

1.

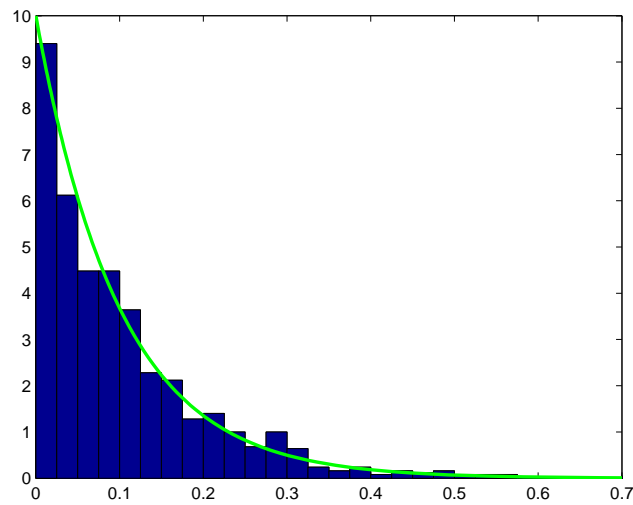
$$f(t; \theta) = \frac{1}{\theta} e^{-\frac{t}{\theta}}$$

2. The mean time for the gap between firing times is 0.1s. From knowledge of the exponential distribution,  $E(T) = \theta$ . We therefore set  $\theta = 0.1$ , so that

$$f(t; \theta) = 10e^{-10t}$$

3.

4. The histogram and density function are shown on the plot below



The density function (shown in green) seems to fit well with the experimental data.

5. Use the result that

$$p(T \leq t) = \int_0^t 10e^{-10s} ds = 1 - e^{-10t}$$

So  $p(T \leq 0.15) = 0.777$ .

6.

$$p(T > 0.1) = 1 - p(T \leq 0.1) = e^{-1} = 0.368$$

7.

$$\begin{aligned} p(T > 0.15 | T > 0.05) &= \frac{p(T > 0.15 \cap T > 0.05)}{p(T > 0.05)} \\ &= \frac{p(T > 0.15)}{p(T > 0.05)} \\ &= \frac{e^{-1.5}}{e^{-0.5}} = e^{-1} = 0.368 \end{aligned}$$

8. The last two questions have the same answer. This illustrates the “memoryless” property of the exponential distribution: if we have already waited a time  $r$  to observe an event, the probability of the event happening before some time in the future is just the same as if we had only just started the watching for the event. This is quite counter-intuitive – you’d normally expect it to be higher.