Computational Foundations of Cognitive Science 1 (2009–2010)

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Tutorial 4: Inverses and Determinants

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1. **Computing Inverses**

   Assume the following matrices:

   $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$

   Compute the following matrices, where possible:

   (a) A^{-1}, B^{-1}
   (b) $(A^T)^{-1}$
   (c) $(2A)^{-1}$
   (d) $(AB)^{-1}$

2. **Terms with Inverses**

   Assume that A, B, C, D are invertible matrices, such that the products given in (a) and (b) are defined.

   (a) Simplify the following term as much as possible: $(AB)^{-1}(AC^{-1})(D^{-1}C^{-1})^{-1}D^{-1}$
   (b) Simplify the following term as much as possible: $(AC^{-1})^{-1}(AC^{-1})(AC^{-1})^{-1}AD^{-1}$
   (c) Find all values of c for which $A = \begin{bmatrix} -c & -1 \\ 1 & c \end{bmatrix}$ is invertible.

3. **Matrices with Special Forms**

   Assume the following matrices:

   $C = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix}, D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}, E = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}, F = \begin{bmatrix} 2 & 1 \\ -4 & 1 \\ 2 & 5 \end{bmatrix}, G = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$

   Compute the following matrices, where possible:

   (a) C^{-1}, D^{-1}, G^{-1}
   (b) CD
   (c) EF
   (d) Which of the following is symmetric: C, E, F^T, F^TF, G^{-1}.

4. **Determinants**

   Assume the matrices A to F in Questions 1 and 3, as well as:

   $H = \begin{bmatrix} \lambda - 2 & 1 \\ -5 & \lambda + 4 \end{bmatrix}$

   (a) Compute $\det(A)$, $\det(A^{-1})$, $\det(A^T)$.
   (b) Compute $\det(C)$, $\det(D)$, $\det(G)$.
   (c) Find all values of λ for which $\det(H) = 0$.