# CFCS Tutorial: Entropy 

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Suppose you have a two-sided coin and a four-sided die. You throw the coin and die a number of times and record the following:

|  | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| h | 1 | 1 | 2 | 2 |
| t | 2 | 4 | 8 | 2 |

(for example, you saw a head and a one once; you saw a tail and a three 8 times etc)

- Compute the full joint probability distribution: $P(C, D)$.
- Now, compute the probability distributions for the coin and the probability for the die: $P(C)$ and $P(D)$.
- What is the entropy of $C$ ?
- What is the entropy of $D$ ?
- What is the entropy of $C, D$ ? Is it higher or lower than the two previous entropies?
- Now compute $H(C \mid D)$. Also compute $H(D \mid C)$. Comment on these values, compared to each other and to the joint entropy.

Overleaf is a table of probabilities and logs.

Probs and logs (base 2)

| 0.1 | -3.32 |
| :--- | :--- |
| 0.15 | -2.74 |
| 0.2 | -2.32 |
| 0.25 | -2 |
| 0.3 | -1.74 |
| 0.35 | -1.51 |
| 0.4 | -1.32 |
| 0.45 | -1.15 |
| 0.5 | -1 |
| 0.55 | -0.86 |
| 0.6 | -0.74 |
| 0.65 | -0.62 |
| 0.7 | -0.51 |
| 0.75 | -0.41 |
| 0.8 | -0.32 |
| 0.85 | -0.23 |
| 0.9 | -0.15 |
| 0.95 | -0.07 |

