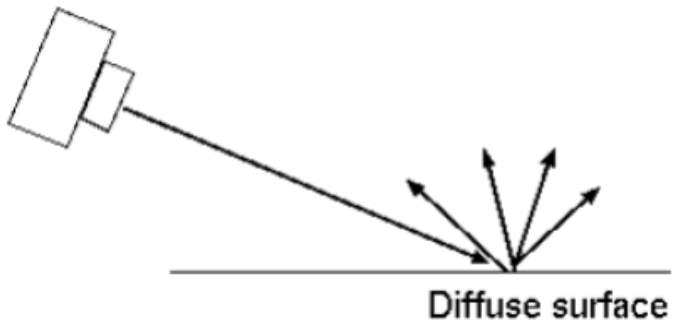
$$\downarrow$$
$$B_j^0 = E_j$$

$$B_j^{k+1} = \rho_j \sum_{i=1}^N B_i^k F_{i,j}$$

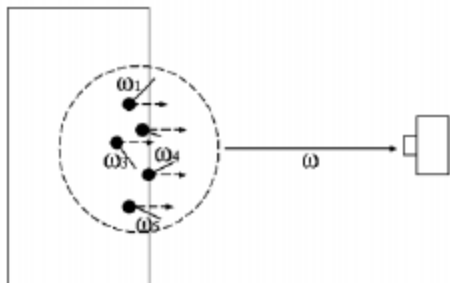
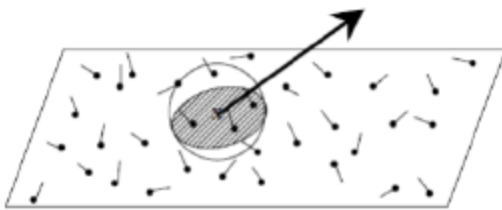
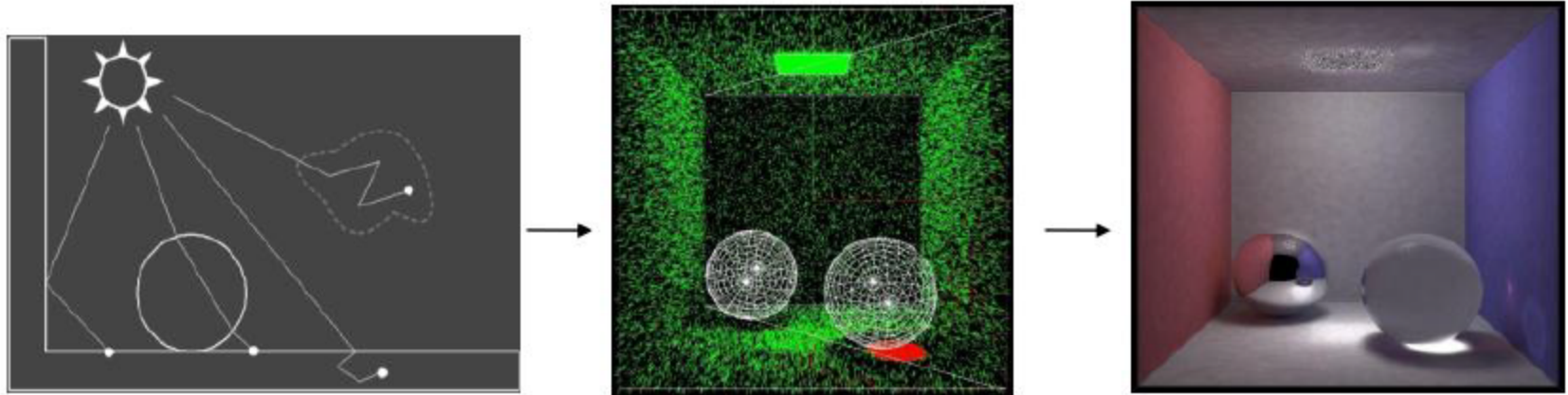
$$B = \sum_k B^k$$



Path Tracing

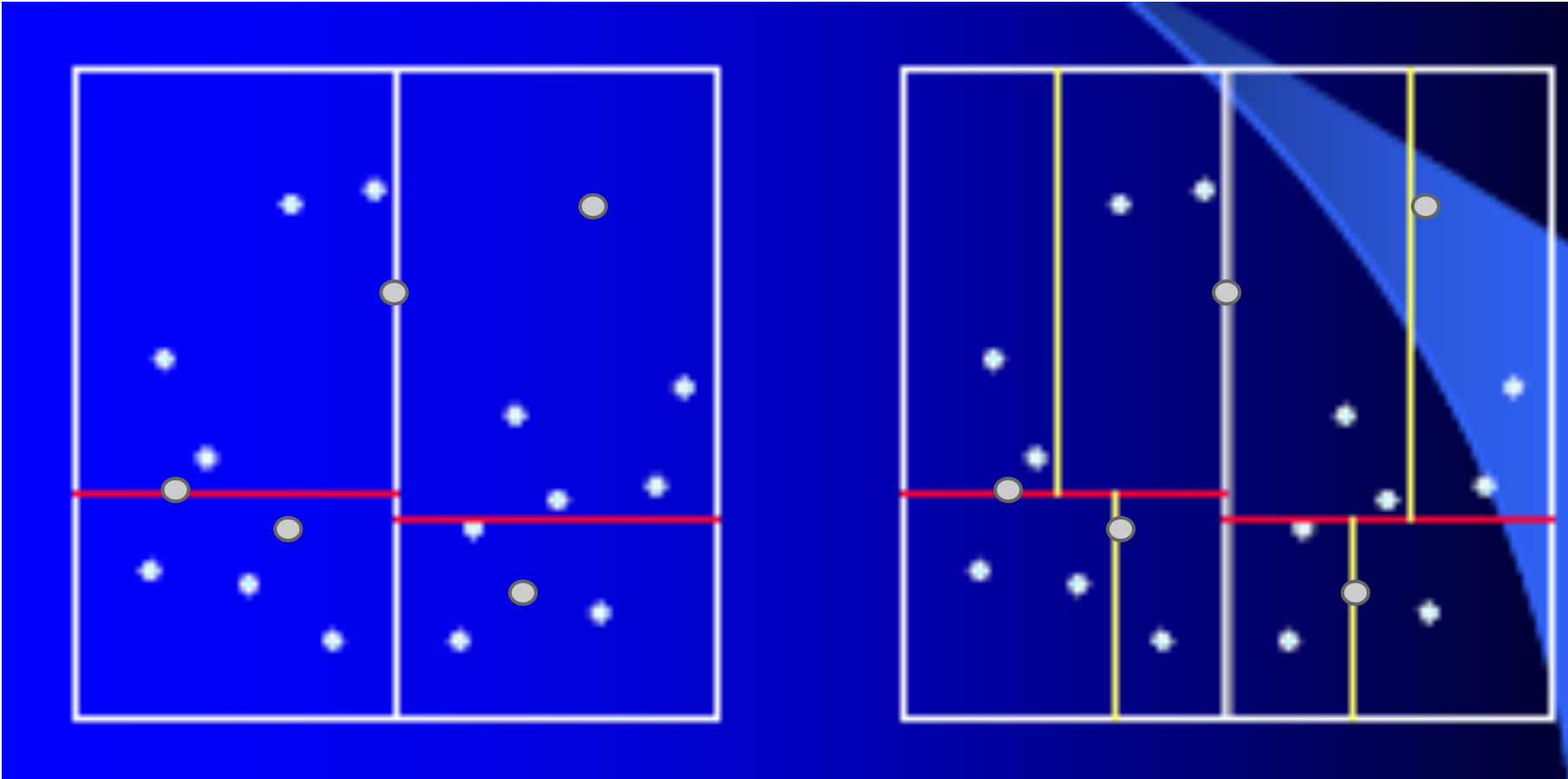


Photon Mapping

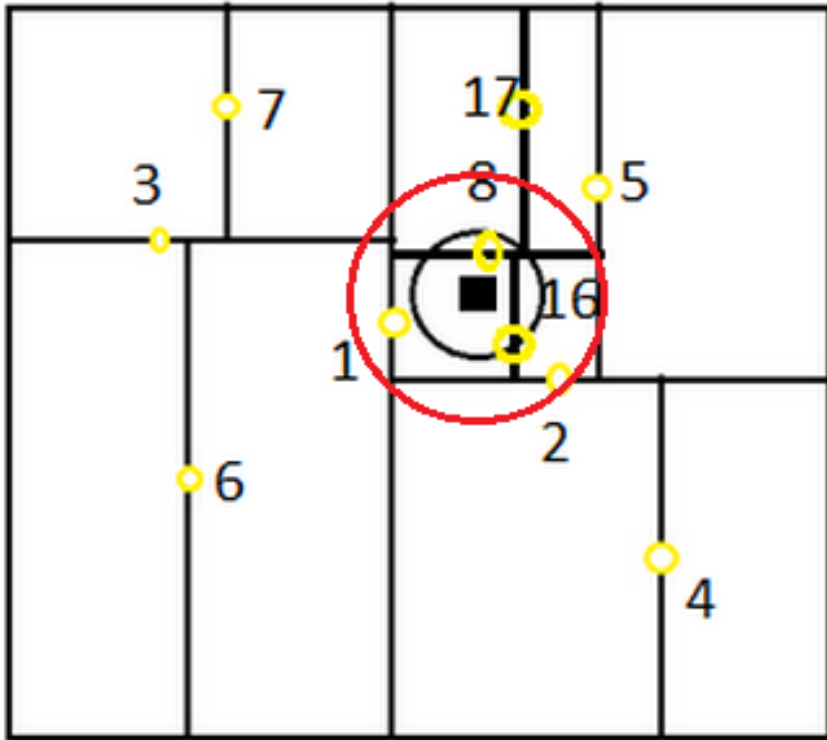


$$L_r(x, \vec{\omega}) = \sum_{p=1}^N f_r(x, \vec{\omega}_p, \vec{\omega}) \frac{\Delta\Phi_p(x, \vec{\omega}_p)}{\Delta A}$$

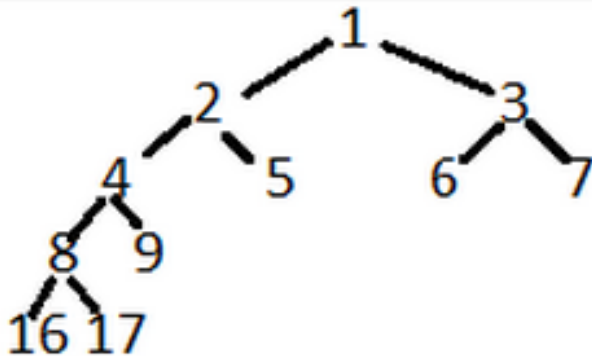
KD-Tree



KD-tree



- For 2 nearest neighbours, what is the order that the photons are added in the KD-tree?
- What are the photons in the heap when the algorithm ends?



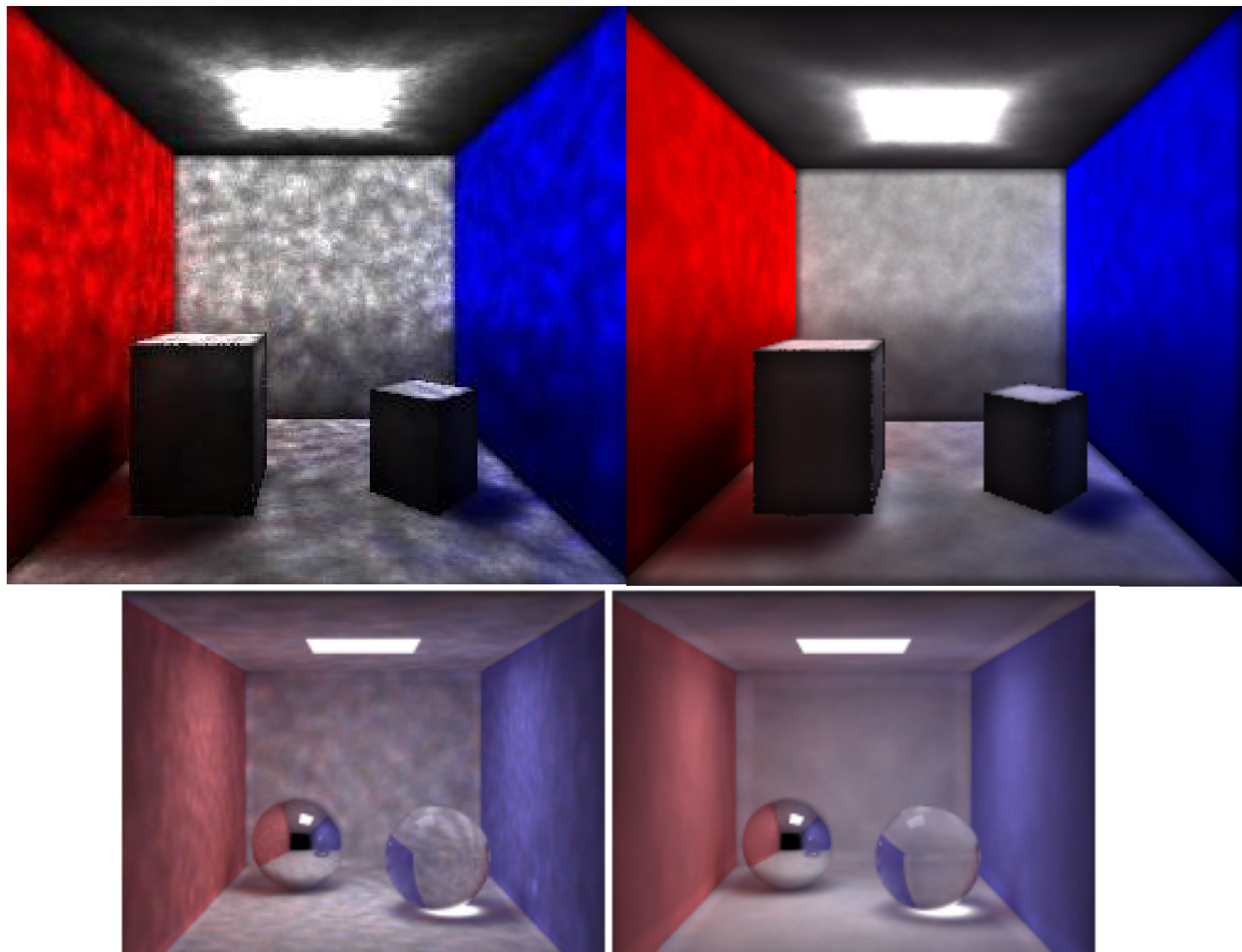
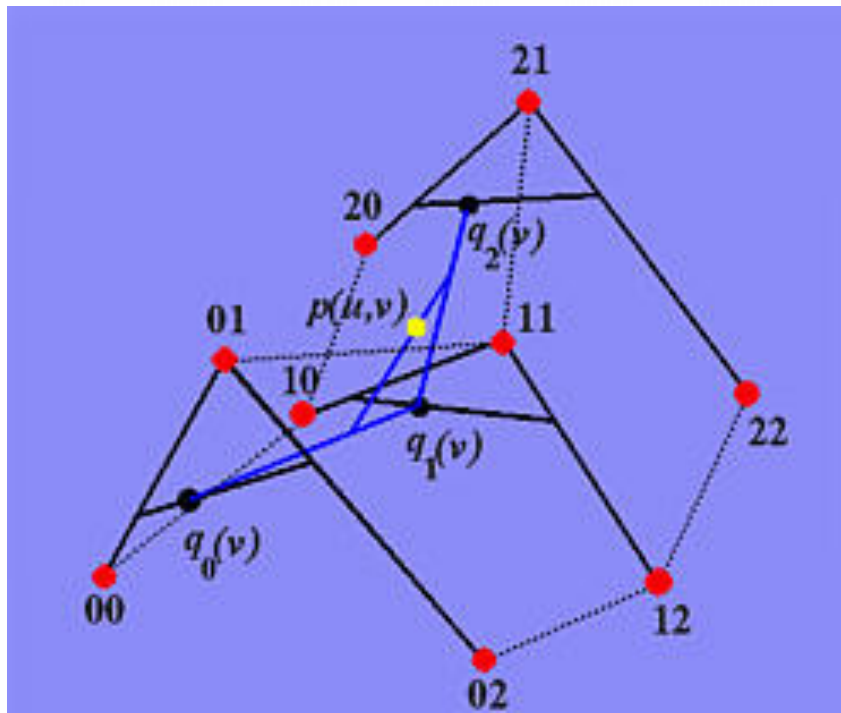
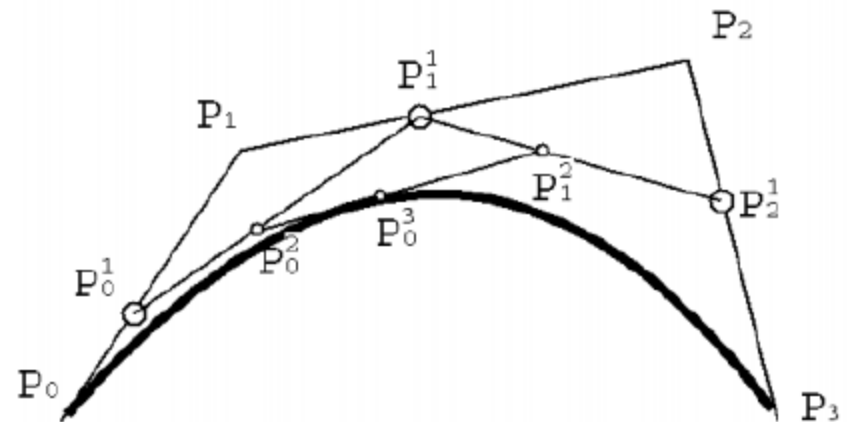
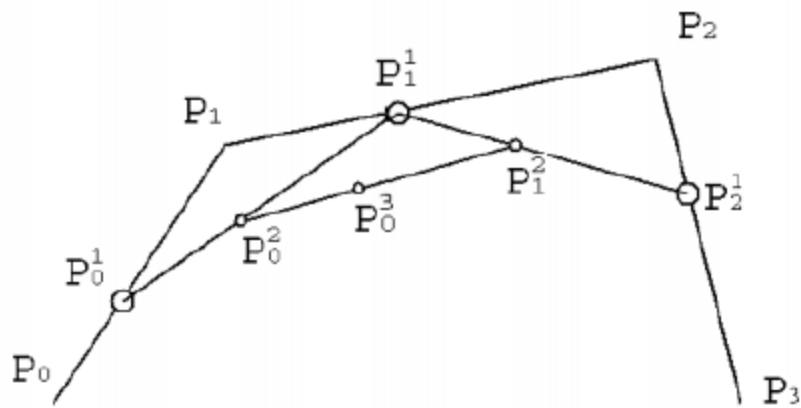


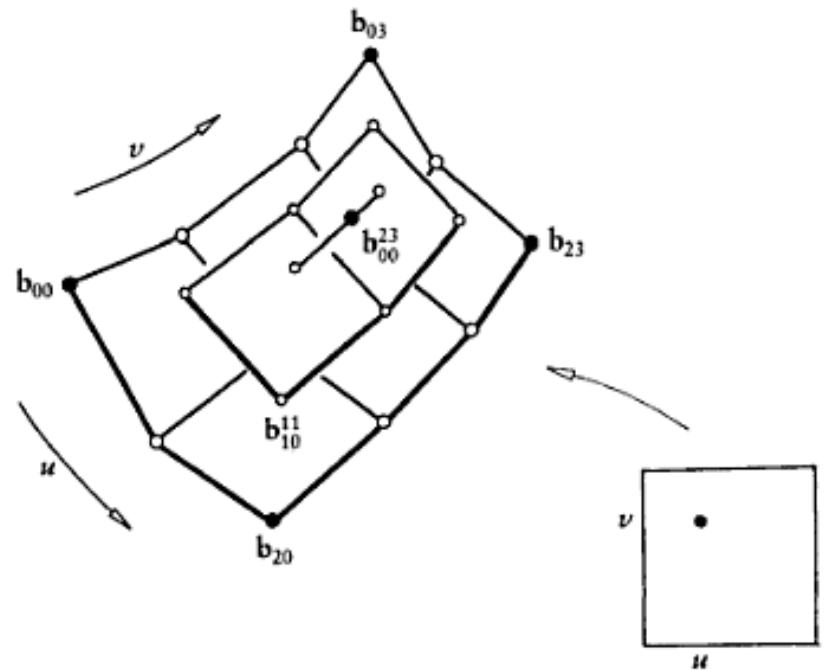
Figure 20: Global photon map radiance estimates visualized directly using 100 photons (left) and 500 photons (right) in the radiance estimate.

Parametric Curves and Surfaces

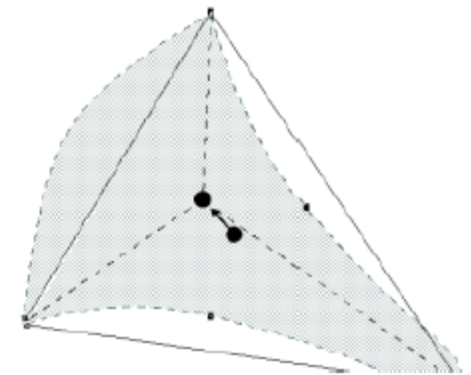
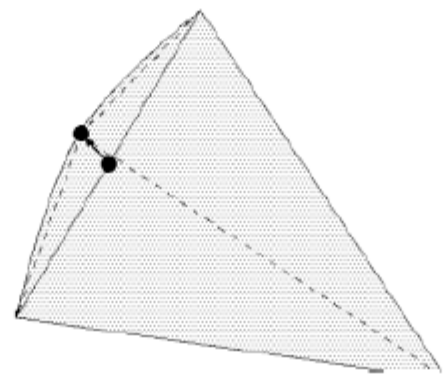
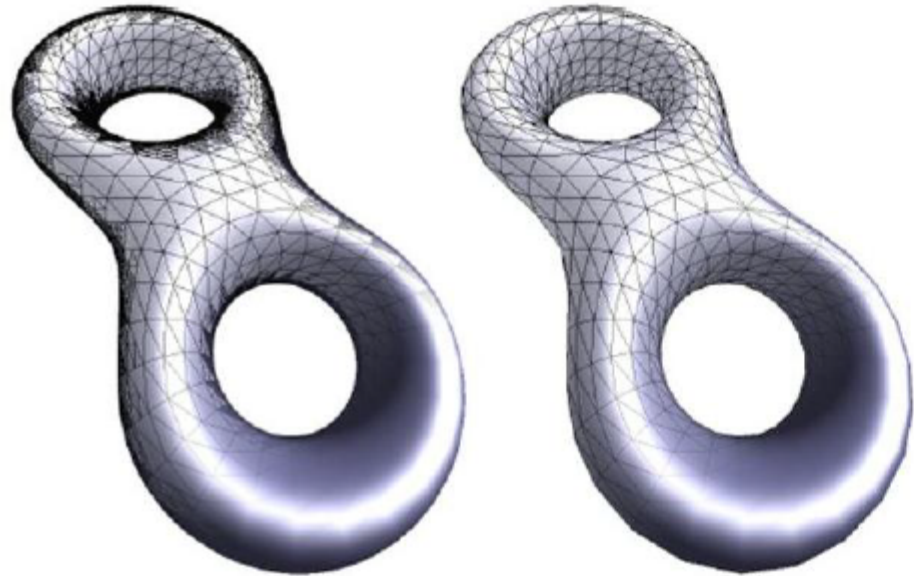


Direct de Casteljau

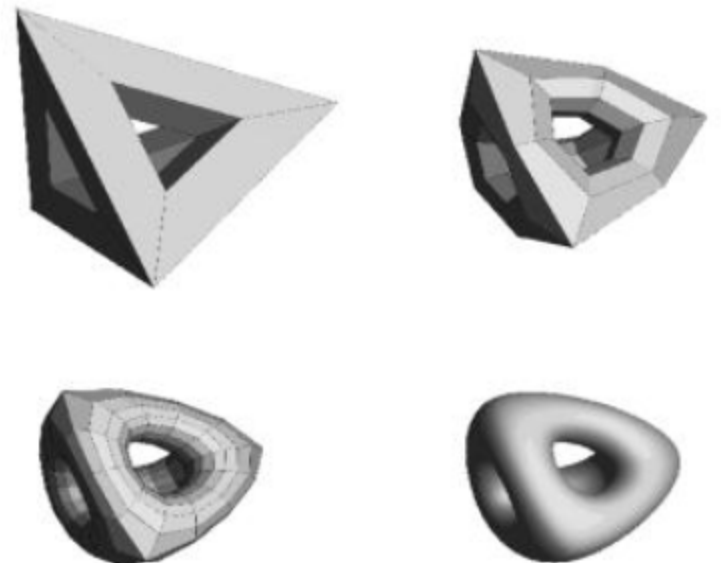
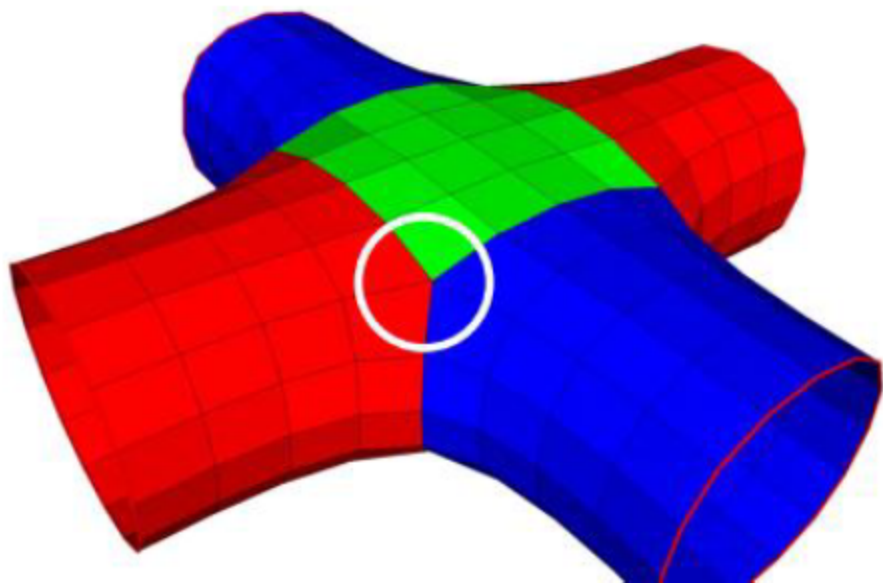
- Given (u,v) , in every adjacent quad area, sample a point by bilinear interpolation
- Iterate this process



Adaptive Tesselation, On-the-fly Tesselation



NURBS, Subdivision Surface



Good Luck!!

