

CFCS1: Practical 6

Conditional Probability & Bayes' Theorem

Peter Bell

February 21, 2008

Introduction

In this practical you will use Matlab to solve problems in conditional probability. The exercises require familiarity with Bayes theorem.

1 Example: lecture attendance

The head of Informatics is worried about poor lecture attendance amongst students. He decides to commission a survey to investigate possible causes. In particular, he is interested in whether the timing of lectures affects attendance, and whether it varies between males and females. The ITO counts attendance at two lectures for near-identical courses, one held at 9am, the other at 10am, and, using the database of students registered for each course, finds the following data:

9am lecture attendance		
	Present	Absent
Males	9	15
Females	12	4

10am lecture attendance		
	Present	Absent
Males	27	9
Females	18	6

Note: this scenario is entirely fictitious!

You can load the above data as two matrices, `data1` and `data2`, from the file `lab6_data.mat`. You should be able to answer the questions that follow by performing computations directly on these matrices using vectorisation techniques. We use M to denote the event that a student is male, F for the event that a student is female; P for the event that a student is present, and A for the event that a student is absent. Obviously these two pairs of events are mutually exclusive!

1. For each lecture, use the data to find matrices giving the joint probability tables of being present or absent from a lecture and being male or female, ie.

	Present	Absent
Males	$p(M \cap P)$	$p(M \cap A)$
Females	$p(F \cap P)$	$p(F \cap A)$

2. For each lecture, find two vectors, one giving $p(M)$ and $p(F)$, the other giving $p(P)$ and $p(A)$.
3. Use your answers to the above two questions to state, for each lecture, whether a student's sex has is a factor affecting lecture attendance, by finding whether the events M and F are independent of the events P and A .
4. Now compute the matrices giving the conditional probabilities of a student attending a lecture, given the student's sex, ie.

	Present	Absent
Males	$p(P M)$	$p(A M)$
Females	$p(P F)$	$p(A F)$

What conclusions would you give to the head of Informatics?

2 Example: tossing biased coins

This example requires you to compute probabilities from Binomial distribution, introduced in Lecture 21.

Suppose there are two coins. One is a fair coin, with the probability of obtaining a head or a tail both equal to 0.5. The second coin is biased: the probability of obtaining a head is 0.6 and the probability of obtaining a tail is 0.4.

1. One of the two coins is selected (we don't know which). The coin is flipped and comes up heads. What is the probability that the coin chosen is the biased one, given that it came up heads?
2. The same coin is flipped a second time. What is the probability that the coin comes up heads, given that it came up heads on the first flip? Why are the two events not independent?
3. Suppose one of the coins is flipped $2n$ times. Write a function in Matlab to compute the probability of obtaining n heads and n tails, given that the coin is fair, and given that the coin is biased (this should be an argument to the function). The function should work for any value of n .
4. The coin has been randomly selected. Use your function to compute the probability that the chosen coin is the biased one, given that n throws were heads and n throws were tails. Plot the the value of this probability for n from 0 to 40. Explain the shape of your plot.
5. (*Optional*) Now suppose that there are **two** biased coins (both the same as before) and one unbiased coin. One coin is selected randomly. Given the same scenario of n heads and n tails being obtained, modify your calculations from Question 4 to calculate the probability that the coin chosen is biased. What is the lowest value of n for which it is more likely that the coin chosen was the unbiased one?