Computational Foundations of Cognitive Science 1 (2009–2010)

School of Informatics, University of Edinburgh
Lecturers: Frank Keller, Miles Osborne

Tutorial 5: Eigenvectors and Convolutions

Week 6 (15–19 February 2010)

1. **Computing Eigenvectors**

   (a) Confirm that $\mathbf{x}$ is a an eigenvector of A, and find the corresponding eigenvalue.
   $$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$$

   (b) Find the characteristic equations of the following matrices, and then find their eigenvalues and eigenvectors.
   $$B = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}, C = \begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix}.$$

   (c) Find the eigenvalues of the following matrices.
   $$D = \begin{bmatrix} 3 & 0 & 0 \\ -2 & 7 & 0 \\ 4 & 8 & 1 \end{bmatrix}, E = \begin{bmatrix} -\frac{1}{3} & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}.$$

   (d) Using the eigenvalues you computed in questions (b) and (c), compute the determinants and traces of matrices A to E.

2. **Properties of Eigenvectors**

   (a) Find some matrices whose characteristic polynomial is $p(\lambda) = \lambda(\lambda-2)^2(\lambda+1)$.

   (b) Suppose that the characteristic polynomial of A is $p(\lambda) = (\lambda-1)(\lambda-3)^2(\lambda-4)^3$. What is the size of A? Is A invertible?

   (c) Suppose that A is a 2×2 matrix with $\mathrm{tr}(A) = \det(A) = 4$. What are the eigenvalues of A?

   (d) Find all 2×2 matrices for which $\mathrm{tr}(A) = \det(A)$.

3. **Convolutions**

   (a) Compute the convolution $\mathbf{k} * \mathbf{a}$ for the following vectors. What is the function of the kernel $\mathbf{k}$?
   $$\mathbf{k} = \begin{bmatrix} -1 & 1 \end{bmatrix}, \mathbf{a} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

   (b) Compute the convolution $f * g$ for the following functions.
   $$g(x) = \begin{cases} 3 & \text{if } 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}, \quad f(x) = \begin{cases} -\frac{1}{2} & \text{if } -1 \le x \le 0 \\ \frac{1}{2} & \text{if } 0 < x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   (c) In image processing, what is the function of the following kernels?
   $$K_1 = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}, K_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 0 \end{bmatrix}, K_3 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}.$$