CFCS Tutorial: Entropy

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Suppose you have a two-sided coin and a four-sided die. You throw the coin and die a number of times and record the following:

	1	2	3	4
h	1	1	2	2
t	2	4	8	2

(for example, you saw a head and a one once; you saw a tail and a three 8 times etc)

- Compute the full joint probability distribution: $P(C, D)$. Sol:

	1	2	3	4
h	1/22	1/22	2/22	2/22
t	2/22	4/22	8/22	2/22

- Now, compute the probability for the coin and the probability for the die: $P(C)$ and $P(D)$. Sol: P(C): C=h = 6/22; C=t = 16/22; P(D): D=1 = 3/22 D=2= 5/22; D=3 = 10/22 D=4=4/22

- What is the entropy of C? Sol: 0.84

- What is the entropy of D? Sol: 1.84

- What is the entropy of C, D? Sol: 2.6 Is it higher or lower than the two previous entropies? Higher

- Now compute $H(C \mid D)$. Also compute $H(D \mid C)$. Comment on these values, compared to each other and to the joint entropy. Sol: use the fact that $H(X \mid Y) = H(X, Y) - H(Y)$ to work out the actual values; as expected, the conditional entropy is lower than the joint; knowing the coin does not really reduce the choices by much, so the entropy is higher than the case when we know the die value first.