# Introduction to Neural Networks

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Automatic Speech Recognition – ASR Lecture 7
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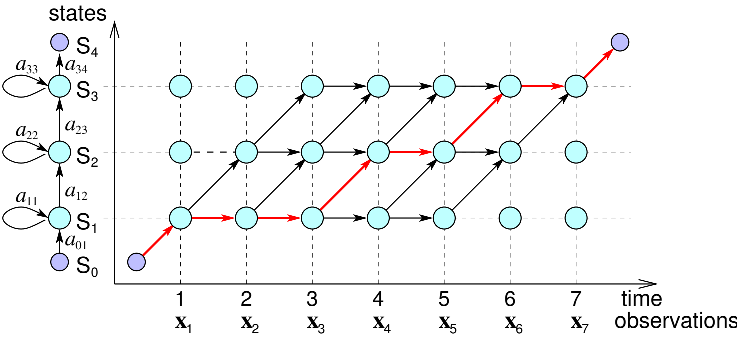
# Local Phonetic Scores and Sequence Modelling

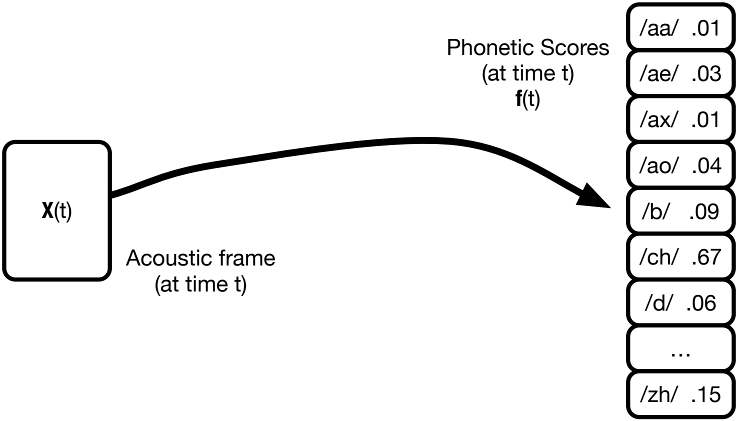
- DTW - local distances (Euclidean)
- HMM - emission probabilities (Gaussian or GMM)




- Compute the phonetic score(acoustic-frame, phone-model) – this does the detailed matching at the frame-level
- Chain phonetic scores together in a sequence - DTW, HMM

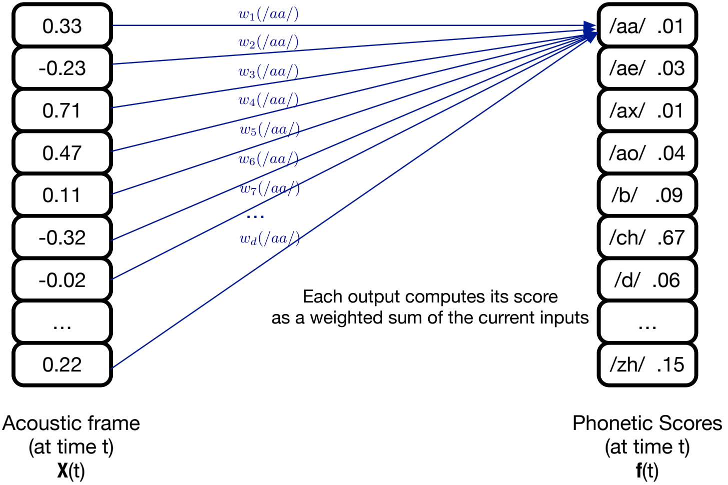
# Phonetic scores

Task: given an input acoustic frame, output a score for each phone



# Phonetic scores

Compute the phonetic scores using a single layer neural network



# Phonetic scores

Compute the phonetic scores using a single layer neural network

- Write the estimated phonetic scores as a vector $\mathbf{f} = (f_1, f_2, \ldots, f_Q)$
- Then if the acoustic frame at time $t$ is $\mathbf{X} = (x_1, x_2, \ldots, x_d)$:

  $$f_j = w_{j1}x_1 + w_{j2}x_2 + \ldots + w_{jd}x_d + b_j$$

  or, write it using summation notation:

  $$f_j = \sum_{i=1}^{d} w_{ji}x_i + b_j$$

  or, write it as vectors:

  $$\mathbf{f} = \mathbf{W}\mathbf{x} + \mathbf{b}$$

  where we call $\mathbf{W}$ the *weight matrix*, and $\mathbf{b}$ the *bias vector*.
- *Check your understanding*:
  What are the dimensions of $\mathbf{W}$ and $\mathbf{b}$?

# Error function

estimated observed

$$\mathbf{f}(t) = \mathbf{W}\mathbf{x}(t) + \mathbf{b}$$

trained

How do we learn the *parameters* $\mathbf{W}$ and $\mathbf{b}$?

- Minimise an Error Function: Define a function which is 0 when the output $\mathbf{f}(n)$ equals the *target output* $\mathbf{r}(n)$ for all $n$
- Target output: for TIMIT the target output corresponds to the phone label for each frame
- Mean square error: define the error function $E$ as the mean square difference between output and the target:

$$E = \frac{1}{2} \cdot \frac{1}{N} \sum_{n=1}^{N} ||\mathbf{f}(n) - \mathbf{r}(n)||^2$$

where there are $N$ frames of training data in total

# Notes on the error function

- $\mathbf{f}$ is a function of the acoustic data $\mathbf{x}$ and the weights and biases of the network ($\mathbf{W}$ and $\mathbf{b}$)
- This means that as well as depending on the training data ($\mathbf{x}$ and $\mathbf{r}$), $E$ is also a function of the weights and biases, since it is a function of $\mathbf{f}$
- We want to minimise the error function given a fixed training set: we must set $\mathbf{W}$ and $\mathbf{b}$ to minimise $E$
- Weight space: given the training set we can imagine a space where every possible value of $\mathbf{W}$ and $\mathbf{b}$ results in a specific value of $E$. We want to find the minimum of $E$ in this weight space.
- Gradient descent: find the minimum iteratively – given a current point in weight space find the direction of steepest descent, and change $\mathbf{W}$ and $\mathbf{b}$ to move in that direction

# Gradient Descent

- Iterative update – after seeing some training data, we adjust the weights and biases to reduce the error. Repeat until convergence.
- To update a parameter so as to reduce the error, we move downhill in the direction of steepest descent. Thus to train a network we must compute the gradient of the error with respect to the weights and biases:

$$\begin{pmatrix} \frac{\partial E}{\partial w_{10}} & \cdot & \frac{\partial E}{\partial w_{1i}} & \cdot & \frac{\partial E}{\partial w_{1d}} \\ & & \cdots & & \\ \frac{\partial E}{\partial w_{j0}} & \cdot & \frac{\partial E}{\partial w_{ji}} & \cdot & \frac{\partial E}{\partial w_{jd}} \\ & & \cdots & & \\ \frac{\partial E}{\partial w_{Q0}} & \cdot & \frac{\partial E}{\partial w_{Qi}} & \cdot & \frac{\partial E}{\partial w_{Qd}} \end{pmatrix} \qquad \begin{pmatrix} \frac{\partial E}{\partial b_1} & \cdot & \frac{\partial E}{\partial b_j} & \cdot & \frac{\partial E}{\partial b_Q} \end{pmatrix}$$

# Gradient Descent Procedure

1. Initialise weights and biases with small random numbers
2. For each batch of training data
   1. Initialise total gradients: $\Delta w_{ki} = 0$, $\Delta b_k = 0$
   2. For each training example $n$ in the batch:
      - Compute the error $E^n$
      - For all $k, i$: Compute the gradients $\partial E^n / \partial w_{ki}$, $\partial E^n / \partial b_k$
      - Update the total gradients by accumulating the gradients for example $n$

$$\Delta w_{ki} \leftarrow \Delta w_{ki} + \frac{\partial E^n}{\partial w_{ki}} \quad \forall k, i$$

$$\Delta b_k \leftarrow \Delta b_k + \frac{\partial E^n}{\partial b_k} \quad \forall k$$

   3. Update weights:

$$w_{ki} \leftarrow w_{ki} - \eta \Delta w_{ki} \quad \forall k, i$$

$$b_k \leftarrow b_k - \eta \Delta b_k \quad \forall k$$

Terminate either after a fixed number of epochs, or when the error stops decreasing by more than a threshold.

# Gradient in SLN

How do we compute the gradients $\frac{\partial E^n}{\partial w_{ki}}$ and $\frac{\partial E^n}{\partial b_k}$?

$$E^n = \frac{1}{2}\sum_{k=1}^{K}(f_k^n - r_k^n)^2 = \frac{1}{2}\sum_{k=1}^{K}\left(\sum_{i=1}^{d}(w_{ki}x_i^n + b_k) - r_k^n\right)^2$$

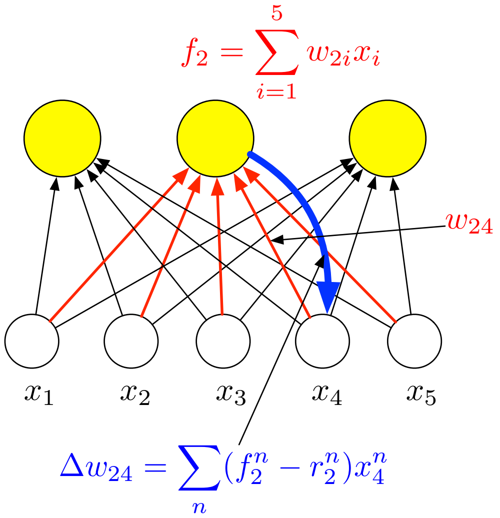
$$\boxed{\frac{\partial E^n}{\partial w_{ki}}} = (f_k^n - r_k^n)x_i^n = \boxed{\delta_k^n x_i^n} \qquad \boxed{\delta_k^n = f_k^n - r_k^n}$$

The **delta rule**: gradient of the error with respect to a weight $w_{ki}$ is the product of the error (delta) at the output of the weight ($r_k$) multiplied by the value of the unit at the input to the weight ($x_i$).

*Check your understanding*: Show that

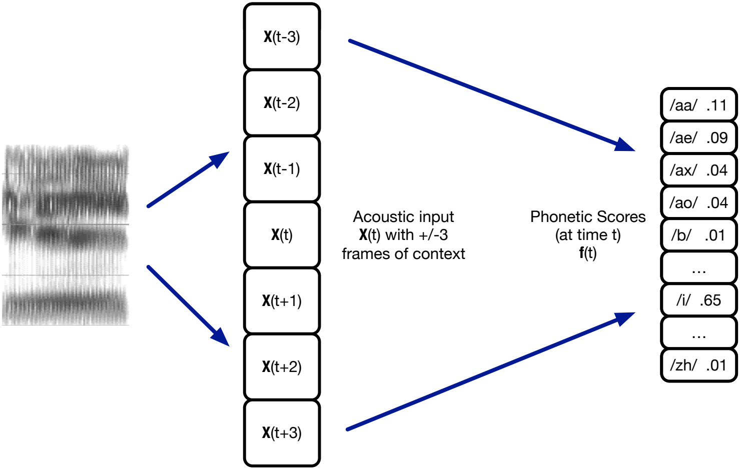
$$\frac{\partial E^n}{\partial b_k} = \delta_k^n$$

# Applying gradient descent to a single-layer network



# Acoustic context

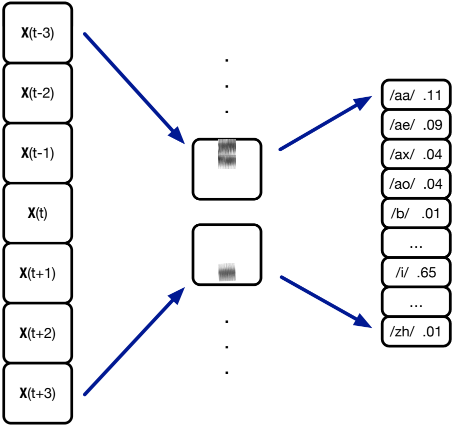
Use multiple frames of acoustic context



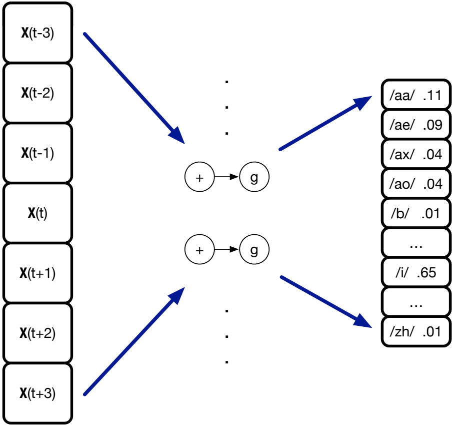
# Hidden units

- Single layer networks have limited computational power – each output unit is trained to match a spectrogram directly (a kind of discriminative template matching)
- But there is a lot of variation in speech (as previously discussed) – rate, coarticulation, speaker characteristics, acoustic environment
- Introduce an intermediate feature representation – "hidden units" – more robust than template matching
- Intermediate features represented by hidden units

# Hidden units extracting features

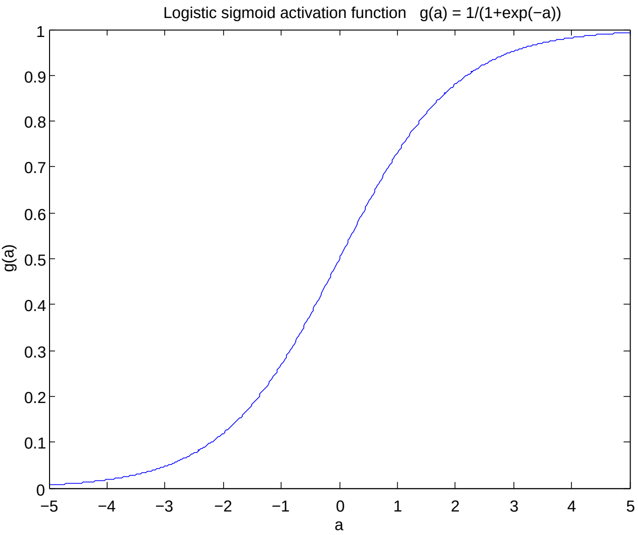


# Hidden Units




$$h_j = g\left(\sum_{i=1}^{d} w_{ji} x_i + b_j\right) \qquad f_k = \text{softmax}\left(\sum_{j=1}^{H} v_{kj} h_j + b_k\right)$$

# Sigmoid function



# Softmax

$$\boxed{y_k = \frac{\exp(a_k)}{\sum_{j=1}^{K} \exp(a_j)}}$$

$$a_k = \sum_{j=1}^{H} v_{kj} h_j + b_k$$

- This form of activation has the following properties
  - Each output will be between 0 and 1
  - The denominator ensures that the $K$ outputs will sum to 1
- Using softmax we can interpret the network output $y_k^n$ as an estimate of $P(k|\mathbf{x}^n)$

# Cross-entropy error function

- Cross-entropy error function:

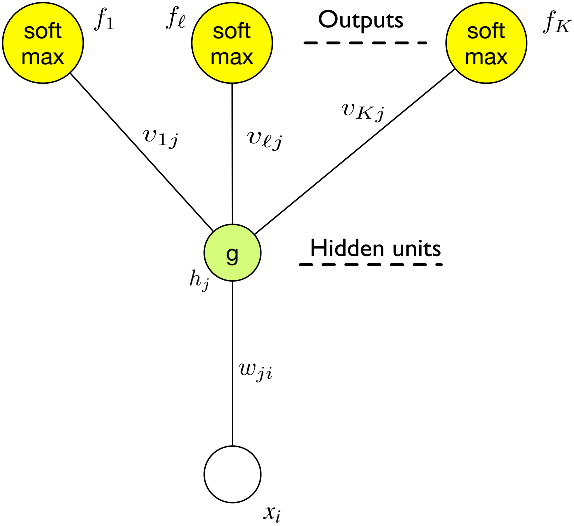
$$E^n = -\sum_{k=1}^{C} r_k^n \ln f_k^n$$

  Optimise the weights **W** to maximise the log probability – or to minimise the negative log probability.

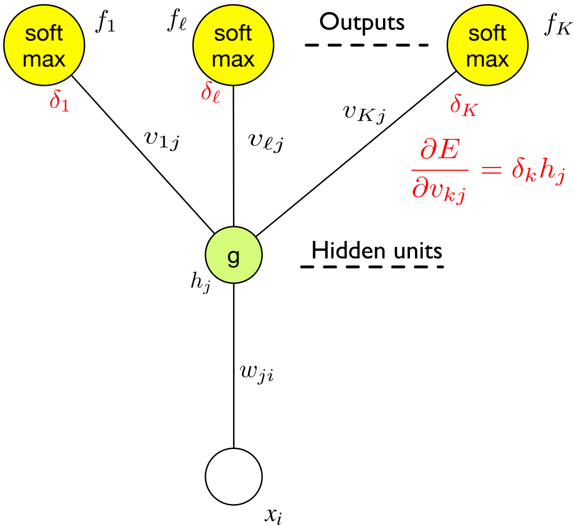
- A neat thing about softmax: if we train with cross-entropy error function, we get a simple form for the gradients of the output weights:

$$\boxed{\frac{\partial E^n}{\partial v_{kj}}} = \underbrace{(f_k - r_k)}_{\delta_k} h_j$$

# Training multilayered networks – output layer



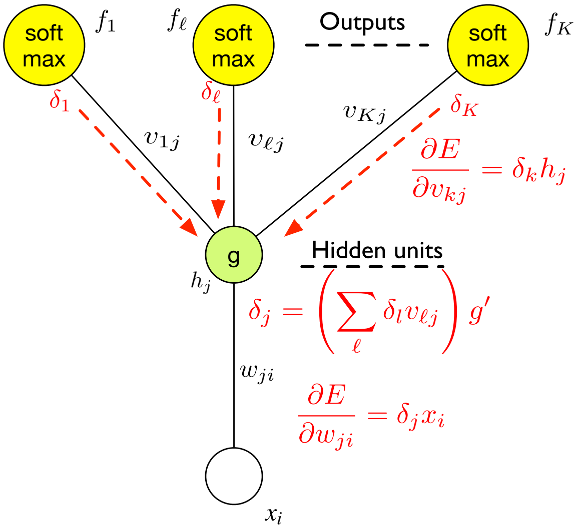
# Training multilayered networks – output layer



# Backprop

- Hidden units make training the weights more complicated, since the hidden units affect the error function indirectly via all the outputs
- The Credit assignment problem: what is the "error" of a hidden unit? how important is input-hidden weight $w_{ji}$ to output unit $k$?
- Solution: *back-propagate* the deltas through the network
- $\delta_j$ for a hidden unit is the weighted sum of the deltas of the connected output units. (Propagate the $\delta$ values backwards through the network)
- Backprop provides way of estimating the error of each hidden unit

# Backprop



soft max
soft max
soft max
$f_1$
$f_\ell$
$f_K$
Outputs
$\delta_1$
$\delta_\ell$
$\delta_K$
$v_{1j}$
$v_{\ell j}$
$v_{Kj}$
$\frac{\partial E}{\partial v_{kj}} = \delta_k h_j$
g
$h_j$
Hidden units
$\delta_j = \left( \sum_\ell \delta_l v_{\ell j} \right) g'$
$w_{ji}$
$\frac{\partial E}{\partial w_{ji}} = \delta_j x_i$
$x_i$

# Back-propagation of error

- The back-propagation of error algorithm is summarised as follows:
  1. Apply an input vectors from the training set, $\mathbf{x}$, to the network and forward propagate to obtain the output vector $\mathbf{f}$
  2. Using the target vector $\mathbf{r}$ compute the error $E^n$
  3. Evaluate the error signals $\delta_k$ for each output unit
  4. Evaluate the error signals $\delta_j$ for each hidden unit using back-propagation of error
  5. Evaluate the derivatives for each training pattern
- Back-propagation can be extended to multiple hidden layers, in each case computing the $\delta$s for the current layer as a weighted sum of the $\delta$s of the next layer

# Summary and Reading

- Single-layer and multi-layer neural networks
- Error functions, weight space and gradient descent training
- Multilayer networks and back-propagation
- Transfer functions – sigmoid and softmax
- Acoustic context
- M Nielsen, *Neural Networks and Deep Learning*, http://neuralnetworksanddeeplearning.com (chapters 1, 2, and 3)

**Next lecture:** Neural network acoustic models