Computational Foundations of Cognitive Science 1 (2009–2010)

School of Informatics, University of Edinburgh
Lecturers: Frank Keller, Miles Osborne

Tutorial 8: Joint Distributions; Expectation and Variance

Week 9 (08–12 March, 2010)

1. **Joint and Marginal Distributions**

   Two psychometric tests are administered to a group of 10 experimental subjects. The first test is a standardized aptitude test used for university admissions. It returns a score between 0 and 100. Let X be the random variable denoting the aptitude test score. The second psychometric test is a memory test. It measures the digit span, i.e., the number of digits in a sequence that a subject is able to remember before they make a mistake. Let Y be the random variable that denotes the digit span, ranging from 1 to 5. The results of the experiment are given in the following table:

Subject	1	2	3	4	5	6	7	8	9	10
x	60	50	40	50	60	30	30	20	30	50
y	5	3	3	3	3	3	3	4	4	4

   (a) Compute the distributions of X and Y.
   (b) Compute the joint distribution of X and Y.
   (c) Compute the marginal distributions of X and Y.
   (d) Are X and Y independent?
   (e) Compute the conditional distributions of X given $Y = 3$.

2. **Expectation and Variance**

   (a) For the discrete random variable X with the following probability distribution:

$$f(x) = \frac{|x-2|}{7} \text{ for } x = -1, 0, 1, 2, 3$$

   determine $E(X)$ and $\text{var}(X)$. Now assume the functions $g(X) = 3X + 2$ and $h(X) = X^2$ and determine $E(g(X))$ and $E(h(X))$.

   (b) In Chebyshev's theorem, which form does the inequality take for $k = 1, 2, 3, 4$?

3. **Covariance**

   The *covariance* of two random variables X and Y with the joint distribution $f(x, y)$ is:

$$\text{cov}(X,Y) = E((X-\mu_X)(Y-\mu_Y)) = \sum_x \sum_y (x-\mu_X)(y-\mu_Y) \cdot f(x,y)$$

   where μ_X and μ_Y are the means of X and Y. Assume that X and Y have the following joint distribution:

(x,y)	0	1	2
0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{12}$
1	$\frac{2}{9}$	$\frac{1}{6}$	0
2	$\frac{1}{36}$	0	0

   (a) Compute the marginal distributions of X and Y.
   (b) Use the marginal distributions to compute μ_X and μ_Y.
   (c) Now compute the covariance of X and Y.