CFCS1

Vectors

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Motivation

We often want to group together data:

- Measurements of an experiment.
- All students who take CFCS.
- Properties of a word:
  - Frequency in some large file.
  - Length in characters.

Vectors allow us to package data together.

Notation

- Vectors have a variety of representations:
  - Row: $[a \; b \; c]$ Column: $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$
- These are mathematically equivalent to each other.
- Conventionally, vectors are written with a **bold font**: $\mathbf{a}$.
- Ordinary numbers (*scalars*) are written in a usual font: 10.

Motivation

Word Frequencies

The	10
cat	6
laughed	2

Vector representation: $[10, 6, 2]$

Motivation

Notice each vector element has the same type:

- They are all integers.
- They all have the same semantics.

We can define operations over vectors which apply to all elements.

Motivation

Word Properties

Consider the word *the*:

Frequency	10
Length	3

Vector representation: $[10, 3]$

Motivation

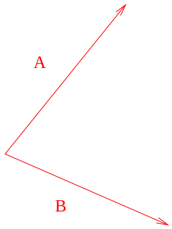
Now, our elements do not have the same semantics as each other:

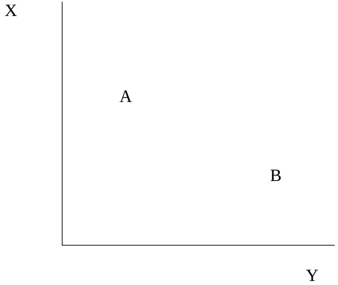
- They are still all integers.
- The meaning of the first element is not the same as the second element.

We must now be careful when manipulating such vectors.

Arrows and Co-ordinates

Vectors can be seen in terms of *arrows* or *co-ordinates*:





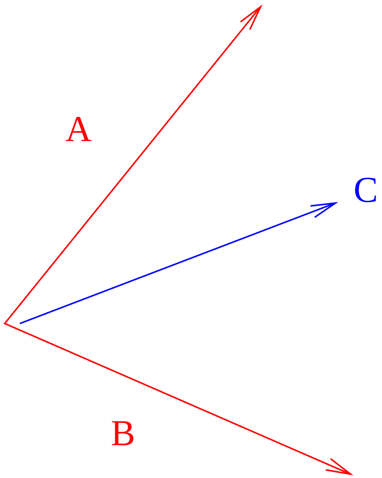
N-Space

If all components of our vectors are real numbers, then we have an *n-space*:

- For $n = 1$, we describe a line (R^1)
- For $n = 2$, we describe a plane (R^2).
- For $n = 3$, we describe a normal space (R^3).

Components of an n-space yield *subspaces*.

Operations: Addition

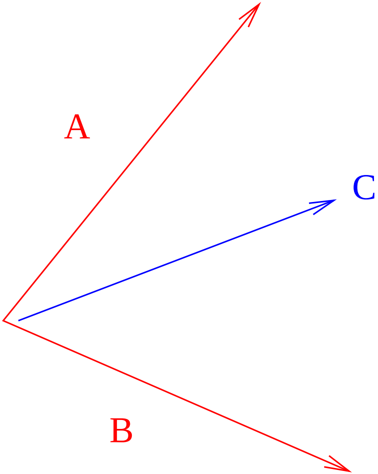


Operations: Addition

Addition

$$[1\ 2] + [1\ 3] = [2\ 5]$$
$$[1\ 2] + [0\ 0] = [1\ 2]$$
$$[1\ 2] + [1] = ?$$

Operations: Subtraction



Operations: Subtraction

Subtraction

$$[1\ 2] - [1\ 3] = [0\ \ -1]$$
$$[1\ 2] - [0\ 0] = [1\ 2]$$
$$[1\ 2] - [1] = ?$$

A vector with negative values has an opposite direction to the corresponding vector with positive values.

Operations: Scalar Multiplication

Multiplying a vector by a scalar yields another vector:

Scalar Multiplication

$$
\begin{aligned}
[1\ 2]\,.2 &= [2\ 4] \\
[1\ 2]\,.1 &= [1\ 2] \\
[1\ 2]\,.-1 &= [-1\ \ -2] \\
[1\ 2]\,.0 &= [0\ 0]
\end{aligned}
$$

Vectors Properties

The following identities can be useful:

$$
\begin{aligned}
\mathbf{u} + \mathbf{v} &= \mathbf{v} + \mathbf{u} \\
(\mathbf{u} + \mathbf{v}) + \mathbf{w} &= \mathbf{u} + (\mathbf{v} + \mathbf{w}) \\
\mathbf{v} + \mathbf{0} = \mathbf{0} + \mathbf{v} &= \mathbf{v} \\
\mathbf{v} + (-\mathbf{v}) &= \mathbf{0} \\
(k + l)\mathbf{v} &= k\mathbf{v} + l\mathbf{v} \\
k(\mathbf{u} + \mathbf{v}) &= k\mathbf{u} + k\mathbf{v} \\
k(l\mathbf{v}) &= (kl)\mathbf{v} \\
1\mathbf{v} &= \mathbf{v}
\end{aligned}
$$

Summary

- Vectors are useful objects for grouping and manipulating data.
- Care needs to be taken that vector operations are meaningful, given the semantics of the components.
- Background reading: Anton and Busby: section 1.1