

1 Lecture attendance

1. To find this probability matrix, divide by the total number of students registered for the course:

$$\text{jointProb1} = \text{data1}/\text{sum}(\text{sum}(\text{data1})) = \begin{bmatrix} 0.2250 & 0.3750 \\ 0.3000 & 0.1000 \end{bmatrix}$$

$$\text{jointProb2} = \text{data2}/\text{sum}(\text{sum}(\text{data2})) = \begin{bmatrix} 0.4500 & 0.1500 \\ 0.3000 & 0.1000 \end{bmatrix}$$

2. $[\text{p}(P) \text{ p}(A)] = \text{sum}(\text{jointProb}) = [0.5250 \ 0.4750]$ and $[0.7500 \ 0.2500]$
 $[\text{p}(M) \ \text{p}(F)] = \text{sum}(\text{jointProb}') = [0.6000 \ 0.4000]$ for both lectures
3. We need to check whether $p(M \cap P) = p(M)p(P)$, etc. The matrix of products is given by

$$[\text{p}(M) \ \text{p}(F)]' * [\text{p}(P) \ \text{p}(A)] = \begin{bmatrix} 0.3150 & 0.2850 \\ 0.2100 & 0.1900 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0.4500 & 0.1500 \\ 0.3000 & 0.1000 \end{bmatrix}$$

Comparing with the answer to Question 1, it can be seen that sex *is* a factor affecting lecture attendance for the 9am lecture, but not for the 10am lecture.

4. Use the formula $p(P|M) = p(P \cap M)/p(M)$, etc. The answers are:

$$\begin{bmatrix} 0.3750 & 0.6250 \\ 0.7500 & 0.2500 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0.7500 & 0.2500 \\ 0.7500 & 0.2500 \end{bmatrix}$$

From this you would conclude that female students are equally likely to attend a 9am lecture and a 10am lecture. Male students are much less likely to attend 9am lectures than females, and are less likely to attend a 9am lecture than a 10am lecture. A student is equally likely to attend a 10am lecture, regardless of sex.

2 Tossing biased coins

1. Use Bayes' theorem:

$$\begin{aligned} p(B|H) &= \frac{p(B \cap H)}{p(H)} = \frac{p(H|B)p(B)}{p(H|B)p(B) + p(H|F)p(F)} \\ &= \frac{0.6 \times 0.5}{0.6 \times 0.5 + 0.5 \times 0.5} = 0.545 \end{aligned}$$

2. We can use the answer to the previous question – this is the new probability of the coin being biased, given that the previous throw was heads:

$$\begin{aligned} p(H_2|H_1) &= p(H_2 \cap B|H_1) + p(H_2 \cap F|H_1) \\ &= p(H_2|B)p(B|H_1) + p(H_2|F)p(F|H_1) \\ &= 0.6 \times 0.545 + 0.5 \times (1 - 0.545) = 0.555 \end{aligned}$$

Note: this uses the fact that $p(H_2|B, H_1) = p(H_2|B)$, in other words, successive throws are independent of each other, if we know which coin has been chosen.

3. `function [prob] = CoinFlipProb(n, coinType)`

```
if coinType == 1 % biased
    p = 0.6
else
    p = 0.5
end
```

```
prob = factorial(2*n)./factorial(n).^2 .* p.^n .* (1-p).^n;
```

```
end
```

(The dots are just there to allow the function to be called with several different values of `n` simultaneously.)

4. Use a similar calculation to Question 1.

```
n = 0:40;
probs = CoinFlipProb(n, 1)./(CoinFlipProb(n, 1)+CoinFlipProb(n, 2));
plot(probs);
```