Modelling speech with HMMs – Context-dependent phone models

Steve Renals

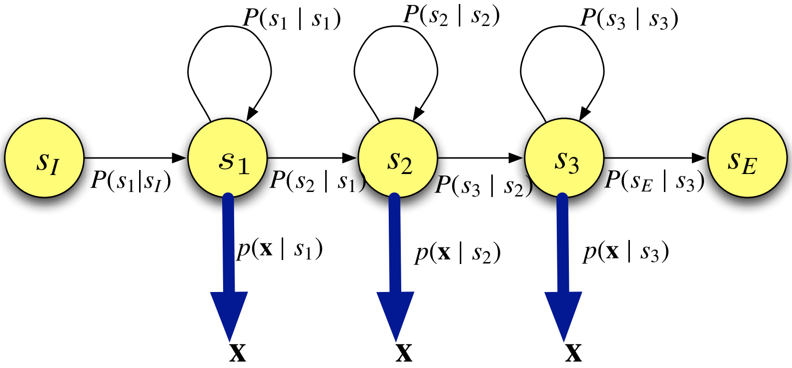
Automatic Speech Recognition
ASR Lectures 6&8
28 January, 4 February 2016

Overview

Phone models

- Modelling phones with HMMs
- The need to model phonetic context
- Triphone models
- Smoothing – interpolation and backing-off
- Parameter sharing – tied mixtures, generalised triphones, state clustering
- Choosing which states to share – phonetic decision trees

Recap: Continuous Density HMM

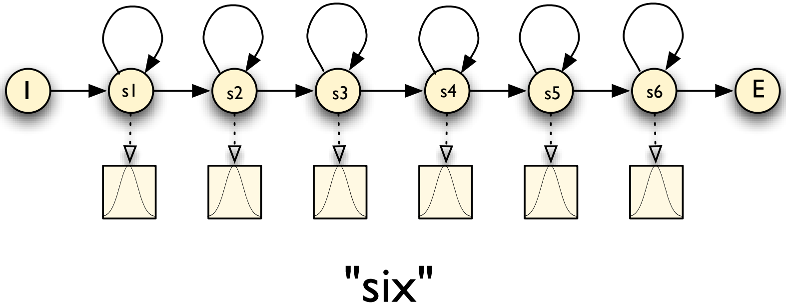


Probabilistic finite state automaton

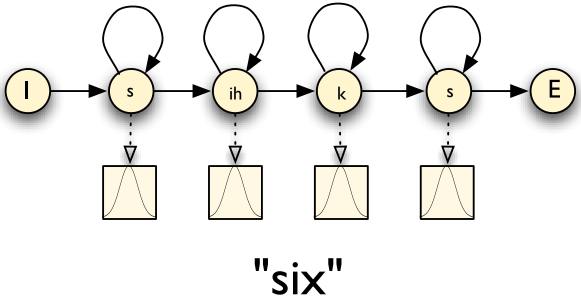
Paramaters $\boldsymbol{\lambda}$:

- Transition probabilities: $a_{kj} = P(s_j \mid s_k)$
- Output probability density function: $b_j(\mathbf{x}) = p(\mathbf{x} \mid s_j)$

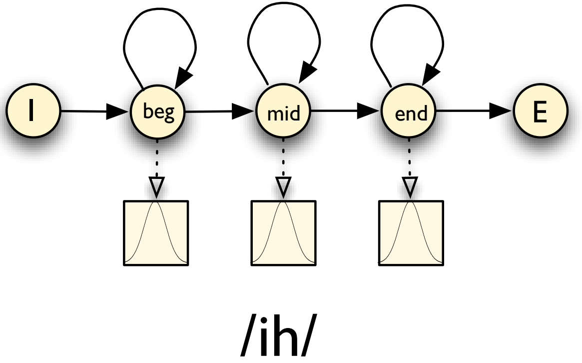
Whole word models



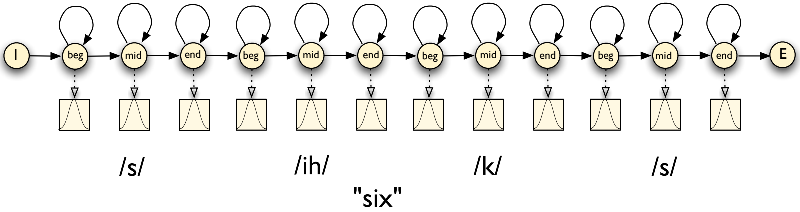
One state per phone models



Three-state phone models



Word model made of phone models



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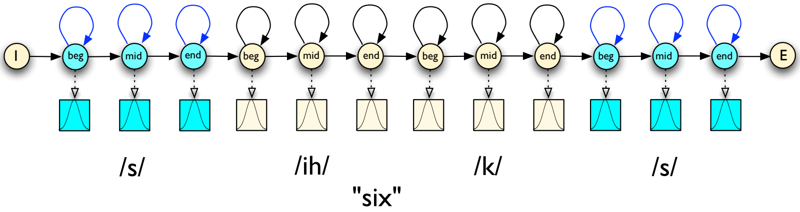
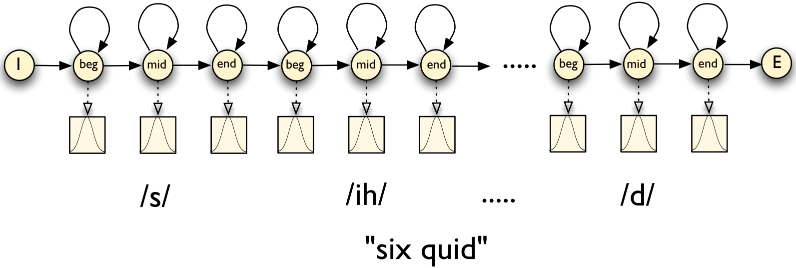
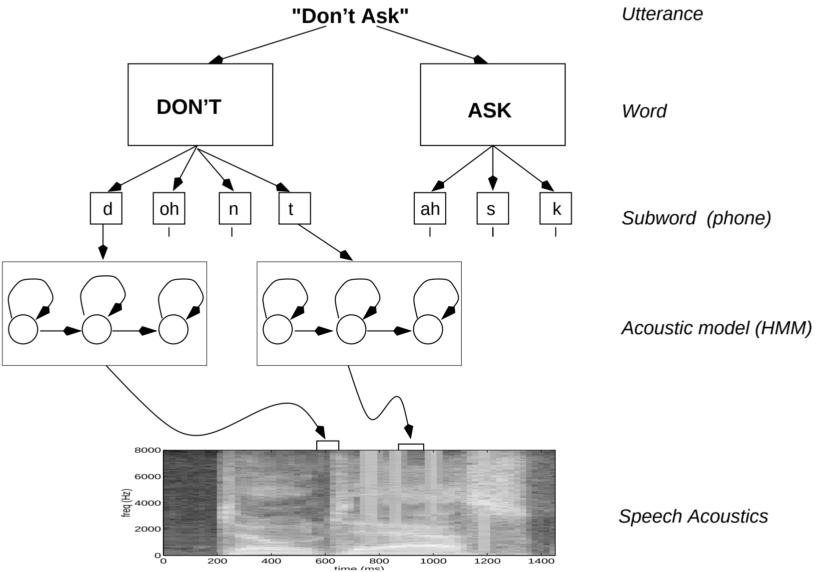


Word sequence models



Hierarchical Modelling in Speech Recognition



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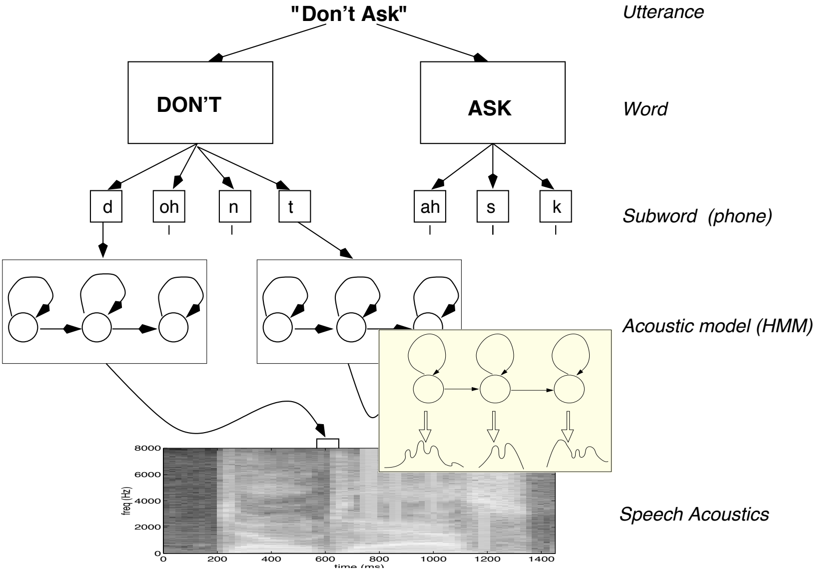

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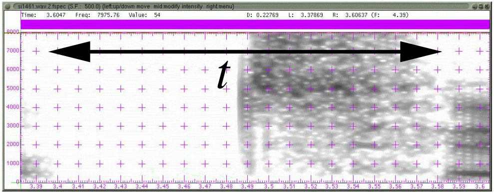
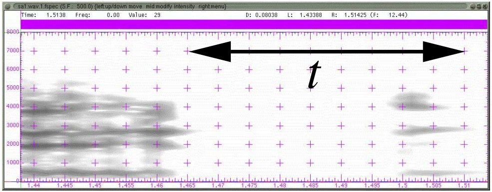
- Consider /n/ in ten and tenth
  - dental in ten
  - alveolar in tenth

Phonetic Context Example



"tube"



"suit"

Modelling Context

- Subword units Individual phone units need to deal with a lot of variability
  - Use longer units that incorporate context, eg: diphones, demisyllables, syllables
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- Pronunciations
  - "did you" `d ih jh y ah`
  - "around this" `ix r aw n ih s`

Divide and conquer

- Context-dependent models are more specific than context-independent models
- Increase the detail of modelling by extending the state space – but by defining multiple context dependent models, rather than more complex context independent models
- Divide and conquer: as more context-dependent models are defined, each one becomes responsible for a smaller region of the acoustic-phonetic space
- Let the data tell us how many contexts to model

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- Cross-word triphones "don't ask" is represented by:
  `sil sil-d+oh d-oh+n oh-n+t n-t+ah t-ah+s ah-s+k s-k+sil sil`
  Note that triphone context extends across words (eg unit `n-t+ah`)

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  - 39-dimension feature vectors (12 MFCCs + energy), deltas and accelerations
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- We would need a very large amount of training data to train such a system
  - to enable robust estimation of all parameters
  - to ensure that all possible triphones are observed (more than once) in the training data

Modelling infrequent triphones

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- All approaches are data driven

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- But training data is sparse (especially when cross-word triphones are used) so relatively few specific triphone models

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- Estimate the interpolation parameters α using deleted interpolation
- This enables more triphone models to be estimated, but adds robustness by sharing training data from other contexts (through the biphone and monophone models)

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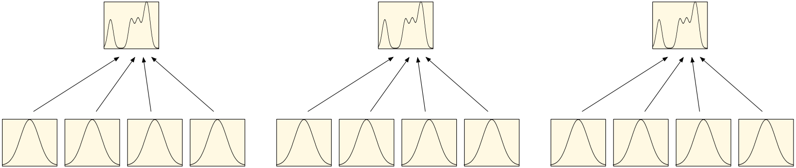
Tied Mixture Model



Gaussian mixture model

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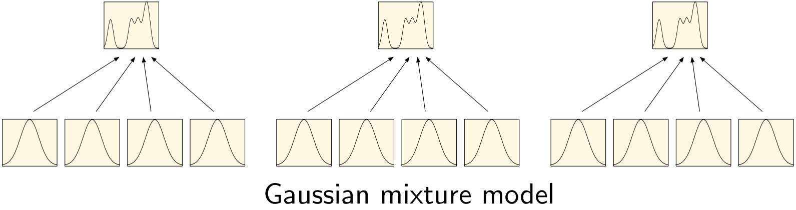

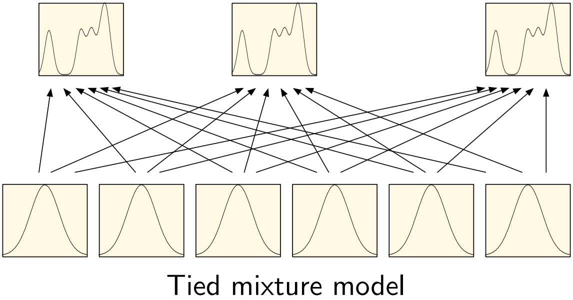

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- But we can do better (state clustering)!
- Tied mixtures are still used when time and memory efficiency is important (eg embedded systems)

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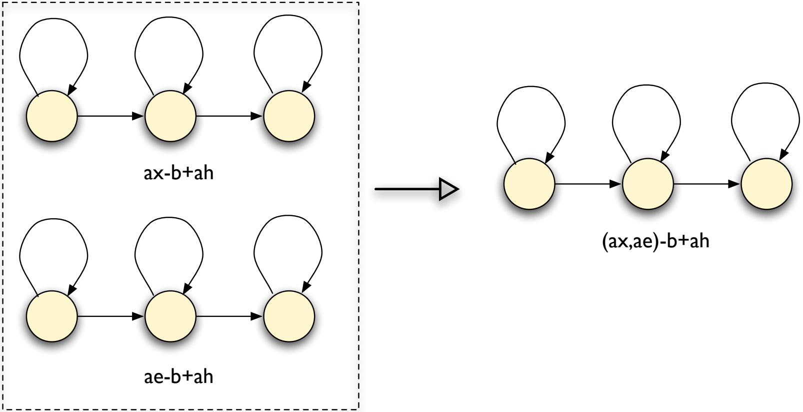
- The resultant merged models are referred to as generalized triphones

Example: Generalized Triphones



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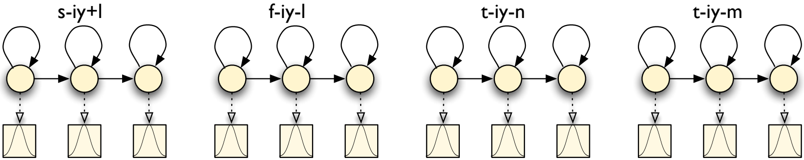
- More flexible than clustering whole models: left and right contexts may be clustered separately

Generalized triphones



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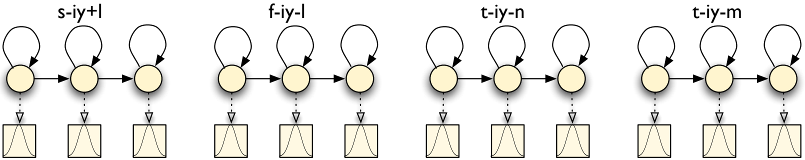

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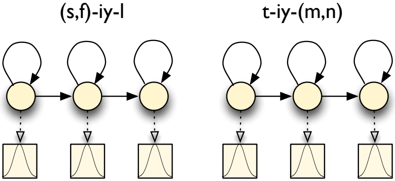

Generalized triphones (model sharing)

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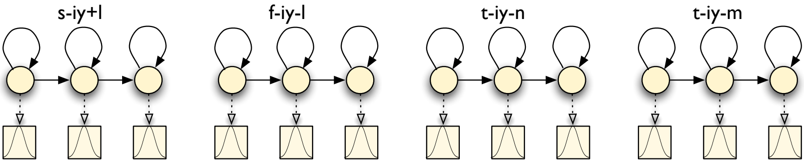

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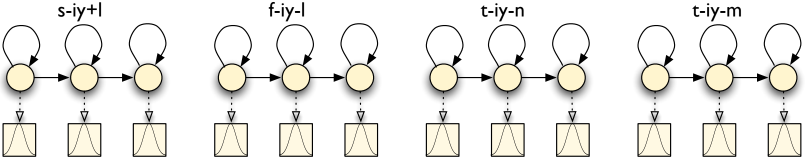

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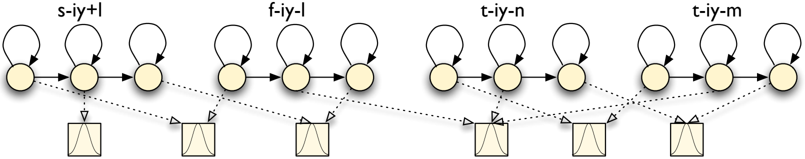

State-clustered triphones (state sharing)

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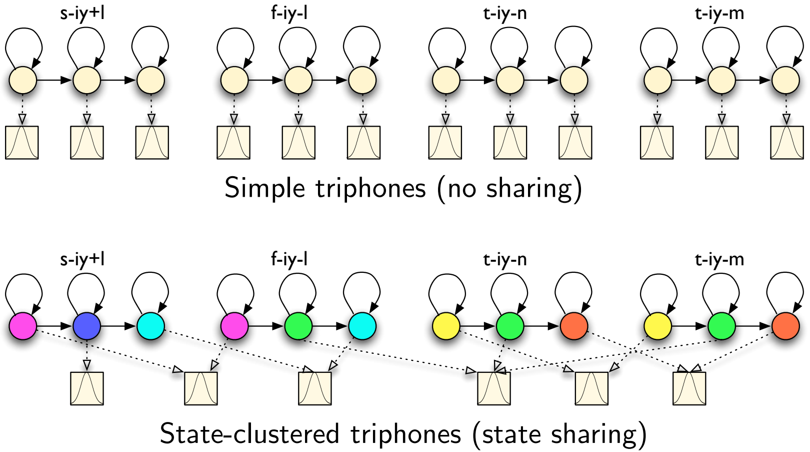

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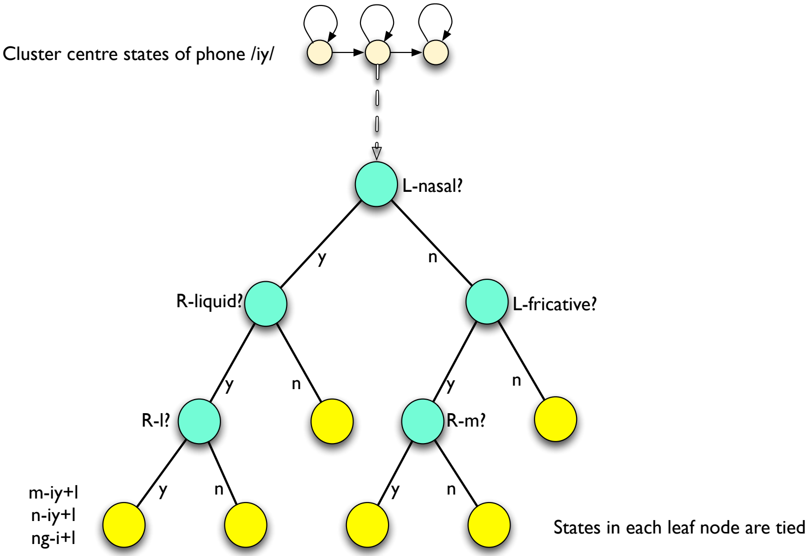
- Stop splitting if either: (a) the likelihood does not increase by more than a predefined threshold; or (b) the amount of data associated with a split node would below a threshold

Phonetic Decision Tree



Phonetic questions

- Ask questions of the form: does phone at offset s have feature f?
- Offsets are $+/-1$ for triphone context
- Example general questions:
  - Stop: `b d g p t k`
  - Nasal: `m n ng`
  - Fricative: `ch dh f jh s sh th v z zh`
  - Liquid: `l r w y`
  - Vowel: `aa ae ah ao aw ax axr ay eh er ...`
- Example consonant questions: Un/voiced, front/central/back, fortis (`ch f k p s sh t th`), lenis (`b d dh g jh v z zh`), voiced stop, ...
- Example vowel questions: front, central, back, long, short, diphthong, rounded, ...

Most useful phonetic questions

- All states of all models:
  `+Vowel -Vowel +Unrounded -UnFortisLenis +UnFortisLenis +r`
- Entry state of all models:
  `-UnFortisLenis -Vowel -Nasal -CentralFront -Unrounded -Fortis`
- Exit state of all consonants:
  `+Vowel +Unrounded +High +ee +Rounded +Syllabic`

(for Wall St Journal read speech – Young, Odell and Woodland 1994)

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- The log likelihood of the data associated with cluster $\mathbf{S}$ is:

$$L(\mathbf{S}) = \sum_{s \in \mathbf{S}} \sum_{\mathbf{x} \in \mathbf{X}} \log P(\mathbf{x} | \boldsymbol{\mu}_S, \boldsymbol{\Sigma}_S) \gamma_s(\mathbf{x})$$

Likelihood of a state cluster (2)

- Don't need to iterate through all data for each state
- If the output pdfs are Gaussian it can be shown that

$$L(\mathbf{S}) = -\frac{1}{2}\left(\log(2\pi)^d|\boldsymbol{\Sigma}_S)|) + d\right)\sum_{s\in\mathbf{S}}\sum_{\mathbf{x}\in\mathbf{X}}\gamma_s(\mathbf{x})$$

  where d is the dimension of the data
- Thus $L(\mathbf{S})$ depends on only
  - the pooled state variance $\boldsymbol{\Sigma}_S$ - can be computed from the means and variances of the individual states in the pool
  - and the state occupation probabilities already computed when forward-backward was carried out

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- The total likelihood of the partitioned data will increase by

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- State occupation count: sum of state occupation probabilities for a state over time

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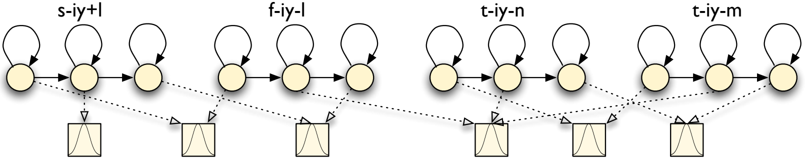
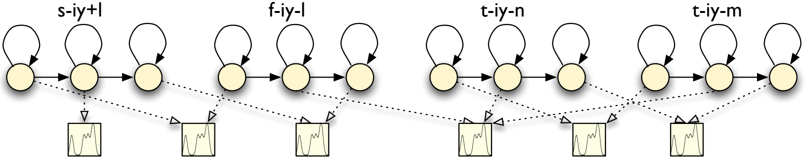
- The above methods for state clustering assume that the state outputs are Gaussians – this makes the computations much simpler
- BUT: Gaussian mixtures offer much better acoustic models than Gaussians
- Solution:
  - Perform state clustering using Gaussian distributions
  - Split the Gaussian distributions in the clustered states, by cloning and perturbing the means by a small fraction of the standard deviation, and retrain.
  - Repeat by splitting the dominant (highest state occupation count) mixture components in each state

"Mixing up"



State-clustered triphones (Gaussians)



State-clustered triphones (GMMs)

Summary: Context-dependent acoustic modelling

- Share parameters through state clustering
- Cluster states using phonetic decision trees for each state of parent phone
- Use Gaussian distributions when state clustering
- Then split Gaussians and retrain to obtain a GMM state clustered system

References: context-dependent acoustic modelling

- *c*1980: First proposed by Bahl et al (IBM)
- Schwartz et al (1985): first paper using triphone models
- Lee (1990): generalized triphones
- Bellegarda (1990), Huang (1992): tied mixture modelling
- Bahl et al (1991): phonetic decision trees first proposed
- Young and Woodland (1994): state clustering
- Young et al (1994): decision tree-based state clustering