Computational Foundations of Cognitive Science 1 (2009–2010)

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Tutorial 3: Matrix Operations

Week 4 (01–05 February 2010)

1. **Matrix Addition and Subtraction**

   Assume the following matrices:

   $$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ -3 & 1 \\ 4 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}, D = \begin{bmatrix} 1 & 1 \\ -3 & 3 \end{bmatrix}, E = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

   Compute the following matrices, where possible:

   (a) $A + 2B$
   (b) $A - B^T$
   (c) $4D - 3C^T$
   (d) $D - D^T$

2. **Multiplying a Matrix with a Vector**

   Assume the following: $A = \begin{bmatrix} 1 & 5 & 2 \\ -4 & 9 & 1 \\ 2 & 0 & 3 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$. Compute $A\mathbf{x}$.

3. **Matrix Multiplication**

   Assume the same matrices as in Question 1. Compute the following matrices, where possible:

   (a) CD
   (b) AE
   (c) BB^T
   (d) DA

4. **Inner and Outer Product**

   Let $\mathbf{u} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$.

   (a) Find the matrix inner product of $\mathbf{u}$ with $\mathbf{v}$.
   (b) Find the matrix outer product of $\mathbf{u}$ with $\mathbf{v}$.
   (c) Find the dot product of $\mathbf{u}$ with $\mathbf{v}$.

5. **Application**

   The following tables shows a record of unit sales for a clothing store. Let M denote the 4×3 matrix of sales.

	Small	Medium	Large
Shirts	45	60	75
Jeans	30	30	40
Suits	12	65	45
Raincoats	15	40	25

(a) Find the column vector $\mathbf{x}$ for which $M\mathbf{x}$ provides a list of the number of shirts, jeans, suits, and raincoats sold.

(b) Find the row vector $\mathbf{y}$ for which $\mathbf{y}M$ provides a list of small, medium, and large items sold.

(c) What does $\mathbf{y}M\mathbf{x}$ represent?

6. **Image Processing**

Assume that you have two matrices A and B representing greyscale images. A represents a picture of a lake and B a picture of a ship. If you compute $A+B$ and $A+B^T$, what do they represent? What do AB and BB^T represent?